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An Empirical Analysis of Portfolio Performance Using Maximin and Fuzzy Linear Programming

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Abstract. Fuzzy decision is the alternative way to help a problem in mathematical modelling for selection of portfolio. For this purpose, we will show how the portfolio constructing from Maximin method into portfolio modelling using fuzzy approach to obtain additional information for the portfolio. Through fuzzy, the portfolio modelling will able to cope two or more objectives in one step. The numerical example will be included to give an illustration from a practical view.

1. Introduction

Numerous formulations can be considered by investors to construct their portfolios due to the expected return and risk. These two parameters are the most impotant factor to derive the optimal portfolios. The well-known method is mean variance from Markowitz. The classical formulation from Harry Markowitz was the pioneer application when variance or standard deviation was used as a measure of risk. Even though this model is simple but the emerging of mean variace model for portfolio can be called as the modern portfolio era. The formulation is as follows [1]:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \quad (1)$$

Subject to these conditions :

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0 ;\ 0 \leq x_j \leq u_j,\ j = 1,2,\ldots,n \text{ ; and } \sum_{j=1}^{n} x_j = M_0 \quad (2)$$

where x_j represents the amount of money to be invested in stock s_j and u_j is a maximum proportion allowed for investment in s_j, r_j is is the average of rate of return of stock and ρ is the minimum rate return stated by investor. In this approach, investor is assumed to avoid the risk or it is called risk averse. One alternative of the mean variance model is the goal of function is to maximize the portfolio return at a certain risk. In this paper, we use a minimize risk as a goal function. We can called it as the minimum variance model.

The classical MV model was developed by Konno and Yamazaki [2] where variance is subtituted with mean absolute deviance to reduce the weakness of the MV model from the strategic perspective. The problem of MV model is a quadratic problem in its objective function and it worked with covariance matrix in large scale when the portfolio involved the huge set of assets. It might take some time to find the solution. So, MAD from Konno and Yamazaki is more preferable when we are working with large assets. Both portfolio models were formulated by using opimization criteria and depend on historical data. Experience and analyst's subjective should be introduced in the model. Thus, Fuzzy Sets Theory can be applied to solve the modelling problem in portfolio selection.

The similar form of fuzzy linear programming and maximin was discussed by Dyson [3]. Many authors published the development of fuzzy in problem of selecting portfolio such as Liu [4], Retno and Rosita [5] exposed the fuzzy with two objective functions with numerical example resulted the fuzzy and parametric form with the advantages for investor to obtain the future information of risk level in study case of Indonesian Stock market. On the other side, based on Papahristodoulou and Dotzauer [1], they compare three formulas, i.e Mean Variance, Mean Absolute Deviance and Maximin in Sweden Market and it performed that the maximin portfolio has its merits since it is more robust to the true decline in price of stock perspective. Another comparison of three other models was done in Brazil Stock Market [6]. The portfolios with three periods in Brazil using MV, MAD and Maximin were resulted in two main statements, one result is Maximin is more optimal with highest sharpe index when portfolio was involving large amount of assets, more than 50 assets.

The goal of this paper is to continue the empirical analysis for the problem of portfolio selection associated with estimating return and risk separately and simultaneously from two models, Maximin and fuzzy bi-objective linear programming (FBLP). In this paper, we used data from Retno and Rosita [5]. The strategy to obtain the advantages and weakness of two models, we will use Sharpe Index as a measure of portfolio performance.

2. Maximin Programming

Authors proposed minimax as the parametric model and it is known in the previous article [5] to discuss the bi-objective with kuhn-tucker operator, we called as Minimax model. It is different with this research, we use Papahristoudoulou result to change the strategy of minimax formulation without alpha coefficient and we follow the procedure as same as in their article. This alternative method need additional variable Z that is defined as the minimum return for every period and non-negative property.

