Performa (2015) Vol. 14, No.1: 53-58

Maintenance Contract With Imperfect Preventive Maintenance

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Abstract

In this paper, we study performance based maintenance contracts for heavy equipment operated in a mining industry. A performance based service contract uses an attractive incentives to motivate the agent to increase the maintenance effort such that the resulting equipment's performance is well above the target. This will in turn give benefits for both the owner of the trucks and the Agent of service contract. In this paper, we study performance based maintenance service contracts for the equipment sold with warranty and the perfomances considered includes availability and safety measures. We find the optimal strategy (i.e. price of each contract and maintenance effort) for the agent and the optimal option for the owner using a non-cooperative game formulation.

Keywords: *imperfect preventive maintenance, maintenance contract, non-cooperative game formulation*

1. Introduction

We consider heavy equipment such as a crane and a loader used in a mining industry to support its business. The equipment deteriorates with usage and age and finally fails to operate as intended. If the equipment is down, no revenue is generated and hence a high availability of the equipment is needed for achieving the production target of a company. To acheive a high availability of the equipment, Preventive Maintenance (PM) actions are performed to reduce the likelihood of failure and down time. Corrective Maintenance (CM) actions are taken after failure, which restores the failed equipment to the operational state.

Most heavy equipments are sold with warranty and often the manufacturer of Original Equipment Manufacturer (OEM) offers the warranty and PM in one package in order to provide more assurance that the equipment will function as promised to the owner. After the warranty expires, the maintenance actions are born by the owner and can be done either in house or by independent agents or the OEM. For complex and expensive equipments used in a mining industry and also remote areas of mining fields, it is very expensive for the owner to have maintenance facility and high skill maintenance crew. Hence, it is not economical to do PM and CM in house after the warranty ends. As the OEM or an external agent normally offers a variety of maintenance service contracts, then the maintenance actions (PM and/or CM) can be outsourced to the OEM. From the owner's viewpoint, maintenance programs are aimed at not only to reach the performance target (e.g. 90% availability) but also to acheive an optimal profit. In order to reach the optimal profit, the maintenance service contract offered by the OEM should not just to ensure the performance target but also to get a higher performance which is beyond the target. This in turn will results in optimal profits for both the owner and the OEM. The decision problems for the owner are to select (i) the maintenance contract option that assures the performance target of the equipment with reasonable maintenance costs, and determine (ii) an actractive incentive.

Maintenance service contract has received much attention in the literature. Jackson and Pascual (2008), and Wang (2010) studied maintenance service contract for repairable items, which involve preventive maintenance policies. All maintenance service contracts studies consider a penalty cost based on down time for each failure. In most mining companies, the

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attractive maintenance service contract is the one which ensures a high availability of the equipment at reasonable cost.

In a conventional service contract with availability as a key measure, a penalty cost incurs when the actual availability falls below the target have studied by Iskandar.et.al (2014). But their result did not give any reward to the agent when the performance (actual availability) well above the target. As a result, this service contract does not give any incentive which motivates the agent to keep improving the performance. In contrast, a performance based service contract uses an attractive incentives to motivate the agent to increase the equipment's performance beyond the target Mirzahosseinian and Piplani (2011). In this paper, we study performance based maintenance service contracts with imperfect PM from both the manufacturer's and the owner's perspectives involving incentive for the agent. This paper is organized as follows. Section 2 gives model formulation and model analysis is presented in Section 3. In Sections 4 and 5, we present optimal soltion, and finally conclude with topics for further research.

2. Model Formulation

2.1 Notation

The following notation will be used in model formulation.

W	: Warranty period	K	: Revenue ($/hour) the owner
T	: Product age	C_m	:Repair cost done by OEM
$\tilde{A}$	: Availability target	C_s	:Repair cost owner for option O_1
		C_a	:Compensation cost by OEM
C_{im}	: original PM cost	C_0	:cost of an improved PM that achieves a 100% reduction in r(t)
ζ	: Total downtime target	C_{pm}	:PM cost per unit of time.
$Y(t)$	: Total downtime in $(0,t]$	C_p	: Penalty cost
$EP(t)$	: Expected penalty cost	C_b	: Price of the product.
$F(t)$	: Dist. function of downtime	$r(t,)$ $R(t)$	: Hazard function, Cum hazard fc.
$F^{[k]}(t)$	: The *k-fold Stieltjes convolution* of *F(t).*	α,β	: Scale and shape parameter
P_0	: PM cost over the contract period		

2.2 Warranty Policy and Maintenance Service Contract

A repairable product is sold with a non-renewing warranty with length W. The manufacturer rectifies all failures (with minimal repair) under warranty, and performs PM without any charge to the owner as the PM and warranty are in one package. After the warranty ceases, all CM and PM actions will be borne to the owner. The owner of the equipment wants to set a comprehensive maintenance program of the equipment over $[W, W+\tau]-$ which can give a maximum performance of the equipment. Several maintenance service contracts are offered by the OEM at the end of the warranty. Each maintenance contract offered is characterised by the maximum length for τ (e.g. 1 year). Three service contract options are considered as follows.

