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Algebra Learning Processes Using Scientific Approach and Direct Instruction

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Abstract: Algebra is one of the mathematical contents which has an essential role in the mathematics field. Nevertheless, most students still find some problems and difficulties in algebra lessons. This study describes the process of algebra learning using the framework of scientific approach and direct instruction. An observational qualitative study involved 71 students and three mathematics teachers from three different public or private senior high schools in West Java, Indonesia. The field notes were used as the instruments. The observational data were analyzed using the framework of the syntaxes of the scientific approach and directed instruction model. Results showed that performing the scientific approach and directed instruction as a learning approach and model in the process of mathematics learning, specifically in algebra learning, should fully pass away all the phases of the learning approach and model. Consequently, by learning algebra contents, such as arithmetic and geometric sequence, matrix determinant, and compound interest, students can optimize some cognitive domains in mathematics, such as communication and conceptual understanding. Particularly, the optimization of questioning activities as the second phase of the scientific approach and demonstrating knowledge and skill as the second phase of the directed instruction model is essential to cultivate and enhance students' critical thinking in mathematics. The implications to mathematics education are discussed comprehensively.

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INTRODUCTION

The fast development of technology and science in the 21st century requires individuals to adapt to emerging challenges, such as problems and difficulties. Consequently, individuals have to hold the 21st century skills to deal with many appearing problems and challenges. Some literature mentioned that the 21st century skills consist of problem-solving, critical thinking, creativity, communication, collaboration, and innovation (e.g., Sanabria & Arámburo-Lizárraga, 2017; Silber-Varod et al., 2019; Voogt & Roblin, 2012). Moreover, Nazarian et al. (2023) stated that 21st-century skills cover cognitive, intrapersonal, and interpersonal domains. Particularly, the interpersonal domain consists of collaboration and teamwork, leadership, intercultural and social skills, and accountability, whereas the intrapersonal domain consists of self-regulation, adaptation and flexibility, emotions, and intellectual openness. Additionally, a few of literature also mentioned that technological literacy is a skill that individuals must master in the 21st century (e.g., Hasse, 2017; Skophammer & Reed, 2014). All 21st-century skills can be generated and developed by various learning environments, including mathematics learning.

The process of mathematics learning can facilitate the enhancement of these skills in the perspective of the interpersonal domain. In the literature, Olanoff et al. (2014) stated that algebra is one of the mathematical contents that has an essential role in the mathematics field. National Council of Teachers of Mathematics (NCTM) (2000) explained that algebra has its historical roots in the study of general methods for solving equations. Moreover, it emphasizes relationships among quantities containing functions, ways of representing mathematical relationships, and the analysis of change. Nevertheless, some empirical studies revealed that students' algebra achievement has not been optimal (e.g., Das & Ali, 2023; Koza Çiftçi & Yıldız, 2019; Lee & Mao, 2021; Leite et al., 2022; Ojo, 2022). Moreover,

a few of empirical studies showed that some students still have low achievement in algebra learning (e.g., Dougherty et al., 2015; Ezeugwu et al., 2016). This indicates that they find some problems and difficulties in algebra lesson.

Many learning approaches, such as realistic mathematics education, science, technology, engineering, and mathematics (STEM), constructivism, open-ended, inductive-deductive, problem-solving, and scientific, have been applied in mathematics education (e.g., Ampadu & Danso, 2018; Chau Nguyen & Hai Pham, 2023; Fatah et al., 2016; Goos et al., 2023; Ilyas et al., 2022; Melaibari & Ismail, 2023; Siswono et al., 2020; Thompson, 2020). Moreover, a lot of learning models, such as discovery learning, problem-based learning, project-based learning, cooperative learning, contextual teaching learning, inquiry learning, and direct instruction have also been implemented in mathematics learning, especially in algebra lesson (e.g., Alenezi, 2023; Asomah et al., 2023; In'am & Hajar, 2017; Rakes et al., 2023; Wang et al., 2022; Yeung & Ng, 2023). However, in Indonesia, the implementation of the mathematics learning approach and model is mainly dominated by a scientific approach combined with direct instruction.

The scientific approach refers to a student-centered learning approach using scientific principles that contain a series of collecting data activities, such as observation, questions, experiments, data analysis, and communication (Krogsgaard et al., 2011). Actually, this approach has been applied in the American educational field at the end of the 19th century in which it points out formalistic laboratory and directed to scientific facts (Hubka & Eder, 1987; Reif, 1981). In a literature, Nenotaek et al. (2019) stated that it has some purposes, such as enhancing students' thinking skills, generating the ability to solve problems systematically, creating a learning environment that is suitable to students' needs, training students to state ideas, enhancing students' academic achievement, and developing students' character. Additionally, Nenotaek et al. (2019) explained that there are five activities in the scientific approach such as observing, questioning, collecting data, associating, and communicating. These activities, which are then understood as hierarchical steps, are used as an analysis tool to observe and analyze the process of algebra learning and teaching in the topic of arithmetic and geometric sequence, matrix determinant, and compound interest.

Directed instruction refers to a learning model using teachers' demonstration and explanation embedded with students' practice and feedback to promote them in getting the knowledge and skill required for the next learning (Stockard et al., 2018). It can be understood as a learning model that can generate students to learn and hold basic skills and gradually get the information (Ewing, 2011). In a literature, McMullen and Madelaine (2014) argued that there are some characteristics of directed instruction, such as: (1) there is a model effect on students including in learning purpose and assessment; (2) it has a syntax based on overall learning trajectory; and (3) management system and learning environment are adjusted to promote the success of learning activities. Additionally, Eppley and Dudley-Marling (2019) mentioned that there are five main phases in directed instruction: (1) presenting learning purposes and preparing students to learn, (2) demonstrating knowledge and skill, (3) guiding students in doing the practice, (4) checking students' understanding and giving the feedback, and (5) providing the opportunity for students to do advanced practice and apply it. These steps are used as an analysis tool to observe and analyze the process of algebra learning and teaching on the topic of arithmetic and geometric sequence, matrix determinant, and compound interest.