The problem is we have to be aware that Z value will be another part of optimal solution and differ from the return requirement from investors, ρ, that is stated explicit by investor. Even though it quite differrent with minimax programming but it has similar goal of two sets, in this model we have the alternative objective function is to maximize the minimum return. Logically, when the investor have a goal to maximize the portfolio return from their investement, it means invetors have to be prepared to face the maximum risk. So, it is clearly that maximin model have a maximum risk to reach maximum return using the minimum return as a risk measure in its goal function. The way to derive the information of minimum return will be investigated from the assumption of observation result at every periods. So we need to acocount for portfolio return periodically. In the model, we add the constraints of finding minimum return as the extra restrictions in the model. It might be the assumptions of every period's return will be at least equal to Z.

In this discussion, we proposed the procedure to model the portfolio using maximin criterion.

Step 1: compute realized return $R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$;

Step 2 : compute expected return , we use qeometric mean $\mathrm{M_G} = \left(\prod_{t=1}^{n}(1+R_t)\right)^{1/n} - 1$

Step 3 : define the goal function in the maximin model is to maximize the minimum risk, where a risk is defined as minimum value of portfolio return so it can be write as Z

Maximize $\phi(w) = Z$

Step 4 : define constraints in the model. Several constraints were developed from the investor preferences to gain return from each stocks and each period minimal or equivalen with minimum return. It can be write as $\sum_{i=1}^{n} r_{it}\, w_i \geq Z,\ t = 1,2,\dots,T$. This equation of constraint can be rewritten as follows :

$$\sum_{i=1}^{n} r_{it}\, w_i - Z \geq 0 \quad (3)$$

Another part of constraint set is the expected return, where it is restricted by minimal return obtained from expected return each stocks (R_M). This can be rewritten as :

$$\sum_{i=1}^{n} E(R_i) x_i \geq R_M \quad (4)$$

The next constraint is the weight total that is equal to one, $\sum_{i=1}^{n} x_i = 1$ and the other set of constraint is maximum allocation of capital for each asset u_i. The value of u_i is depend on investor.

$$0 \leq x \leq u_i \text{ , } i = 1, 2, 3, \ldots, n \quad (5)$$

3. Fuzzy Bi-Objective Linear Programming

Integration of the vagueness and ambiguity condition can be cope by fuzzy theory and it can be expanded for solving in problem with two objectives through these following steps [5]:

Step 1 : vector minimization problem

minimize

$$Z_1(x) = -\left(\sum_{j=1}^{n} r_j x_j\right) \quad (6a)$$

$$Z_2(x) = \sum_{j=1}^{n}\left(\frac{1}{T}\sum_{t=1}^{T} q_{tj}\right) x_j \quad (6b)$$

subject to

$$\sum_{j=1}^{n} x_j = M_0 \text{ and } 0 \leq x_j \leq u_j,\ j = 1, 2, \ldots, n$$

Step 2: Solve vector minimization problem partially for each objective function and let v_i be the optimum solution of objective function $Z_i, i = 1, 2$.

Step 3: pay-off matrix as follows:

$$\begin{array}{ccc} & v_1 & v_2 \\ Z_1 & \left|Z_1(v_1)\right. & \left.Z_1(v_2)\right| \\ Z_2 & \left|Z_2(v_1)\right. & \left.Z_2(v_2)\right| \end{array},$$

and then find $U_i = \max\left[Z_i(v_1), Z_i(v_2)\right], i = 1, 2$ and $L_i = \min\left[Z_i(v_1), Z_i(v_2)\right], i = 1, 2$

Step 4: the characteristic functions for each of the objective functions as follow:

$$\mu_{Z_i}(x) = \begin{cases} 1, Z_i(x) \leq L_i \\ \dfrac{U_i - Z_1(x)}{U_i - L_i}, L_i \leq Z_1(x) \leq U_i \\ 1, Z_1(x) \geq U_i \end{cases}$$

for $i = 1, 2$.