Option O_1 : After the expiry of warranty at W, the owner carries only PM in-house but CM is outsourced. In other words, when the equipment fails in Ω_S, the owner calls the OEM to fix the equipment. The OEM will charge the owner a fixed cost C_S for each repair (CM). No penalty cost incurs the OEM if the availability falls below the target as the OEM only performs CM.

Option O_2: For a fixed price of service contract P_{G2}, the OEM agrees to carry out PM and CM in [W, W+t). If the availability of the equipment over the contract A is less than the availability target $\tilde{A}$, then the OEM should pay a penalty cost. The amount of the penalty cost is proportional to $\delta = \tilde{A} - A$. The penalty cost, C_P is viewed as a penalty given by the OEM. Here, the OEM performs PM and CM over a given period. Here PM is full coverage. Under Option O_1, the OEM provides a service covering PM and CM to the owner with a fixed cost for a contract period –e.g. 1 to 5 years.

Option O_3 : For a fixed price of service contract P_{G3}, the OEM agrees to perform both PM and CM (full coverage) for a period of time, τ. The contract starts at the end of warranty, W. Here, the OEM assures a minimum availability target stated in the contract. As the maintenance service is full coverage (PM and CM), then a penalty cost incurs the OEM if the actual availability falls below the target. But if it falls above the target, the OEM will earn an incentive. Furthermore if δ<0 or availability is greater then the target $(A > \tilde{A})$, then the OEM earns some incentives. The incentives cost, C_I is viewed as an incentive gotten by the OEM.

To keep the equipment in good condition, PM is conducted at discrete time instant τ_1,.., τ_j,…and τ_1=0, Kim *et. al* (2004). The effect of PM is that it results in a rejuvenation of the item so that it effectively reduces the age of the item. The reduction in the age depends on the maintenance effort (m) used. The maintenance effort is constrained so that $0 \le m \le M$. The value of $m = 0$ corresponds to no PM and M is the upper limit of maintenance effort. Following Kijima *et.al* (1998), with $m = M$ at every PM action, then the item is restored back to as good as new after each PM action. If $m = 0$, then $v_j = \tau_j, j \ge 1$. The item's virtual age at time t is given by $v_j = v_{j-1} + t - \tau_{j-1},\ \tau_{j-1} \le t < \tau_j$, j=1,2,... Since failure are repaired minimally and repair times are negligible, the rate of occurance (ROCOF) of failures is given by $r[v(x)] = r[v_{j-1}^k + x - \tau_{j-1}], \tau_{j-1} \le x < \tau_j$, j=1,2,...The plot of $r(t)$ for the three options as in Fig.1.

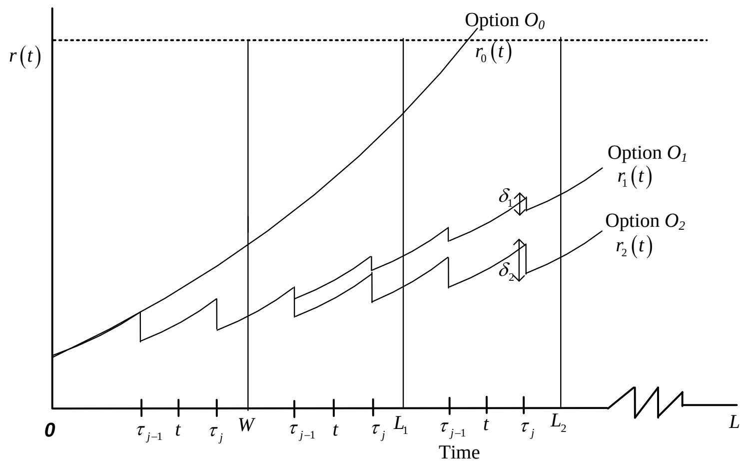

Fig.1. ROCOF for options O_0, O_1, O_2, O_3

2.3. Preventive Maintenance and Repair Options

PM effort differs for each option and hence the effect of PM for the three options is given by δ_1(m)> δ_2(m)> δ_3(m) with $\delta_i(m) = (1+m)^{-\rho_i m}$, $i = 1, 2$, and $\rho_1 > \rho_2 > \rho_3$. Let n_1, n_2, n_3 denote the number of PM actions under options 1, 2 and 3 respectively (See Table 1).