In recent two decades, a lot of observational research related to the mathematics learning process has been studied widely (e.g., Ayuwanti et al., 2021; Bostic et al., 2021; McKenna et al., 2015; Warwick et al., 2016). More particular, many observational studies regarding the process of algebra learning and teaching have been conducted in various countries in the world (e.g., Darling-aduana & Shero, 2023; Drijvers, 2004; Drijvers et al., 2012; Harel, 2017; Litke, 2020b, 2020a; Moss & Lamberg, 2019; Pierce & Stacey, 2001; Veith et al., 2022; Wilkie, 2016; Wilkie & Clarke, 2015; Yerushalmy, 2000). More specifically in Indonesian context, some observational studies related to the process of algebra instruction have been carried out for a few of certain topics, such as absolute value equations (e.g., Jupri & Gozali, 2021), linear program-minimum and maximum problems (e.g., Jupri et al., 2021), quadratic equations (e.g., Mastuti & Prayitno, 2023), and functions (e.g., Lestariani et al., 2018; Widodo et al., 2020). However, This current study observes some algebra instruction processes in a few topics, such as arithmetic and geometric sequence, matrix determinant, and compound interest in three different senior high schools. Hence, this

study describes the process of algebra learning and teaching and analyzes it using a scientific approach and directed instruction considering teachers' TPACK and teaching methods. This study is expected to provide descriptions regarding how the algebra learning process is implemented in the mathematics classroom and make some suggestions for algebra instruction in the future.

METHOD

Research Design and Approach

To investigate the process of algebra learning and teaching for some topics, such as arithmetic and geometric sequence, matrix determinant, and compound interest in mathematics classrooms, an observational qualitative study was carried out. Algebra was selected as one of the mathematical contents in this observation because some empirical studies found that students' algebra achievement had not been optimal (e.g., Das & Ali, 2023; Lee & Mao, 2021; Leite et al., 2022). Moreover, a few of empirical studies showed that some students still had low achievement in algebra learning (e.g., Dougherty et al., 2015; Ezeugwu et al., 2016). Consequently, students' difficulties in algebra learning had to be solved, and they found its solutions by observing their process comprehensively. Additionally, the learning topics in algebra content were selected in that it was adjusted to teachers' schedule in teaching and the regulation of school institutions to permit us in observing the learning activities in mathematics classrooms.

Participant

The observation of this current study involved three different public or private senior high schools in West Java, Indonesia. Moreover, this study involved 71 students consisting of 27 tenth-grade students (18 males and 9 females) from public vocational school, 34 eleventh-grade students (13 males and 21 females) from public senior high school, and 10 eleventh-grade students (3 males and 7 females) from private senior high school. Additionally, this study also involved three different female mathematics teachers that were Mrs. T.H who had ten years of mathematics teaching experience and she taught the topic of arithmetic and geometric sequence, followed by Mrs. W.S who had eight years of mathematics teaching experience and she taught the topic of matrix determinant, and Mrs. A.D who had six years of mathematics teaching experiences and she taught the topic of compound interest. Students and teachers involved in this study were selected by using purposive sampling with the reasons such as teachers' schedule in teaching and the regulation of school institutions.

Instrument

This study used field notes focusing on the steps of algebra learning and teaching for case topics of arithmetic and geometric sequence, matrix determinant, and compound interest. In detail, the field notes contained three parts of the algebra learning process: introduction, core, and closing. In particular, the field notes focused on the scientific approach in the core activities and direct instruction from introduction activities to closing activities. The framework in field notes had been discussed and validated by one of the experts conducting the observational study regarding the process of mathematics learning and teaching, who was a mathematics lecturer in the Department of Mathematics Education at the University of Indonesia.

Procedure

To do this observational study, firstly, the author designed the framework for the field notes. Secondly, the author requested permission from the leader of the target schools by using a formal cover letter from our institution. Thirdly, the author observed the process of algebra learning and teaching in three different schools and schedules in which the author sat down in the back of the class, made some notes regarding the steps of learning activities, and took a minimum of pictures. Fourthly, the author qualitatively analyzed the observational data.

Data Analysis

The observational qualitative data were analyzed using the framework of the steps of the scientific approach mentioned by Nenotaek et al. (2019) and directed instruction mentioned by Eppley and Dudley-Marling (2019). Previously the data were firstly explained describing the process of algebra learning and

teaching for each of case topic of arithmetic and geometric sequence, matrix determinant, and compound interest. Generally, there were three main parts of the learning process such as introduction, core, and close.

RESULT AND DISCUSSION

The Process of Algebra Learning: In Case of Arithmetic and Geometric Sequence

The teaching of algebra learning in the topic of arithmetic and geometric sequences was carried out by Mrs. T.H and followed by 27 tenth-grade vocational school students in one of public vocational schools in Tasikmalaya, a regency in West Java Province, Indonesia. They consisted of 18 male students and 9 female students. Additionally, the teacher has had about ten years of teaching mathematics lessons in the vocational school since 2014. The implementation of algebra learning in this topic was conducted during two hours of lessons (80 minutes) which started at 8.20 AM to 9.40 AM on November 2nd, 2023 (See Figure 1).

