# Application of Min-Plus Algebra in Determination Shortest Route for Distribution of Pindang Fish by Pindang Muri Company

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**Abstract**

Min-plus algebra can be applied to the problem of determining the shortest route. This article discusses the determination of the shortest route using min-plus algebra with a case study on the pindang fish distribution route from Juwana, Pati to Legi market, Surakarta. From the research results, a weighted directed graph consisting of 15 nodes is obtained as a representation of the distribution route. By using manual calculation of min-plus algebra, obtained the shortest route that can be traveled, namely Pindang Muri Company → Juwana Old Terminal → entrance lane Pati outer ring road→ Purwodadi intersection → Raffa Monggot Store → Sumberlawang Pharmacy → Agung Rahayu Kadipiro workshop → Major Achmadi Monument → Legi market, Surakarta. The mileage of the shortest route is 128.1 km.

*Key words and Phrases*: min-plus algebra, shortest route, pindang fish distribution

## 1 Introduction

Max-plus algebra is one of the branches of mathematics in the field of algebra. Max-plus algebra can be used to solve several problems in everyday life. These problems include flexible manufacturing systems, telecommunications networks, parallel process systems, traffic control systems, and logistics systems (Rafflesia [8]). Besides max-plus algebra, there is min-plus algebra which is similar to max-plus algebra. Min-plus algebra can be applied to the problem of determining the shortest route.

In 2001, Bacceli et al. [2] have researched about max-plus algebra. In addition to max-plus algebra, there is also research on min-plus algebra conducted in 2008 by (Gondran and Minoux [3]). Then in 2013, Rudhito [9] studied the system of min-plus linear equations and its application to the shortest path problem. Furthermore, in 2014, Nowak [6] conducted research on min-plus algebra to the upper matrix of min-plus algebra. The research was also conducted by Putri [7] in 2016 about the application of the shortest path using min-plus algebra with a case study of potato distribution on the Pangalengan, Bandung-Jakarta route. In 2017, Suwanti et al. [12] conducted research on the application of min plus algebra in determining the fastest route for milk distribution.

This research discusses the determination of the shortest route using min-plus algebra with a case study on the pindang fish distribution route from Juwana, Pati to Legi market, Surakarta. The distribution of pindang fish is one of the important aspects in the industrial chain of pindang fish production, so a shortest route search method is needed. This is important to determine the level of timeliness in the delivery journey to maintain the quality of pindang fish to the hands of consumers properly and to optimize the journey to be more efficient.

According to research conducted by Gunawan et al. [4], Pati Regency has considerable potential to be developed into a modern pindang processing industry supported by various resource facilities that include fish farmers, processors, marketers, fishermen, and a large number of laborers. Marketing channels for pindang products spread to the Central Java area (Pati Regency, Rembang, Jepara, Kudus, Solo, Wonogiri, Klaten, Semarang), Yogyakarta, East Java (Malang, Surabaya, Prigi), West Java (Tasikmalaya) and Jakarta. Pindang Muri company is one of the pindang fish processing companies located in Pati. The researcher used data based on the distribution route of pindang fish from Juwana, Pati to Legi market, Surakarta by Pindang Muri company.

This research is different from Putri [7] who formulated a numerical solution by using scilab software. It is also different from Suwanti et al. [12] who made a prototype of the Matlab Min-plus algebra program. In this article, the researcher performs manual calculations in the process of finding the shortest route for pindang fish distribution using min-plus algebra based on the observation data that has been obtained. Things to consider in determining the shortest route before applying min-plus algebra is the route traveled in the delivery process, the distance traveled in the fish distribution process. The data is represented in the form weighted directed graph. Then will be formed a matrix which elements are the weights of the graph. Next, a lifting operation will be performed using the operation on min-plus algebra to determine the result of the shortest route with minimum travel distance.

## 2 Definition on Min-Plus Algebra

Based on Nowak [6], the definition of min-plus algebra is given as follows.

Definition 2.1. *The min-plus algebra* $\mathbb{R}_{min}$ *is the set of* $\mathbb{R}_{min} = \mathbb{R} \cup \{\varepsilon'\}$ *with* $\mathbb{R}$ *is the set of all real numbers and* $\varepsilon' = \infty$ *equipped with operations minimum* $(\oplus')$ *and addition* $(\otimes)$*, so that* $\forall a, b \in \mathbb{R}_{min}$ *applies*

$$a \oplus' b = min\{a, b\}$$

$$a \otimes b = a + b$$

Operation $\otimes$ and $\oplus^{'}$ are associative and commutative, so it follows that

$$a \oplus' (b \oplus' c) = (a \oplus' b) \oplus' c$$

$$a \otimes (b \otimes c) = (a \otimes b) \otimes c$$

$$a \oplus' \ b = b \oplus' \ a$$

$$a \otimes b = b \otimes a$$

for every $a, b, c \in \mathbb{R}_{min}$. It can be shown that $(\mathbb{R}_{min}, \oplus^{'}, \otimes)$ is an idempotent commutative semiring with elements neutral to the operation $\oplus^{'}$ is $\varepsilon^{'} = \infty$ and to the $\otimes$ operation is 0.