Table 1. Number of PM Actions

Number	interval	Value
n_1	$[0, W)$	$\lfloor W/\Delta \rfloor$
n_2	$[0, W+\tau)$	$\lfloor (W+\tau)/\Delta \rfloor$
n_3	$[W, W+\tau)$	$\lfloor \tau/\Delta \rfloor$

Option O_1 : periodic PM [0,*W*) and in house PM over [*W*, *L*) by the owner,

$$r_m^1(t)=\begin{cases} r_0(t), & \text{for } 0\le t<\tau_1,\\ r_0(v_j^2+t-\tau_j), & \text{for } \tau_j\le t<\tau_{j+1},\ j=1,\dots,(n_1-1)\\ & \text{with}, \tau_j=j\Delta,\ j=1,\dots,n_1\\ r_0(v_{n_1}^2+t-\tau_{n_1}), & \text{for } \tau_{n_1}\le t<w,\\ [r_m(\tau_j)-r_0(\tau_j)]+r_m(v_j^1+t-\tau_j), & \text{for } \tau_j\le t<\tau_{j+1},\ j=1,\dots,(n_3-1)\\ & \text{with}, \tau_j=w+j\Delta,\ j=1,\dots,n_2\\ r_0(v_{n_3}^1+t-\tau_{n_3}), & \text{for } \tau_{n_3}\le t<W+\tau \end{cases} \quad 1)$$

Option O_2 : periodic PM [0, $W+\tau$) with $v_j^2=v_{j-1}^2+\delta_2(m)(\tau_j-\tau_{j-1})$ by the OEM,

$$r_m^2(t)=\begin{cases} r_0(t), & \text{for } 0\le t<\tau_1,\\ r_0(v_j^2+t-\tau_j), & \text{for } \tau_j\le t<\tau_{j+1},\ j=1,\dots,(n_2-1)\\ r_0(v_{n_2}^2+t-\tau_{n_2}), & \text{for } \tau_{n_2}\le t<W+\tau, \text{with}, \tau_j=j\Delta,\ j=1,\dots,n_2 \end{cases} \quad 2)$$

Option O_3 : periodic PM [0, $W+\tau$) with $v_j^3=v_{j-1}^3+\delta_3(m)(\tau_j-\tau_{j-1})$ by the OEM,

$$r_m^3(t)=\begin{cases} r_0(t), & \text{for } 0\le t<\tau_1,\\ r_0(v_j^3+t-\tau_j), & \text{for } \tau_j\le t<\tau_{j+1},\ j=1,\dots,(n_2-1)\\ r_0(v_{n_2}^3+t-\tau_{n_2}), & \text{for } \tau_{n_2}\le t<L, \text{with}\,\tau_j=j\Delta,\ j=1,\dots,n_2 \end{cases} \quad (3)$$

3. Model Analysis

We consider the case where OEM and owner will negotiate the pricing of service contract and the cost of repair through a non-cooperative game theory.

3.1 Owner's Decision Problem

Option O_1 **:** the expected profit is given by

$$E[\phi(O_1;C_s)]=K\{\tau-E[D_1(W,W+\tau)]\}-C_sN_1(W,W+\tau)-P_0-C_b \quad (4)$$

Where, $N_1(W,W+\tau)=\int_W^{\tau_{n_1+1}} r_0(v_{n_1}^2 t-\tau_{n_1})dx+\sum_{j=1}^{n_3-1}\int_{\tau_j}^{\tau_{j+1}}[r_m^1(\tau_j)-r_0(\tau_j)+r_0(v_j^1+t-\tau_j)]dt$

$$+\int_{\tau_{n_3}}^{W+\tau}[r_m^1(\tau_j)-r_0(\tau_j)+r_0(v_{n_3}^1+t-\tau_{n_3})]dt$$

And $\tau_j=W+j\Delta, j=1,2,\cdots n_3$ and n_3 is the largest integer less than τ/Δ . N_1 *(W, W+τ)* is expected number of failures and $r_0(t)$ is given in (4). In many cases, failure of heavy equipment can cause accident. We consider that the probability of failure induces accident is *p*, and does not (1-*p*). The value of p may be dependent on the land contour where $p_1<p_2<p_3$, i=1,2,3 are referred to light incline , high incline and very hilly, respectively.