Figure 1. The Implementation of Algebra Learning in a Public Vocational School

Generally, the process of algebra learning in the topic of arithmetic and geometric sequences was split into three main parts: introduction, core, and close. The process started with the teacher preparing students to learn algebra topic, particularly arithmetic and geometric sequences whereby she instructed them to make ready learning materials, such as mathematics book and note book. Then, she conducted learning apperception by recalling number patterns in which she in detail wrote some sequences on the whiteboard and asked students related to what are the next three numbers. (See Figure 2)

2 3 5 7 11 13 17 19 … … …	(1st Sequence)
1 3 4 7 11 18 29 47 … … …	(2nd Sequence)
3 8 13 18 23 28 33 … … …	(3rd Sequence)
2 6 18 54 162 486 … … …	(4th Sequence)

Figure 2. Some Sequences in the White Board

Most students could complete the 1st, 2nd, and 3rd sequences, but only a few could complete the 4th sequence. This shows that students have been able to complete arithmetic sequence but have not known that it is called as arithmetic sequence. Moreover, they can't complete geometric sequence and know that it is called as geometric sequence.

In the core part, the teacher recalled the 3rd and 4th sequences and said those to students as arithmetic sequence (3rd sequence) and geometric sequence (4th sequence). Moreover, she asked students the following questions: "What number is in the 100th position of the 3rd and 4th sequence? How to determine it simply?" There were no students who could answer these questions. Spontaneously, she stated the formula to determine the n^{th} term of arithmetic and geometric sequence by writing those on the whiteboard (See Figure 3).

nth Term of Arithmetic Sequence

$$U_n = U_1 + (n-1) \times b$$

nth Term of Geometric Sequence

$$U_n = U_1 \times r^{n-1}$$

Figure 3. The Formula of nth Term of Arithmetic and Geometric Sequence

Moreover, she explained that U_1 is the first term both in an arithmetic sequence and in a geometric sequence, and it is usually symbolized as a. In addition, she also explained that the variable b in an arithmetic sequence is the difference among two consecutive terms or it means that $b = U_n - U_{n-1}$, whereas the variable r in the geometric sequence is the ratio among two consecutive terms or it means that $r = \frac{U_n}{U_{n-1}}$. After those, the teacher demonstrated the way to solve the previously given questions to students using these formulas, and students tried to understand those by writing it in their note book.

Furthermore, the teacher gave two problems to students, consisting of one problem about arithmetic sequence and one problem about geometric sequence (See Figure 4). Then, she instructed students to make groups, with each group consisting of 5 to 6 members, so there were three groups containing five students, followed by two groups containing six students. In the discussion process, besides using mathematics books, she also allowed students to utilize smartphones to promote them to solve problems. In every group, students jointly discussed how to solve the problems, and the teacher walked around each group to guide them if they were stuck or found some difficulties. After twenty-five minutes of going on, each of group had solved the problems.

1. U_7 and U_{13} of an arithmetic sequence are consecutively 45 and 87. Find the sum of U_4 and U_{16}.
2. U_3 and U_7 of a geometric sequence are consecutively 12 and 192. Find the U_{10}.

Figure 4. Sequence Problems Given to Students

Subsequently, the teacher instructed and provided the opportunity for one group to explain the solution of the arithmetic sequence problem and one group to explain the solution of the geometric sequence problem. From the first group explaining how to solve arithmetic sequence problem, it could be seen that they broke down the given information and adjusted it to the formula of arithmetic sequence. Consequently, they presented a + 6b = 45 and a + 12b = 87, and then using elimination method in the linear equation of two variables to find the value of a and b in which they found that the value of a was 3 and the value of b was 7. Students then, used those values to find U_4 and U_{16} whereby the value of U_4 was 24 and the value of U_{16} was 108, as a consequence, the value of U_4 + U_{16} was 132. The teacher also made sure that the process and final answer of this arithmetic sequence problem were exactly true. Regarding the arithmetic sequence problem, there were no other students who asked to the first group.

Moreover, from the second group explaining how to solve geometric sequence problem, it could be seen that they decomposed the given information and adjusted it to the formula of geometric sequence. As a consequence, they presented $ar^2 = 12$ and $ar^6 = 192$, and then breaking down $ar^6 = 192$ to be $ar^2 \times r^4 = 192$, so they found that the value of a was 3 and the value of r was 2. Finally, they found that the value of U_{10} was 1,536. Regarding the geometric sequence problem, there was a student from another group who asked to this problem as follows: "why $ar^6 = 192$ can be represented as $r^4 = 16$?" One student of the second group explained that since $ar^2 = 12$ and $ar^2 \times r^4 = 192$, consequently, $12 \times r^4 = 192$ and even more, $r^4 = 16$. The teacher also justified that the algorithm and final answers presented by the second group were exactly true.

Moreover, in the closing part, the teacher made some conclusions about arithmetic and geometric sequence. Particularly, she reinforced related to the concepts and the procedure to find the value of U_n for both arithmetic sequence and geometric sequence. She also said that there was homework regarding

this topic to be carried out by students as the task. It was delivered by her to students through the WhatsApp Group in which students could read and check the task containing some problems regarding the arithmetic and geometric sequence.

The Process of Algebra Learning: In the Case of Matrix Determinant

The teaching of algebra learning in the topic of matrix determinant was carried out by Mrs. W.S and followed by 34 eleventh-grade senior high school students in one of the public senior high schools in Bandung, a city in West Java Province, Indonesia. They consisted of 13 male students and 21 female students. Additionally, the teacher has had about eight years of teaching mathematics lessons in the senior high school since 2016 to date. The implementation of algebra learning in this topic was conducted during two hours of lessons (80 minutes) which started at 7 AM to 8.20 AM on October 24th, 2023 (See Figure 5).