Step 5: Solve fuzzy model for problem (5). This is stated as follow:

$$\text{Maximize } \left\{Min\left(\mu_{\max}(x)\right), Min\left(\mu_{\min}(x)\right)\right\} \quad (7)$$

subject to

$$\sum_{j=1}^{n} x_j = M_0 \text{ and } 0 \leq x_j \leq u_j,\ j = 1, 2, \ldots, n$$

And the model can be rewritten :

$$\text{Maximize } \lambda \tag{8}$$

subject to

$$\frac{U_1 - Z_1(x)}{U_1 - L_1} \geq \lambda$$

$$\frac{U_2 - Z_2(x)}{U_2 - L_2} \geq \lambda$$

$$\sum_{j=1}^{n} x_j = M_0 \text{ and } 0 \leq x_j \leq u_j,\ j = 1, 2, \ldots, n$$

The first and second constraints of problem (7) can be reduced into the following form, $\frac{\lambda(U_1 - L_1) + Z_1(x)}{U_1} \leq 1$ and $\frac{\lambda(U_2 - L_2) + Z_2(x)}{U_2} \leq 1$.

Therefore, model (8) above can be rewritten as follow:

$$\text{Maximize } \lambda \tag{9}$$

subject to

$$\frac{\lambda(U_1 - L_1) + Z_1(x)}{U_1} \leq 1 ; \frac{\lambda(U_2 - L_2) + Z_2(x)}{U_2} \leq 1$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq u_j,\ j = 1, 2, \ldots, n$$

4. Data and Result

To ilustrate our strategy, we discuss the Maximin portfolio using data as portfolio 1 and continue with FBLP as portfolio 2. In this research, we still use two types of maximum allocation 50% and 30%. In order to compare the performance of the result empirically, we apply a real data set monthly from January 1, 2013 until May 1, 2014 with 5 stocks namely UNVR(U), PGAS(P), SMGR(S), KLBF(K) and CPIN(C) based on previous data in [5] as the original data to accomplish the investigation of fuzzy bi-objective linear programming with other model, especially the similar form maximin programming.

The fluctuation of price can be observe from the line graph below for 5 stocks in the portfolio and we may get an information that UNVR Price is higher than other stocks. CPIN, KLBF and PGAS are tend to have a constant movement.

To provide a better understanding of the purpose of this research, we perform two portfolios from maximin result and FBLP also its Sharpe-ratio performance in the end of discussion.

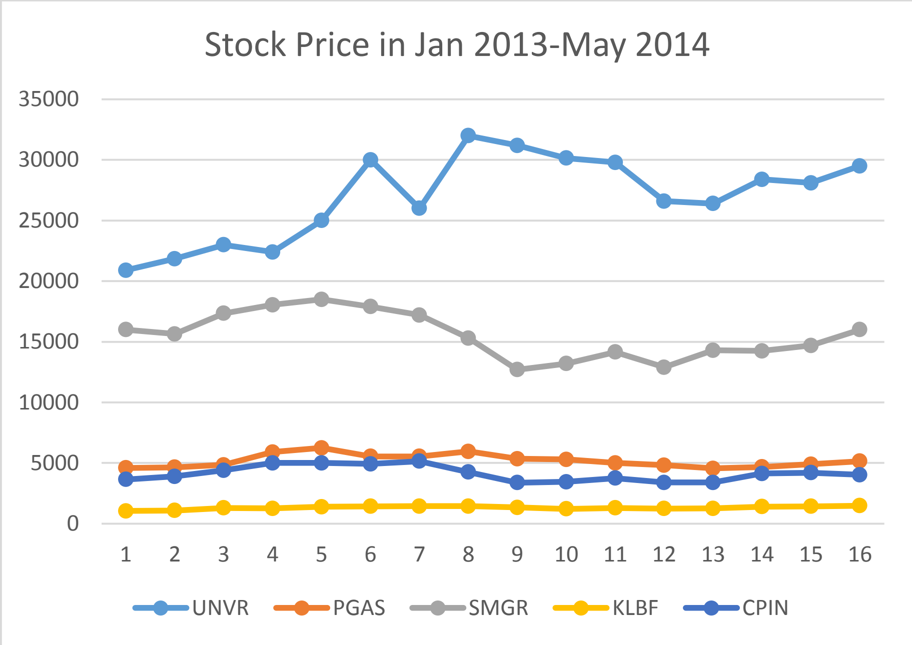

Figure 1. Monthly Stock Price 2013-2014

Table 1. Risk/Return of each stocks

	U	P	S	K	C
Mean Absolute Deviance	0.074367	0.057442	0.066073	0.049674	0.132475
Arithmetic mean	0.028	0.011	0.003	0.025	0.013
Geometric Mean	0.020241	0.009485	-0.00486	0.024018	0.001691