There are three types of downtimes namely, downtime 1 after a failure, downtime 2 after an accident caused by a failure, and downtime 3 after a preventive maintenance. It is assumed that downtime 3 is relatively small, and it can be ignored. The downtime 2 of the truck is the sum of repair time and lost time due accident whilst downtime 1 is only a repair time. It is assumed that down times 1 and 2 are distributed exponentially with parameter λ_1, λ_2, respectively. We consider that $\lambda_1<\lambda_2$. As a result,

$$E[D_1(W,W+\tau)]=N_1(W,W+\tau)[(1-p)1/\lambda_1+p1/\lambda_2] \quad (5)$$

Option O_2 : the expected profit of the owner is

$$E[\phi(O_2;P_{G2})]=K\{\tau-E[D_2(W,W+\tau)]\}+EP(\tau)-P_{G2}-C_b \quad (6)$$

Where $EP(\tau)$ is the expected penalty received by the owner.

Option O_3 : the expected profit of the owner is

$$E[\phi(O_3;P_{G3})]=K\{\tau-E[D_3(W,W+\tau)]\}+EP(\tau)-E[\text{Incentive cost}]-P_{G3}-C_b \quad (7)$$

3.2 OEM's Decision Problem

Option O_1 **:** here with on call repair option, two costs incurs OEM i.e. repair cost and accident cost due to accident induced by failure. The expected profit of OEM is given by

$$E\left[\pi(O_1;C_s)\right]=\left(C_s-(C_m+C_a p)\right)N_1(W,W+\tau)+C_b \quad (8)$$

Option O_2 : when failure does induce an accident, then the OEM incurs two costs – repair cost and penalty cost paid to the owner for each accident. As a result, the revenues received by the OEM consists of price of the contract and incentive earned when availability above the target, penalty and repair cost and PM cost. Hence, the expected profit is given by $E\left[\pi(O_2;P_{G2})\right]= P_{G2}+C_b-E[\text{ Pen. cost}]-E[\text{ PM cost}]-E[\text{ Rep. \& Comp.cost}]$.

Expected of Penalty Cost: as in Iskandar et.al (2014), $EP(\tau)=C_p\int_{\zeta}^{\infty}(y-\zeta)g(y)dy$. Expected of repair and compensation cost: $EC_2(\tau)=(C_m+C_a p)\,\mathrm{N}_2(W,W+\tau)$

N(t) follows a NHPP with intensity function $p.r(t)$. $N_2(W,W+\tau)$ is defined as

$$=\int_{W}^{\tau_{n_1+1}} r_0(v_{n_1}^2+t-\tau_{n_1})dt+\sum_{j=n_1+1}^{n_2-1}\int_{\tau_j}^{\tau_{j+1}} r_0(v_j^2+t-\tau_j)dt+\int_{\tau_{n_2}}^{W+\tau} r_0(v_{n_2}^2+t-\tau_{n_2})dt$$

Expected of PM cost: $EC_{pm}=(C_{im}+D\delta^p)\mathrm{n}_3$ with $0\le\delta\le1$, p>1 and $D=C_0-C_m$, as a result, the total expected revenue of the OEM is

$$E[\pi(O_2)]=P_{G2}-EP(\tau)-\mathrm{EC}_2(\tau)-EC_{pm}+C_b \quad (9)$$

Option O_3 : the revenues received by the OEM consists of price of the incentive earned when availability above the target. Hence, the expected profit is given by

$$E\left[\pi(O_3;P_{G3})\right]=P_{G3}+C_b+E[\text{Incentive earned}]$$
$$-E[\text{Penalty cost}]-E[\text{Repair and Compensation cost}]-E[\text{PM cost}].$$

Expected Incentive Cost: The OEM earns some incentives when the availability is greater than the target. Let δ_i denote the excess availability from the target. It is assumed that the incentive earned is linear function of δ_i , hence the expected of incentive earned in $(W,W+\tau)$ is given by $EI(\tau)=C_I\int_0^{\zeta}G(y)dy$.

