Beliefs in Problem Solving: Case Study in Circles Tangents Line Materials

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Abstract. The study aimed to reveal the belief profile of student' who had medium and low math ability in solving problems of circle tangent material. This type of research was qualitative research. Subject selection based on the ability of students, feedback from professors and fluency in communicating both verbally and writing. Data collection techniques used are the method of semi-structured interview with technique of think aloud method, students are asked to express clearly everything was thought out and asked to write directly. The results showed that subjects with high, medium and low mathematical ability: believed that the mathematical problems given have difficulty level of moderate and high. It was due to the subject does not have an initial picture of problems resolution, and there were several ways, and there were many stages to be solved. The time required in problem solving is about 20 minutes. Further subjects believed the concept of tangent is used to find the length of the chain that was not attached to the wheel, while the concept of the circumference was used to find the length of the chain attached to the wheel. The subject believed it could be completed in one way that is using the circumference of the circle and the external tangents. Subjects believed stages of problem solving begin by finding the circumference of a half circle for the big circle and half circle for a small circle. Next is finding the length of external tangents between circles. Although the subject believed the concept, relation between concepts, step-by-step solution; subjects are not sure of the answers that would be obtained from a solution that will be done. Finding in problem solving practices was the subject making the same mistake that is they could not represent the mathematical problems into mathematical images. This was due to all three subjects draw a diameter line on the big circle and a small circle thus when drawing a line connecting two points on each circle, the line was not tangent of both circles.

Keywords: Profile, Belief, Problem Solving, Circle Tangents Line.

1. Introduction

Belief affects student achievement (see, e.g., [4], [8] and [18]). Students' belief about the importance of math had a significant impact on the achievement and had increased motivation [1]. As an illustration, the experience is used to shape the behavior of students in solving particular mathematical problems. Students can understand the problem after solving some or lots of math problems. Previous experience will determine what information should be relevant and what concept is right for the students thinking in solving the problem.

Problem solving is a process of problems reception as challenges to solve. Problem solving is a manifestation of a mental activity that consists of a variety of cognitive skills and actions that are intended to get the correct solution of the problem. Due to activity of problem solving involves a person's cognitive activity, it is resulting in the ability of each person to solve the problem would be different. To complete a math problem, students need to manage their thoughts properly to take advantage of prior knowledge, control and reflect the process of the results of their own thinking, because what is thought can help in resolving the problem. Positive beliefs toward math problem solving is an important thing that should be inculcated in children from an early age given that belief can be the basis for the disposition, the basis for action, the basis for change and the basis for the study [2]. This was caused by the advantages that can be obtained when the problem solving was done with involve thinking and the ability of self-regulation, allowing the establishment of understanding and strong beliefs to the problem with a logical reason [10]. This kind of understanding and belief must always be emphasized in the learning of mathematics at all levels of education, for compliance with the strong mathematical mindset.

Belief was justified by reasoning without having to involve the rules of proof, usually in the form of cognition that is stable and forming a strong structure. Although many aspects affect, the students' mathematical beliefs can be established through activities in the classroom. One effective way to gain Belief in solving problems is through teachers, textbooks, instructional strategies, and that the main use of the problems that exist around the student's learning activities [7]. During the math lesson, students not only learn math concepts and procedures, but they also learn how to interact in the classroom, they learn about the series of beliefs, and they learn how to behave in math.

The description above shows how important the belief is towards mathematics. Therefore, teachers play an important role in developing belief of students towards mathematics. What is the students believed, mostly based on the experience gained while studying mathematics [22]. Student of teacher candidates who became professional teachers should also have belief in mathematics. They should also be able to master all the material taught in school. The results of the initial survey showed that 62% of candidate teachers believed that they master the material of plane and 40% of students believed they didn't master the material of tangent to the circle.

Related with mathematical ability, a person who has the different background and the mathematical skills also has the capability of solving mathematical problems differently. The mathematical abilities affect the ability of solving a mathematical problem [19]. Students who have higher mathematics ability have high ability in solving mathematical problems, students with medium mathematical ability is quite good in problem-solving ability, and students who have lower math ability is not good in math problem-solving ability. The ability of students, especially related to the mathematical ability in a classroom, is usually divided into three groups, namely the ability level of high, medium and low. Based on the description above, researchers are interested in conducting research studies on belief of student with medium and low math ability in solving problems related to the tangent to the circle and its implications.