Figure 5. The Implementation of Algebra Learning in a Public Senior High School

Mostly, the process of algebra learning on the topic of matrix determinant was split into three main parts, such as introduction, core, and close. The process started with the teacher asking students regarding their news, such as health and weekend holidays. She, then, prepared students to learn the algebra topic, particularly matrix determinant whereby she instructed them to make ready learning materials, such as mathematics book and note book. Then, she conducted learning apperception by recalling students' knowledge regarding matrix operation and matrix transpose which she, in particular, asked students as follows: "Could you explain the concept of matrix transpose, especially for the matrix with 2 × 2 ordo and 3 × 3 ordo?". One of the students responded as follows: "Matrix transpose referred to a matrix operation whereby the position of row elements was changed to the position of column elements and vice versa". She then justified that the statement of the student was true.

Subsequently, the teacher explained the concept of matrix determinant and the procedure to compute the determinant of matrix with 2 × 2 ordo and 3 × 3 ordo. She stated that the matrix determinant is the difference between the multiplication of elements in the main diagonal and the multiplication of elements in the secondary diagonal. Moreover, she argued that the operation of matrix determinant can only be done in a square matrix. It could be illustrated that the teacher's explanation is similar to the following sentences. Suppose that the matrix of A is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the deteminant of matrix A can be symbolize as $|A|$ and it is formulated as $a \times d - b \times c$. For example, $A = \begin{bmatrix} 3 & 5 \\ 2 & -4 \end{bmatrix}$, the value of $|A|$ was $3 \times -4 - 2 \times 5 = -12 - 10 = -22$. Furtermore, suppose that the matrix of B is $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, the determinant of matrx B can be symbolized as $|B|$ and it is formulated as $(a \times e \times i + b \times f \times g + c \times d \times h) - (g \times e \times c + h \times f \times a + i \times d \times b)$. For example, $B = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 2 \\ 1 & -2 & 1 \end{bmatrix}$, the value of $|B|$ was $(1 \times 1 \times 1 + 2 \times 2 \times 1 + -1 \times -2 \times -2) - (1 \times 1 \times -1 + -2 \times 2 \times 1 + 1 \times -2 \times 2) = -8$.

Moreover, the teacher gave two problems regarding the matrix determinant to be carried out by students. She wrote down those on the whiteboard (See Figure 6). The teacher instructed students to solve the problems individually, and each student who had solved those were instructed to directly collect

it from her. The teacher, however, also provided guidance for students in solving the problems of matrix determinants by visiting each of the students' tables. After facilitating them through guidance, she stayed in front of the class to wait for students to collect their work. Each of the students gradually went to the front of the class and collected those for her. She then checked the students' algorithm thinking in solving these problems and provided the justification for the truth of students' answers to matrix determinant problems.

1. Suppose that $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$, find the determinant of A × B.
2. The value of $|C|$ is 21 and $C = \begin{bmatrix} 3 & 3 \\ -2 & a \end{bmatrix}$, find the value of a.

Figure 6. The Problems of Matrix Determinant Given to Students

The Process of Algebra Learning: In Case of Compound Interest

The teaching of algebra learning in the topic of compound interest was carried out by Mrs. A.D and followed by 10 eleventh-grade senior high school students in one of private senior high schools in Bandung, a city in West Java Province, Indonesia. They consisted of 3 male students and 7 female students. Additionally, the teacher has had about six years of teaching mathematics lessons in the senior high school since 2018 to date. The implementation of algebra learning in this topic was conducted during two hours of lessons (80 minutes) which started at 8.15 AM to 9.35 AM on November 6th, 2023 (See Figure 7).

Figure 7. The Implementation of Algebra Learning in a Private Senior High School

In most cases of the topic of compound interest, the process of algebra learning in this topic was split into three main parts, such as introduction, core, and close. The process started with the teacher asking students one by one regarding their news, such as health and weekend holidays. She, then, prepared students to learn the algebra topic, particularly compound interest whereby she instructed them to make ready learning materials, such as mathematics book, note book, and digital calculator. She then conducted a learning apperception by recalling students' knowledge regarding single interest, in which she specifically asked students as follows: "Could you explain the concept of single interest? Moreover, please give examples of single interest". One of the students responded as follows: "Single interest refers to the interest which is given by considering the calculation of the first asset, so it only has one variation from the first period to the last period. For example, when depositing money in the bank, the customers will get the constant interest in every period". The teacher then justified that the statement of the student was true.

Furthermore, the teacher explained the concept of compound interest and the procedure to find compound interest and accumulative assets after the asset has been deposited for some period. She stated that compound interest refers to the interest which is given by considering the first asset and the accumulative interest on the previous period. Moreover, she argued that compound interest has many variations and always changes in each period. It could be illustrated that the teacher's explanation is similar to the following sentences. Suppose that M_0 is the first asset, *i* is the interest rate, *n* is the period,

and M_n is the accumulative asset after *n* periods, consequently, the formula to compute the accumulative asset after *n* period can be represented as $M_n = M_0(1+i)^n$.

She then explained the use of the formula in a daily life problem regarding compound interest (See Figure 8).

> The customer of a bank deposits the asset as much as IDR 500,000.00, if the interest rate is 10% for each of year, find the accumulative asset after seven years.