4.1 The maximin program

Mathematic model form maximin program is

$$\text{Maximize } \phi(x) = Z$$

s.t $\sum_{i=1}^{5} r_{it}\, x_i - Z \geq 0,\ t = 1,2,\dots,16$

For $t = 1 : 0{,}04545x_1 + 0{,}01087x_2 + \cdots + 0{,}06849x_5 - Z \geq 0$

$t = 2\ : 0{,}05263x_1 + 0{,}04301x_2 + \cdots + 0{,}12821x_5 - Z \geq 0$

Until for $t = 16$ analog with $t = 1,2$

1. $\sum_{i=1}^{5} E(R_i)x_i \geq R_M$

$0{,}02024x_1 + 0{,}00948x_2 + \cdots + 0{,}00169x_5 \geq 0{,}01011$

2. $\sum_{i=1}^{5} x_i = 1$
3. $0 \leq x_i \leq 0{,}3$ or $0 \leq x_i \leq 0{,}5$ where i = 1, 2, 3, ..., *5*

Using WinQSB we get the result for each u_j as follows :

Tabel 2. Composition each stock from maximin result

	U	P	S	K	C
30%	0,3	0,3	0,1	0,3	0

50%	0,3256	0,1744	0	0,5	0

The portfolio return and risk from Maximin model $E(R_p)$ and σ_p are

$$E(R_p) = (0{,}3 \times 0{,}02041) + \cdots + (0 \times 0{,}001691) = 0{,}015637$$

$$\sigma_p^2 = \boldsymbol{x^t \sigma x} = [x_1 \dots x_5]\begin{bmatrix}\sigma_{11} & \sigma_{21} & \cdots & \sigma_{51}\\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{52}\\ \vdots & \vdots & & \vdots\\ \sigma_{15} & \sigma_{25} & \cdots & \sigma_{55}\end{bmatrix}\begin{bmatrix}x_1\\ \vdots\\ x_5\end{bmatrix}$$

$$= [0{,}3 \quad 0{,}3 \quad \dots \quad 0]\begin{bmatrix}0{,}008926 & 0{,}000262 & \dots & 0{,}001161\\ 0{,}000262 & 0{,}005803 & \dots & 0{,}004469\\ \vdots & \vdots & \ddots & \vdots\\ 0{,}001161 & 0{,}004469 & \dots & 0{,}046288\end{bmatrix}\begin{bmatrix}0{,}3\\ 0{,}3\\ \vdots\\ 0\end{bmatrix}$$

$= 0{,}002616$ so we get the result of risk, $\sigma_p = \sqrt{0{,}002616} = 0{,}05115$

Also we get for the other maximum allocation 50%, the portfolio return $E(R_p) = 0{,}02025$ and the portfolio risk is $\sigma_p = 0{,}05636$

3.2 FBLP

We propose fuzzy bi-objective linear programming in this research with MAD basic. Based on the step by step in the previous discussion, we find the composition v_1 and v_2 for each goal function particularly then we work on that composition on each goal. So we get pay-off matrix as follows :

Table 3. Pay off matrix for each maximum alllocation

max 30%	v_1	v_2
z_1	-0.01666	-0.1313
z_2	0.061044	0.0594
max 50%	V_1	V_2
z1	-0.0221	-0.01675
z_2	0.06202	0.0536