2. Research Methods

This type of research is qualitative-explorative conducted on students who have medium and low math ability. The data in this study is in the form of words or sentences and images. The subjects in this study are students who have medium and low math ability. Subject selection was based on mathematical ability test scores, consideration on grade point average of students and students' ability in oral and written communication. Instruments in this study were divided into two parts: 1) the researchers themselves as the main instrument and; 2) The assignment sheet. The data were collected using interviews and task-based think aloud method, students were asked to express clearly everything was thought out and asked to write directly. The research procedure is undertaken in phases as follows:

2.1. The preparation phase of research

The activities conducted by researchers were: 1) examine the theory of belief in solving math problems, 2) determine the location of research, 3) designing research instruments and 4) validate and revise the research instruments in accordance with a suggestion from the validator.

2.2. The implementation phase of research

Activities to be conducted at this phase were: 1) determining the subject of research in accordance with predetermined criteria; 2) collecting data through task-based interviews, video analysis and field notes; 3) analyzing the study data and triangulation. The collected data were analyzed by the following steps: (a) Data reductions, those are the selection process, focusing simplification, abstraction and transformation of raw data in the field. If there are invalid data, this data was collected separately which may be used as a complementary data or sideline findings; (b) the presentation of data, those are to classify and to identify the data soit is properly organized and categorized; (c) draw conclusions based on the results of exposure data. After the data presented with properly categorized, the next step is to infer or interpret the meaning of the exposure data. The data analysis is performed on any data obtained from each method of data collection (task analysis, interviews).

3. Main Result

Students with low mathematics ability who was the subject of this research are given the initials ADC, while the students with medium mathematics ability who was the subject of this research are given the initials IS. Based on research data it could be said that the subject of ADC believes that the mathematical problem given have moderate difficulty level, it was based because the subject not yet have an initial picture of problem solving and there are several ways and there are a lot phases to be solved, the time needed in problem solving was 12 minutes 18 seconds. Further subject ADC believes that the concept of tangent is used to find the length of the chain that is not attached to the wheel, while the concept of the circumference will be used to find the length of the chain attached to the wheel. The subject ADC believes that the problem can be solved in two ways but subject only mentions that that is one way of solving the same circumference of external tangents because it already has a description of way of solving problems such as the subject belief to the number of ways of solving problems. Here is interview excerpts with the subject ADC:

TR_11   : Why do you believe you can finish in 15 minutes?
ADC_11 : That is including calculations too
TR_12   : There are calculations as well.
ADC_12 : The calculation included big numbers too
TR_13   : The numbers are big so yeah, it took about 15 minutes huh?
ADC_13 : Yes
TR_14   : Do you believe 15 minutes huh?
ADC_14 : (nodding)
TR_15   : Well, it's okay. Then, in order to resolve the problem, how do you believe the concepts that can be used to solve this problem?
ADC_15 : it's that concept of the circle tangent, and also looking outside the tangent line, around the circle too.
TR_18   : Two yeah? The same tangents?
ADC_18 : Circumference
TR_19   : What circumference?
ADC_19 : Circles
TR_20   : Circumference of a circle. Are you convinced by that concept?
ADC_20 : Yes
TR_21   : Yes. Will be used for what the two concepts?
ADC_21 : To calculate the length of that chain that cuts off to the wheels.
TR_22   : Chain that cuts the wheel yeah. And what do you believe in relation of the concept, what was that... circumference of a circle with tangents? Relation in the context of problem solving?

ADC_22   : For the tangent… so the circles are different, so the tangents are… chain that is not attached in circles. Just a tangent, but if that is for the circumference of the circle it is the one that definitely sticks... stuck to the side of the circle
TR_23   : So it is used for it yeah?
ADC_23   : (nodding)

Subject ADC believed problem solving steps begin with finding the circumference of a half circle for the big circle and half circle for a small circle. Next is to search the length of external tangents between circles. Although the subject ADC has believed the concept, relation between concepts, step-by-step completion; the subject does not believe the right answers that would be obtained from a completion that will be done. Here are interview excerpts with the subject ADC:

ADC_34   : The... which use the outside tangent line and the circumference of a circle.
TR_35   : Outside tangent and circumference of a circle. You believe it huh?
ADC_35   : (Nodding)
TR_36   : How do you believe it? How must we use it?
ADC_36   : Because the formula can also be used (softly)
TR_37   : So, you believe that the way to resolve this problem in only one way, by looking for external tangents
ADC_37   : And the circumference
TR_38   : And the circumference of the circle yeah. Okay. Could you possibly elaborate the phases of completion?
ADC_38   : I could.
TR_39   : What has the first to be found please!
ADC_39   : ….
TR_40   : It's okay, just explain it
ADC_40   : The first is looking around the half-circle for the big circle and half circle for a small circle. Then looking for external tangents between the lengths of the circle.
TR_41   : After that
ADC_42   : summed up
TR_43   : summed up. First, looking for what?
ADC_43   : Circumference
TR_44   : Circumference of the big circle.
ADC_44   : and circumference of the small circle
TR_45   : Next, looking for tangent, after that
ADC_45   : summed up
TR_46   : It was the result of the solution?
ADC_46   : That's half of the circumference
TR_47   : Oh. Half of the circumference yeah, which was summed before, what will it be? Is that the solution?
ADC_47   : The solution