Figure 8. The Problems of Compound Interest Given to Students

She and the students jointly solved this problem which she started by substituting each variable value to the formulas as follows: $M_n = 500{,}000 \times (1+0.1)^7$. Finally, they got that the value of M_n was 974,358.55, so the accumulative asset the customer of a bank had after depositing it for seven years was IDR 974,358.55.

Moreover, the teacher gave two problems regarding the compound interest to be solved by students. She wrote down those on the whiteboard (See Figure 9).

1. A businessman deposits his asset to a conventional bank. After ten years, the accumulative asset is IDR 16,288,946.27, if the interest rate for every year is 5%, find the first asset that he deposits to the bank.
2. A customer of a bank charged the asset to complete her business needs whereby she needs it amount IDR 20,000,000.00, if the interest rate is 10% for every year, how much money that she has to pay to the bank after five years?

Figure 9. The Problems of Matrix Determinant Given to Students

The teacher instructed students to solve the problems individually, and each student who had solved those was instructed to directly collect it to her. The teacher, however, also provided guidance for students in solving the problems of compound interest by visiting each of the students' tables. After facilitating them through guidance, she stayed in front of the class to wait for students to collect their work. Each student gradually went to the front of class and collected those to her. She then checked the students' procedure for solving these problems and provided the justification for the truth of the students' answers to compound interest problems.

The Analysis of Algebra Learning Processes in the Perspective of Scientific Approach and Direct Instruction

The framework of steps of the scientific approach mentioned by Nenotaek et al. (2019), consisting of observation, question, data collection, association, and communication were used to analyze the process of algebra learning and teaching for three different topics, such as arithmetic and geometric sequence, matrix determinant, and compound interest. Briefly, the difference of algebra learning and teaching performed by three different female mathematics teachers is shown in Table 1.

Table 1. Algebra Learning and Teaching in the Perspective of Scientific Approach

Steps of Scientific Approach	Activities of Algebra Learning
Mrs. H. T (Case Topic of Arithmetic and Geometric Sequence)	
Observing	- The teacher provided four sequences of integers (See Figure 3). - Students observed those sequences and tried to complete the three next numbers.
Questioning	- The teacher asked students as follows: "What number in the 100th position of the 3rd and 4th sequence is? How to determine it simply?"

Observing	- The teacher stated the formula to determine the nth term of arithmetic and geometric sequence by writing those on the whiteboard (See Figure 4). - The teacher explained parts of the formulas, such as U_1, *b, r,* and *n*. - The teacher demonstrated how to solve the previously given questions to students using these formulas. - Students tried to understand those by writing it in their notebook.
Questioning	- The teacher gave two problems to students, consisting of one problem about arithmetic sequence and one problem about geometric sequence (See Figure 5).
Collecting the Data	- The teacher instructed students to make groups containing 5 – 6 members. - Students used smartphones to promote them to solve problems besides using mathematics book.
Associating	- Students jointly discussed how to solve the problems. - The teacher walked around each group to guide them if they were stuck or had difficulties. - They had to solve the problems about twenty-five minutes.
Communicating	- The teacher instructed and provided the opportunity for one group to explain the solution of arithmetic sequence problem and one group to explain the solution of geometric sequence problem. - Students presented and explained how to solve the problem of arithmetic and geometric sequence. - The teacher also made sure that the process and final answer of this arithmetic and geometric sequence problem were exactly true.
Mrs. W. S (Case Topic of Matrix Determinant)	
Observing	- The teacher conducted learning apperception by recalling students' knowledge regarding matrix operation and matrix transpose. - Students paid attention to the teacher's explanation regarding matrix operation and transpose.
Questioning	- The teacher asked students regarding the concept of matrix transpose for the matrix with 2 x 2 ordo and 3 x 3 ordo. - Students gave the response by explaining the concept of matrix transpose. - The teacher justified students' explanation regarding the concept of matrix transpose.
Observing	- The teacher explained the concept of matrix determinant and the procedure to compute the determinant of matrix with 2 x 2 ordo and 3 x 3 ordo. - Students observed the teacher's explanation regarding the concept of matrix determinant.
Questioning	- The teacher gave two problems regarding the matrix determinant to be carried out by students (See Figure 7).
Collecting the Data & Associating	- The teacher instructed students to solve the problems individually whereby each student who had solved those was instructed to collect it to her in the front of class directly. - Students solved those problems individually. - The teacher provided guidance for students in solving the problems of matrix determinants by visiting each of the students' tables.
Mrs. A. D (Case Topic of Compound Interest)	
Observing	- The teacher conducted learning apperception by recalling students' knowledge regarding a single interest. - Students paid attention to the teacher's explanation regarding single interest.
Questioning	- The teacher asked students regarding the examples of single interest. - Students gave the response by mentioning the examples of single interest.

	- The teacher justified students' explanations regarding the examples of single interest.
Observing	- The teacher explained the concept of compound interest and the procedure to find compound interest and accumulative assets after the asset has been deposited for some period. Moreover, she explained the use of the formula in a daily life problem regarding the compound interest. - Students observed the teacher's explanation regarding the concept of compound interest.
Questioning	- The teacher asked with providing the problem regarding the compound interest. - The teacher and students jointly solved the problem whereby she started by substituting each variable value to the formula of compound interest.
Collecting the Data & Associating	- The teacher gave two problems regarding the compound interest to be solved by students. - The teacher instructed students to solve the problems individually. - Students solved the problems regarding the compound interest. - The teacher provided guidance for students in solving the problems of compound interest by visiting each of the students' tables. - Students directly collected the answer of those problems to the teacher in the front of class.