Then we define Lower and Upper bound for each Z for portfolio with uj = 50% and 30 %

$$\mu_{z_1} = \begin{cases} 0 & , Z_1(x) \geq -0.01675 \\ \dfrac{-0.01675 - Z_1(x)}{0.00535} & -0.0221 \leq Z_1(x) \leq -0.01675 \\ 1 & , Z_1(x) \leq -0.0221 \end{cases}$$

$$\mu_{z_2} = \begin{cases} 0 & , Z_2(x) \geq 0.06202 \\ \dfrac{0.06202 - Z_2(x)}{0.00842} & , 0.0536 \leq Z_2(x) \leq 0.06202 \\ 1 & , Z_2(x) \leq 0.0536 \end{cases}$$

For u_j = 30%

$$\mu_{z_1} = \begin{cases} 0 & , Z_1(x) \geq -0.0166 \\ \dfrac{-0.01675 - Z_1(x)}{0.00535} & , -0.1353 \leq Z_1(x) \leq -0.0166 \\ 1 & , Z_1(x) \leq -0.1353 \end{cases}$$

$$\mu_{z_2} = \begin{cases} 0 & , Z_2(x) \geq 0.061044 \\ \dfrac{0.06202 - Z_2(x)}{0.00842} & , 0.0594 \leq Z_2(x) \leq 0.061044 \\ 1 & , Z_2(x) \leq 0.0594 \end{cases}$$

Table 4. The result composition of FBLP

	Lambda	U	P	S	K	C
u_j=50%	1	0.0018	0.5	0	0.4982	0
u_j=30%	1	0.3	0.1	0.3	0	0.3

Analog with computation for expected return portfolio for FBLP and its risk, we summarize with the previous result from maximin in Table 5

Table 5. Expected return, risk and Sharpe index
for portfolio with 30 % maximum allocation

	maximin	FBLP
expected return	0.015637	0.0089861
portfolio risk	0.05115	0.087596
sharpe index	0.0305711	0.102586

The performance index from maximin and FBLP will be interpreted partially, when we compare both models, the sharpe index of maximin is higher than FBLP. In another perspective, when we use fuzzy approach, we can get the additional information about how the models perform the lowest return that the investor will get and the maximum risk will investor reach such as in this portfolio with 30% maximum allocation, investor will account for return 0.01675 until 0.0221 and 0.0536 until 0.06202 for the it risk. It is clear that the assumption of the result of FBLP is violated. This result in line with [3] that the goodness of fuzzy linear programming will disappear when the assumption is invalid.
From modelling construction, we need to input portfolio return historically for completing the extra set of constraints to find minimum return. For a longer time period, it will require higher number of constraints.

4. Conclusion

Choosing the best portfolio is one of problem in portfolio selection based on the goal of investor. The empirical analysis in this research shows that from both models, maximin composition is more optimal than FBLP from its Sharpe ratio.

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References

[1] Papahristodoulou, C and Dautzer, E 2005 Optimal Portfolios Using Linear Programming Models. *Journal of the Operational Research* p 1-13

[2] Konno, H., and Yamazaki, H 1991 Mean Absolute Deviation Portofolio Optimization Model and its Applications to Tokyo Stock Market. *Management Science* p 519-531

[3] Dyson, R. 1980 Maximin Programming, Fuzzy Linear Programming and Multi-Criteria Decision Making. *The Journal of the Operational Research Society*, 31. *No. 3* p 263-267

[4] Liu, H.-W. 2011 Linear Programming for Portfolio Selection Based on Fuzzy Decision-Making Theory. *The tenth International Symposium on Operations Research and Its Applications (ISORA 2011)* p 195-202. (Dunhuang-China)

[5] Retno, S., & Rosita, K. 2015 Portfolio Selection in Indonesia Stock Market with Fuzzy Bi-Objective Linear Programming. *International Conference on Research and Education Mathematics 7th* (Kuala Lumpur: IEEE)

[6] Farias, C. A., Cruz Vieira, W. D., & Dos Santos, M. L 2006 Portfolio Selection models : Comparative Analysis and Its Application to the Brazilian Stock Market. *Journal of Mathematical Finance* p 387-407