Definition 2.2. *For every $x \in \mathbb{R}_{min}$ and $k \in \mathbb{N}$, so the rank in the algebra of min-plus algebra is defined*

$$x^{\otimes k} = k \times x, k \in \mathbb{N}$$

# 3 Definition on Min-Plus Algebra

Based on Subiono [11], the set of all matrices over min-plus algebra is the set of all matrices of size $m \times n$ which are denoted by $\mathbb{R}_{min}^{m\times n}$, with $m, n \in \mathbb{N}$, $defined\ m = 1, 2, \ldots, m$ and $n = 1, 2, \ldots, n$. Matrix elements $A \in \mathbb{R}_{min}^{m\times n}$, in the row $i$ and column $j$ are expressed by $a_{ij}$ , for $i \in m$ and $j \in n$. Matrix $A \in \mathbb{R}_{min}^{m\times n}$ can be written as

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix}$$

The operations of maximum ($\oplus'$) and addition ($\otimes$) of matrices on $\mathbb{R}_{min}$ are similar to the operations of addition and multiplication of matrices on $\mathbb{R}$. The $\oplus'$ operation can be operated if the sizes of both matrices are equal while the $\otimes$ operation can be operated if the column size of the first matrix is equal to the row size of the second matrix. The matrix operation in $\mathbb{R}_{min}$ according to Nowak [6] is defined as follows.

Definition 3.1. *For $A = (a_{ij}) \in \mathbb{R}_{min}^{m\times n}$ and $B = (b_{ij}) \in \mathbb{R}_{min}^{m\times n}$ defined on $\oplus'$ operation, that is*

$$[A \oplus' B] = a_{ij} \oplus' b_{ij} = min\{a_{ij}, b_{ij}\}$$

Definition 3.2. *For $A = (a_{ij}) \in \mathbb{R}_{min}^{m\times n}$ and $B = (b_{ij}) \in \mathbb{R}_{min}^{m\times n}$ defined on $\otimes$ operation, that is*

$$[A \otimes B]_{ij} = \bigoplus_{k=1}^{n} (a_{ik} \otimes b_{kj}) = (a_{i1} \otimes b_{1j}) \oplus' (a_{i2} \otimes b_{2j}) \oplus' \ldots \oplus' (a_{in} \otimes b_{nj})$$

Definition 3.3. *For $A = (a_{ij}) \in R_{min}^{n\times n}$ is defined by the multiplication operation on scalar* $\alpha \in R_{min}^{m\times n}$*, that is*

$$[\alpha \otimes A]_{ij} = \alpha \otimes a_{ij}$$

Definition 3.4. *For $A = (a_{ij}) \in R_{min}^{n\times n}$ is defined by the multiplication operation on scalar* $\alpha \in R_{min}^{m\times n}$*, that is*

$$a^{\otimes\{k} = \underbrace{A \otimes A \otimes \ldots \otimes A}_{k}$$

# 4 Matrix and Weighted Directed Graph

According to Subiono [11], suppose given matrix $A \in R_{min}^{m\times n}$ a directed graph of matrix A is $G(A) = (V, E)$. A graph $G(A)$has $n$ nodes, and the set of all nodes of $G(A)$ is denoted by $V$. A line from vertex $j$ to vertex $i$ exists if $a_{ij} \neq$, this line is denoted by $(j, i)$. thus $(j, i) \in A$. The weight of the line $(j, i)$ is the value of $a_{ij}$denoted by $w(j, i) = a_{ij} \in R_{min}$. If $a_{ij} = \varepsilon$, then the line $(j, i)$does not exist.

Definition 4.1. *A directed graph* $G = (V, E)$ *with* $V = \{1,2, \dots, n\}$ *is said to be strongly connected if for every* $i, j \in V, i \neq j$, *there exists a path from* $i$ *to* $j$
Definition 4.2. *For every weighted directed graph* $G$ *it is always possible to define a matrix* $A \in R_{min}^{m\times n}$ *with*

$$A_{ij} = \begin{cases} w(j, i), & (j, i) \in A \\ \varepsilon, & (j, i) \notin A \end{cases}$$

which is called the weighted matrix of the graph $G$.