Furthermore, the framework of steps of directed instruction model mentioned by Eppley and Dudley-Marling (2019), consisting of presenting learning purposes and preparing students to learn, demonstrating knowledge and skill, guiding students in doing the practice, checking students' understanding and giving feedback, and providing the opportunity for students to do advanced practice and apply it, was used to analyze the process of algebra learning and teaching for three different topics, such as arithmetic and geometric sequence, matrix determinant, and compound interest. Briefly, the difference of algebra learning and teaching performed by three different female mathematics teachers is shown in Table 2.

Table 2. Algebra Learning and Teaching in the Perspective of Directed Instruction

Steps of Directed Instruction Model	Activities of Algebra Learning
Mrs. H. T (Case Topic of Arithmetic and Geometric Sequence)	
Presenting learning purposes and preparing students to learn	- The teacher prepared students to learn algebra topic, specifically arithmetic and geometric sequence in which she instructed students to make ready learning materials, such as mathematics book and note book. - The teacher conducted learning apperception by recalling number patterns (See Figure 3). - Most of students could complete 1st, 2nd, and 3rd sequence but only a few of students who could complete 4th sequence.
Demonstrating knowledge and skill	- The teacher recalled the 3rd and 4th sequences and said those to students as arithmetic sequences and geometric sequences. - The teacher stated the formula to determine nth term of arithmetic and geometric sequence (See Figure 4). - The teacher explained the variables of arithmetic and geometric sequence formula. - The teacher demonstrated how to solve the previously given questions to students using these formulas. - Students tried to understand those by writing it in their notebook.
Guiding students in doing the practice	- The teacher gave two problems to students, consisting of one problem about arithmetic sequence and one problem about geometric sequence (See Figure 5).

	- The teacher instructed students to make groups containing 5-6 members. - Students jointly discussed to solve the problems. - The teacher walked around each group to guide students if they were stuck or found some difficulties.
Checking students' understanding and giving feedback	- The teacher instructed and provided the opportunity for one group to explain the solution of arithmetic sequence problem and one group to explain the solution of geometric sequence problem. - Each group presented an explanation regarding how to solve arithmetic and geometric sequence problems. - The teacher ensured that the process and final answer of the arithmetic and geometric sequence problem were true.
Providing the opportunity for students to do advanced practice and apply it	- The teacher said that there was homework regarding the topic of arithmetic and geometric sequence to be carried out by students as the task in which it was delivered by her to students using WhatsApp Group. - Students could read and check the task containing some problems regarding the arithmetic and geometric sequence.
Mrs. W. S (Case Topic of Matrix Determinant)	
Presenting learning purposes and preparing students to learn	- The teacher prepared students to learn the algebra topic, particularly matrix determinant in which she instructed them to make ready learning materials, such as mathematics book and note book. - The teacher conducted learning apperception by recalling students' knowledge regarding matrix operation and matrix transpose. - The teacher asked students regarding the concept of matrix transpose. - Students gave the response about the concept of matrix transpose. - The teacher justified that the students' statement was true.
Demonstrating knowledge and skill	- The teacher explained the concept of matrix determinant and the procedure to compute the determinant of matrix with 2 x 2 ordo and 3 x 3 ordo. - The teacher gave two problems regarding the matrix determinant to be carried out by students (See Figure 7).
Guiding students in doing the practice	- The teacher instructed students to solve the problems individually in which each of students who had solved those was instructed to directly collect it to her. - The teacher provided guidance for students in solving the problems of matrix determinants by visiting each of the students' tables.
Checking students' understanding and giving feedback	- The teacher checked the students' algorithm thinking in solving these problems and justified the truth of students' answer of matrix determinant problems.
Mrs. A. D (Case Topic of Compound Interest)	
Presenting learning purposes and preparing students to learn	- The teacher prepared students to learn the algebra topic, particularly compound interest in which the teacher instructed them to make ready learning materials, such as mathematics book, note book, and digital calculator. - The teacher conducted learning apperception by recalling students' knowledge regarding a single interest. - The teacher asked students regarding the concept of single interest. - Students gave responses related to the concept of single interest. - The teacher justified that the statement of the students was true.
Demonstrating knowledge and skill	- The teacher explained the concept of compound interest and the procedure to find compound interest and accumulative assets after the asset has been deposited for some period.

	- The teacher explained the use of the formula in a daily life problem regarding the compound interest. - The teacher and students jointly solved this problem in which she started by substituting each of variable value to the formulas of compound interest.
Guiding students in doing the practice	- The teacher gave two problems regarding the compound interest to be solved by students (See Figure 10). - The teacher instructed students to solve the problems individually, and each student who had solved those was instructed to directly collect it to her. - The teacher provided guidance for students in solving the problems of compound interest by visiting each of the students' tables.
Checking students' understanding and giving feedback	- The teacher checked the students' procedure in solving these problems and justified the truth of students' answer of compound interest problems.

From Table 1, it can be stated that the process of algebra learning in the topic of arithmetic and geometric sequence has contained all of the phases of the scientific approach. Nevertheless, in those phases, students do not ask about the concept of arithmetic and geometric sequence and the procedure to solve the problems of arithmetic and geometric sequence. Only the teacher who actively asks students about the concept and the procedure. This indicates that students are not critical in responding to the concept and procedure of arithmetic and geometric sequence. Meanwhile, the algebra process in the topic of matrix determinant and compound interest has passed away four phases of the scientific approach, such as observing, questioning, collecting the data, and associating. It is similar to the algebra learning process in the case of the topic of arithmetic and geometric sequence in which students do not ask something related to the concept of matrix determinant and compound interest, and the procedure to solve the problems regarding matrix determinant and compound interest. This also indicates that they are not critical in reacting to the teacher's explanation regarding the concept and procedure of matrix determinant and compound interest. Furthermore, there are no communication activities in both learning process of matrix determinant and compound interest. This can be understood that there are no teacher's instructions to stimulate communication activities for students in the learning process of matrix determinant and compound interest.