As a result, the total expected revenue of the OEM is

$$E[\pi(O_3)]=P_{G3}-EP(\tau)+EI(\tau)-\mathrm{EC}_3(\tau)-EC_{pm}+C_b \quad (10)$$

Where $EC_3(\tau)=(C_m+C_a p)\,\mathrm{N}_3(W,W+\tau)$ and the expected number of failures $N_3(W,W+\tau)$ is defined as $N_3(W,W+\tau)=\int_{W}^{\tau_{n_1+1}} r_0(v_{n_1}+t-\tau_{n_1})dx$

$$+\sum_{j=1}^{n_3-1}\int_{\tau_j}^{\tau_{j+1}}[r_0(\tau_j)-r_m^3(\tau_j)+r_0(v_j^3+t-\tau_j)]dt+\int_{\tau_{n_3}}^{W+\tau}[r_0(\tau_j)-r_m^3(\tau_j)+r_0(v_{n_3}^3+t-\tau_{n_3})]dt\ .$$

4. Optimal Solution

Situation modeled is for the case of heavy equipment manufactured by a monopolist where neither party (owner or OEM) is more powerfull. As a result, both parties will negotiate and determine jointly the terms and condition of the service contract. To find the optimal solution we use Nash solution of the bargaining game of alternating offers Osborne and Rubinstein (1994).

Lemma 1. In the presence of bargaining between two parties with the probability of negotiations break down at the end of period is sufficiently small, for every preference agreements, the owner and the OEM will receive the same expected profit, $E[\pi(O_i)]=E[\omega(O_i)];\mathrm{i}=1,2,3.$ □

Proof: Following the sub game perfect equilibrium approach by solving the following equation,

$\max_{C_s} E[\pi(O_1)]E[\omega(O_1)] = \max_{C_s}(C_S N_1(W, W+\tau) + Q_\pi)(-C_S N_1(W, L) + Q_\omega)$, then we get C_s and hence $E[\omega(O_1)] = E[\pi(O_1)]$. Using the same argument as in Option 1, this completes the proof that $E[\pi(O_i)] = E[\omega(O_i)]; i = 1,2,3.$ Based on lemma 1 we obtain the following results.

Theorem2.There exist C_S^*, P_{G2}^* and P_{G3}^* such that $E[\omega(O_2)] = E[\pi(O_2)]$ given by

$$C_S^* = 1/2N_1(W,L)\left[K\{\tau - E[D_1(W,W+\tau)]\} + (C_m + C_a p)N_1(W,W+\tau) - P_0 - 2C_b\right] \quad (11)$$

$$P_{G2}^* = 1/2\left[K\{\tau - E[D_{2y}(W,W+\tau)]\} + 2EP(\tau) + EC(\tau) + C_{pm}n_3 - 2C_b\right] \quad (12)$$

(20)

$$P_{G3}^* = 1/2\left[K\{\tau - E[D_{3y}(W,W+\tau)]\} + 2EP(\tau) + EC(\tau) + C_{pm}n_3 - 2EI(\tau) - 2C_b\right] \quad (13)$$

Proof. It is clear from Lemma1. □

Theorem 3. The expected profit of the owner/OEM on option O_1, O_2 and O_3 become,

$$E[\pi(O_1)] = 1/2\left[K\{\tau - E[D_1(W,W+\tau)]\} - P_0 - (C_m + C_a p)N_1(W,W+\tau)\right] \quad (14)$$

$$E[\pi(O_2)] = 1/2\left[K\{\tau - E[D_2(W,W+\tau)]\} - (C_m + C_a p)N_2(W,W+\tau) - C_{pm}n_3\right] \quad (15)$$

$$E[\pi(O_3)] = 1/2\left[K\{\tau - E[D_3(W,W+\tau)]\} - (C_m + C_a p)N_3(W,W+\tau) - C_{pm}n_3\right] \quad (16).$$

Furthermore, all satisfying $E[\pi^*(O_1)] < E[\pi^*(O_2)] < E[\pi^*(O_3)]$

Proof: It is clear from Lemma1. □

5. Conclusions

We have studied performance based maintenance service contracts where an attractive incentive is given when the performance is greater then the target. This in turn will motivate the agent to increase the maintenance effort. One can extend the paper to the case of the two dimensional service contract where the contract is characterised by two limits – i.e. age and usage limits. The other interesting topic is to consider a situation where the agent has more bargaining power then the owner. These topics are currently under investigation.

Acknowledgments

This work is funded partly by Hibah Bersaing 2014 to the first author and Hibah Desentralisasi 2014 to the second author from DGHE Indonesia.

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