The weight matrix of a graph $G$ is a square matrix $A \in R_{min}^{m\times n}$ with the number of rows and columns or the size of the matrix is equal to the number of points in the weighted directed graph $G$ (Siang [10]).

Definition 4.3. *The shortest path from point* $u$ *to v is defined as a path from* $u$ *to v where the total weight of the path is the sum of the minimum weights of the edges on any path originating or starting from point* $u$ *to point* $u$. *If there is no path from* $u$ *to v, then the weight is said to be* ∞ (Lipschutz and Lipson [5]).

Based on Andersen [1], to determine the solution of the shortest route problem is given the following definition

Definition 4.4. *A graph with* $n$ *vertices and matrix representation A, then we get* $A^* = A \oplus' A^{\otimes^2} \oplus' A^{\otimes^3} \oplus' \dots \oplus' A^{\otimes^{n-1}}$. *Then* $a_{ij}$ *of* $A^*$ *is the weight of the shortest route from vertex i to vertex j.*

# 5 Results and Discussions

Pindang Muri company is a pindang fish processing company located at Dukutalit village, Juwana district, Pati regency, Central Java. Pindang Muri company distributes pindang fish to various regions, one of which is to Legi market, Surakarta. A pick-up truck is used for the distribution of pindang fish. Data was collected through interviews with several drivers who distribute pindang fish to Legi market, Surakarta. The route data obtained is a route that is often traveled by several drivers in distributing pindang fish to Legi market, Surakarta.

The distribution route traveled by the pindang fish delivery car can be represented in the form of a directed graph to make it easier to find the shortest route using min-plus algebra. In a weighted directed graph, nodes represent the location of the path traveled, arcs represent a path, while the arc weight represents the distance traveled. Based on the data results from field, the following directed graph is found below

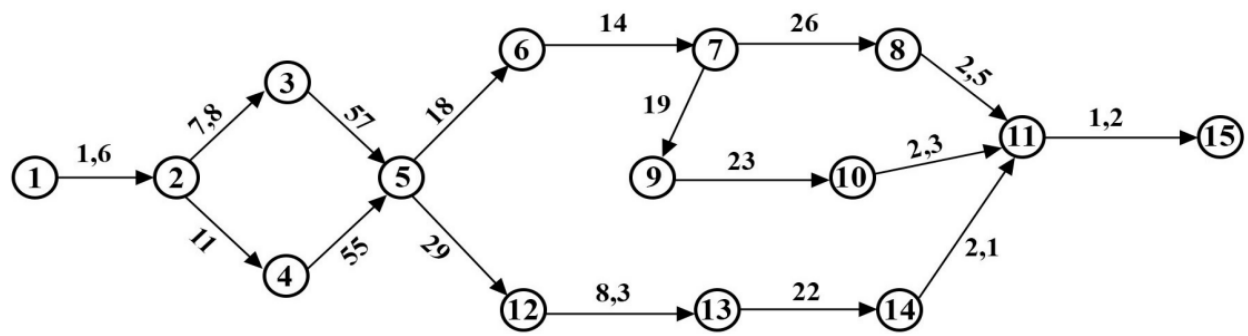


Figure 1: Distribution Route in Graph Shape

Description :

| | | | |
|---|---|---|---|
| Points 1, 2, 3, ..., 15 | : vertex | 7 | : Sumberlawang pharmacy |
| → | : edge | 8 | : Agung Rahayu Kadipiro workshop |
| ← | : the distance between nodes | 9 | : Sukowati Kebayan food court |
| 1 | : Pindang Muri company | 10 | : Cembengan monument |
| 2 | : Juwana old terminal | 11 | : Major Achmadi monument |
| 3 | : Entrance of Pati outer ring road | 12 | : Jaya cell Mondokan |
| 4 | : Gemeces Pati park | 13 | : Gabugan Tanon market |
| 5 | : Purwodadi intersection | 14 | : Dynamo Setya Utama Mojosongo |
| 6 | : Raffa Monggot shop | 15 | : Legi market |