These findings indicate that critical thinking becomes an essential need for students in mathematics learning, especially algebra learning. A few of literature stated that there are two essential thinking processes, such as critical thinking and creative thinking (e.g., Chaffee, 2017; Sanders, 2016). Critical thinking is defined as an ability to process the information that enables each individual can evaluate and justify the information to develop the argument and solve the problem (Kong, 2015; Sanders, 2016). Moreover, critical thinking refers to an ability to reflectively think and skill fully assess, so it can decide what information can be relied on and what actions that have to be taken during reasoning or solving problems (Fung & Howe, 2014; Kwan & Wong, 2014). Additionally, Sanders (2016) argued that it requires students to organize, interpret, analyze, evaluate, infer, and explain the information in mathematics learning. Consequently, critical thinking is an essential ability that must be cultivated and enhanced on students in mathematics learning.

In particular, Kong (2015) mentioned that there are five indications of students who have critical thinking, such as interpreting, analyzing, evaluating, inferring, explaining, and self-regulating. Meanwhile, Ennis (2015; 2016) stated that students with critical thinking can conduct basic and advanced clarification, provide reasons for a decision, conclude, and predict in an integrated manner. A few instructions can be carried out to optimize students' critical thinking in mathematics by implementing problem-based and project-based learning. Some literature mentioned that the implementation of problem-based learning can be one of the effective solutions to optimize students' low critical thinking in mathematics (e.g., Nugraha & Suparman, 2021; Suparman et al., 2021, 2022; Yohannes et al., 2021). Moreover, a few of the literature stated that the design of problem-based learning could develop and enhance students' critical thinking in mathematics, so it can generate students as critical individuals who

can analyze, verify, and justify the truth and validity of the information that they can adjust in a variety of conditions and situations (e.g., Du et al., 2013; Neber & Neuhaus, 2013). Meanwhile, Loyens et al. (2023) explained that project-based learning refers to a learning process that organizes students into a series of collaborative inquiry activities, which helps them get specific new knowledge and critical thinking processes in solving real-life problems. Moreover, a few of the literature also found that project-based learning is one of the effective learning models in optimizing students' critical thinking in mathematics (e.g., Susiyanti et al., 2022; Yunita et al., 2022). Consequently, problem-based learning and project-based learning can be considered effective in optimizing students' critical thinking in mathematics.

Subsequently, communication activity is a fundamental phase in the scientific approach whereby it can generate student communication skills. In a literature, Rapti and Sapounidis (2024) defined communication as a delivery process of information, such as messages and ideas, from an individual to other individuals to achieve a certain purpose. Communication skills are extremely necessary in daily life, especially in intertwining the relation. Moreover, this skill is essential in promoting the success of mathematics learning when verbally presenting mathematics concepts or problems. Several kinds of literature explained that communication skills in mathematics refer to an ability in revealing mathematical ideas in various forms, such as tables, graphics, diagrams, and others (e.g., Kaya & Aydin, 2016; Kosko & Wilkins, 2010; Lomibao et al., 2016; Setiyani et al., 2020). Particularly, mathematical communication contains some abilities, such as stating a situation to mathematics symbol, idea, and model, meaningfully explaining and reading, verbally, non-verbally, or visually stating, understanding, interpreting, and evaluating mathematics ideas and presentation, listening, discussing, and writing about mathematics, and stating an argument in own language (Chasanah et al., 2020; Güçler, 2014; Yaniawati et al., 2019).

Additionally, NCTM (2000) stated that communication skills in mathematics are necessary to be generated for students, so they can reflect and clarify in thinking about mathematics ideas in various situations, convert a situation to verbal or non-verbal, figure, graphic, algebra form, enhance the understanding of mathematics ideas, apply abilities in reading, listening, writing, interpreting, and evaluating mathematics ideas, analyze mathematics ideas by believed conjecture and reason, and understanding the value of mathematics notation in developing mathematics ideas. NCTM points out that mathematical communication skill becomes an important part of mathematics and mathematics education. This indicates that communication ability in mathematics is really important. Moreover, it is highly required to be cultivated and enhanced by selecting suitable learning methods such as discussion and question & answer to promote it in the process of mathematics learning, especially algebra learning.

From Table 2, it can be stated that the process of algebra learning in the case topic of arithmetic and geometric sequence has passed away all of the phases of the directed instruction model. Every phase contains learning activities conducted by teacher and students. The process of algebra learning in the case of the topic of matrix determinant and compound interest, however, only contains the first phase, "presenting learning purposes and preparing students to learn," until the fourth phase, "checking students' understanding and giving the feedback". Meanwhile, the last phase of the directed instruction model, "providing the opportunity for students to do advanced practice and apply it," is not found in the learning process of matrix determinant and compound interest. Moreover, it can be detected in the closing part of algebra learning that there are no learning activities in the closing part of matrix determinant and compound interest. Some learning activities, such as conclusion or evaluation, reinforcement, reflection, and additional instruction, are not found in the case topic of matrix determinant and compound interest.