From the weighted directed graph in Figure 1, a matrix A can be constructed, where the elements of the matrix are the distances between nodes. Based on [5], if there is no travel route in the graph, then the weight is said to be ∞ or ε. The matrix of the weighted directed graph is a matrix with 15 rows and columns because there are 15 nodes in the weighted directed graph in Figure 1. The form of matrix A is

$$A = \begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
1{,}6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & 7{,}8 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & 11 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & 57 & 55 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & 18 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 14 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 26 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 19 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 23 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 2{,}5 & \varepsilon & 2{,}3 & \varepsilon & \varepsilon & \varepsilon & 2{,}1 & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & 29 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 8{,}3 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 22 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 1{,}2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon
\end{bmatrix}$$

Next, there will be a lifting operation on the matrix A based on [1] to obtain $A^*$. The lifting operation of matrix A is performed $(n-1)$ times with $n$ being the size of matrix A. Matrix A is

$A_{15\times 15}$ so the operation of lifting matrix A is 14 times. By performing the lifting operation on matrix A expressed by $A^*$, will obtain a matrix with the minimum weight of the shortest path and the length of the path is the power of matrix A. The following is the result of rank matrix A.

$$A^* = A \oplus' A^{\otimes^2} \oplus' A^{\otimes^3} \oplus' A^{\otimes^4} \oplus' A^{\otimes^5} \oplus' A^{\otimes^6} \oplus' A^{\otimes^7} \oplus' A^{\otimes^8} \oplus' A^{\otimes^9} \oplus' A^{\otimes^{10}} \oplus' A^{\otimes^{11}} \oplus' A^{\otimes^{12}} \oplus' A^{\otimes^{13}} \oplus' A^{\otimes^{14}}$$

$$A^* = \begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
1{,}6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
9{,}4 & 7{,}8 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
12{,}6 & 11 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
66{,}4 & 64{,}8 & 57 & 55 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
84{,}4 & 82{,}8 & 75 & 73 & 18 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
98{,}4 & 96{,}8 & 89 & 87 & 32 & 14 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
124{,}4 & 122{,}8 & 115 & 113 & 58 & 40 & 26 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
117{,}4 & 115{,}8 & 108 & 106 & 51 & 33 & 19 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
140{,}4 & 138{,}8 & 131 & 129 & 74 & 56 & 42 & \varepsilon & 23 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
126{,}9 & 125{,}3 & 117{,}5 & 115{,}5 & 60{,}5 & 42{,}5 & 28{,}5 & 2{,}5 & 25{,}3 & 2{,}3 & \varepsilon & 32{,}4 & 25{,}1 & 2{,}1 & \varepsilon \\
95{,}4 & 93{,}8 & 86 & 84 & 29 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
103{,}7 & 102{,}1 & 94{,}3 & 92{,}3 & 37{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 8{,}3 & \varepsilon & \varepsilon & \varepsilon \\
125{,}7 & 124{,}1 & 116{,}3 & 114{,}3 & 59{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 30{,}3 & 22 & \varepsilon & \varepsilon \\
128{,}1 & 126{,}5 & 118{,}7 & 116{,}7 & 61{,}7 & 43{,}7 & 29{,}7 & 3{,}7 & 26{,}5 & 3{,}5 & 1{,}2 & 33{,}6 & 25{,}3 & 3{,}3 & \varepsilon
\end{bmatrix}$$

Based on $A^*$ calculations, the shortest route that can be taken is obtained through 1 → 2 → 3 → 5 → 6 → 7 → 8 → 11 → 15 with the total distance traveled 128.1 km. The initial path from Pindang Muri company to Juwana old Terminal with a distance of 1.6 km. From Juwana old terminal to the entrance of Pati outer ring road with a distance of 7.8 km. From the entrance of Pati outer ring road to Purwodadi intersection with a distance of 57 km. From Purwodadi intersection to Raffa Monggot shop with a distance of 18 km. From Raffa Monggot shop to Sumberlawang pharmacy with a distance of 14 km. From Sumberlawang pharmacy to Agung Rahayu Kadipiro workshop with a distance of 26 km. From Agung Rahayu Kadipiro workshop to Major Achmandi monument shop with a distance of 2,5 km. From Major Achmandi monument to destination that is Legi market, Surakarta with a distance of 1,2 km.

# 6 Conclusion

Based on the results and discussion, it is concluded that min-plus algebra can be applied in determining the shortest route. The shortest route obtained using the min-plus algebra theory for pindang fish distribution from Juwana, Pati going to Legi Market, Surakarta is from Pindang Muri company → Juwana old terminal → Pati outer ring road → Purwodadi intersection → Raffa Monggot shop → Sumberlawang pharmacy → Agung Rahayu Kadipiro workshop → Major Achmadi monument → Legi Market, Surakarta. The mileage of the shortest route is 128.1 km.

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