The activities in providing the opportunity for students to do advanced practice and apply it is an important part of directed instruction model. In this phase, students can improve their conceptual understanding regarding matrix determinant and compound interest. Several empirical studies found that the advanced practice in directed instruction promotes students in enhancing conceptual understanding, especially in mathematics (e.g., Birgin & Uzun Yazıcı, 2021; Ewing, 2011; Lestari & Mansyur, 2021; Tunde & Listiani, 2021). Moreover, Stein et al. (2018) stated that the phases of directed instruction, especially the advanced practice, promote the enhancement of students' conceptual understanding of mathematics. Additionally, Y. Wang et al. (2019) revealed that direct instruction is one of the effective learning models for students in cultivating their conceptual understanding in science and mathematics. These findings provide adequate proof to state that the advanced practice in directed instruction is able to cultivate and

enhance students' conceptual understanding of mathematics, mainly algebra concepts in matrix determinant and compound interest. Consequently, mathematics teachers in direct instruction must provide the opportunity for students to do advanced practice and apply it as mathematics homework to cultivate and enhance their conceptual understanding of mathematics.

Furthermore, reinforcement in the closing part of a learning process is required to promote the success of students' academic achievement. A few of the literature explained that it is defined as a positive response provided by the teacher to students in the learning process to provide the information or feedback and confirm well-certain things as an action or correction so students can maintain or improve the good behavior (e.g., Humble et al., 2020; Palhares, 2012; Sen & Yıkmış, 2021; Shan, 2021). Particularly, reinforcement in mathematics learning is an important part of the behavioral modification of mathematics teachers to students' behavior that can improve good behavior. In mathematics learning, it is really necessary to optimize students' mathematics achievement (Fang et al., 2023; Terry & McGee, 2012). From the perspective of the affective aspect, it significantly affects students' motivation in mathematics to improve positive behaviors (Sumiati et al., 2019). Additionally, a few of the literature explained that the reinforcement in the behavioral theory proposed by Skinner generally consists of positive and negative reinforcement (e.g., Boorla, 2013; Omomia & Omomia, 2014; Schlinger, 2011). Specifically, Schlinger (2011) explained that positive reinforcement refers to the frequency of response increasing because followed by supporting stimulus like as rewarding, whereas negative reinforcement refers to the frequency of response increasing because followed by removing the detrimental stimulus like as providing additional practice or homework. As a consequence, mathematics teachers as long as possible to reinforce students in the closing part of a learning process on what they have studied, especially the process of algebra learning in case of the topic of matrix determinant and compound interest, so it promotes students in improving their motivation in mathematics and enhancing their mathematics achievement.

CONCLUSION, IMPLICATION, AND LIMITATION

Performing scientific approach and directed instruction as a learning approach and model in the process of mathematics learning, specifically in algebra learning, should fully pass away all of the phases of the learning approach and model. Consequently, by learning algebra contents, such as arithmetic and geometric sequence, matrix determinant, and compound interest, students can optimize some cognitive domains in mathematics, such as communication and conceptual understanding. Particularly, the optimization of questioning activities as the second phase of the scientific approach and demonstrating knowledge and skill as the second phase of the directed instruction model is extremely necessary to cultivate and enhance students' critical thinking in mathematics.

The findings of this study provide some valuable contributions to the field of mathematics education. Performing scientific approach and directed instruction as a learning approach and model in the process of mathematics learning, specifically in algebra learning, should fully pass away all of the phases of the learning approach and model. Consequently, by learning algebra contents, such as arithmetic and geometric sequence, matrix determinant, and compound interest, students can optimize some cognitive domains in mathematics, such as communication and conceptual understanding. Communication skills are extremely necessary in daily life, especially in intertwining the relation. Moreover, this skill is very essential in promoting the success of mathematics learning when verbally presenting mathematics concepts or problems (Kaya & Aydin, 2016; Kosko & Wilkins, 2010; Lomibao et al., 2016). Additionally, conceptual understanding in mathematics is also really required for students to promote them in optimizing other advanced abilities, such as reasoning, communication, connection, problem-solving, critical thinking, and creativity (Andamon & Tan, 2018; Jansen et al., 2017; NCTM, 2000; Schaathun, 2022; Stovner & Klette, 2022).

Particularly, the optimization of questioning activities as the second phase of the scientific approach and demonstrating knowledge and skill as the second phase of the directed instruction model is extremely necessary to cultivate and enhance students' critical thinking in mathematics. Critical thinking becomes an essential need for students in the process of mathematics learning, especially algebra learning (e.g., Chaffee, 2017; Sanders, 2016). Additionally, Sanders (2016) argued that it requires

students to organize, interpret, analyze, evaluate, infer, and explain the information in mathematics learning. This indicates that critical thinking is an extremely essential ability that must be cultivated and enhanced in students in the process of mathematics learning, specifically in algebra learning.

The observation activity in this present study for each school is only conducted for one time. Consequently, the data found in the mathematics classroom regarding the process of algebra learning has not consistently described the actual phenomena related to how students learn in the case topic of arithmetic and geometric sequence, matrix determinant, and compound interest. Furthermore, as data collectors in the mathematics classroom, authors do not have much access to take the picture. On one side, it can disturb the focus of mathematics teachers in teaching algebra content to students. Additionally, on the other hand, the time to take a lot of meanings of the algebra learning process will decrease. Consequently, the authors can't present much-documented evidence highlighting the essential activities in the process of algebra learning. Subsequently, authors can't get access to learn more about the lesson plans designed by the mathematics teachers in teaching the algebra content. If it can be accessed, authors can analyze it and adjust it to the actual implementation in the mathematics classroom. It can be better if the observation activity is performed many times as long as consistent data regarding the process of algebra learning are found. For future studies, it would be better if similar studies involve adequate observers so that they can enrich the data found from the observation activities. Additionally, For the future studies, the researchers should get more access in observing the mathematics activities in the classroom by asking and permitting it more to the institutional authority.

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