Subject IS believed that mathematical problems given have challenging or high difficulty level. It is based on the subject that does not have a preliminary description of problems resolution, and there are several ways, and there are also a lot of stages to be solved. For the beginning, the subject has not been able to determine how long it takes, but finally the subject IS believed that to be able to solve the problem will take between 25 minutes. Subject IS believed that the concept needed to solve the problem is tangent to the circle, then the outside tangent line, and the circumference of the circle. Furthermore, the concept of the tangent line will be used to find the length of chain that does not stick in the wheel, while the circumference of a circle concept will be used to find the length of the chain attached to the wheel. Subject IS believes that the problem can be solved by only one way that is solving the circumference of a circle and external tangents. It is because they already has an idea about problems solving just like the subject believed about the number of ways to solving problems. Subject IS believed the phases of problem solving begin with finding the circumference of a half circle for the

big circle and half circle for a small circle. Next is to search the length of external tangents between circles. Although the subject IS has believed the concept, relation between concepts, step-by-step of completion; the subject did not believe in the truth of the answers that would be obtained from a solution that will be done.

Table 1 Problem Solving Beliefs of Subject IS

| Descriptor	Beliefs in Problem Solving	Problem Solving Practices
Belief in the level of difficulty of the problem	Subject believes that the mathematical problem given has challenging or high difficulty level. It was based because the subject has not had an idea of the initial completion of the problem, there are several ways and phases to be solved	Problem solving practices showed that subjects experienced an error in representing mathematical problems given in the form of a sketch the image of mathematics. This indicates that the subject had difficulty in understanding mathematical problems.
Beliefs to be able to resolve the problem within a certain time	In the beginning the subject has not been able to determine how long it takes, but finally the subject IS believes that in order to solve the problem it takes between 25 minutes.	Problem solving practices showed that the time required approximately 16 minutes and 53 seconds
Beliefs about understanding concepts related to completion of problems	Subject IS believed that the concept needed to solve the problem is a tangent to the circle, then the outside tangent line, and the circumference of the circle. Furthermore, the concept of the tangent line will be used to find the length of chain that does not stick in the wheel, while the circumference of a circle concept will be used to find the length of the chain attached to the wheel.	Problem solving practices indicate that the subject IS used the concept of a tangent to look for the lenght of the-chain that does not stick in the wheel, while the circumference of a circle concept will be used to find the length of the chain attached to the wheel.
Belief in the planning of problem solving	Subject IS believed that the problem can be solved only by one way of solving that is the circumference of the circle and the external tangents because it already has an idea about problems solving such as the belief of subject about the number of ways of solving problems.	Problem solving practices indicate that the subject IS solving mathematical problems using the way circumference of the circle and the external tangents, but the results obtained are not appropriate because of the subject errors in representing problem in the form of a mathematical image.
	Subject IS believed phases of problem solving begins with finding the circumference of a half circle for the big circle and half circle for a small circle. Next is to search the length of external tangents between circles.	Problem solving practices show that the problem-solving phases of the subject IS are looking for the length of external tangents of the circle, looking for circumference of the half-circle for the big circle and half circle for the small circle.
Beliefs about the truth of the answers obtained	Although subject IS has believed the concept, the relations between concepts, step-by-step of completion; the subject does not believe in the truth of answers that would be obtained from a solution that will be done.	The worksheet of subject IS indicated that the subject has less precise because the initial error is made by the subject IS in representing mathematical problems in image form.

4. Discussion

Belief gives the opportunity to predict the decision, and what behaviors will be shown during the problem solving process [6]. Belief affects the success of the subject in the problem solving process (see, e.g., [3], [5], [13], [15], [16], [17], [20], and [21]), the argument is against the findings of this study. In the implications of beliefs in solving mathematical problems practice, the subject of research

(ADC and IS) basically do the same initial mistake of not being able to represent the mathematical problems into mathematical images (Figure 1). This is because the subjects draw a diameter line on the large circle and the small circle so when they draw a line connecting two points on each circle, the line is not tangent to touch both circles. Figure 2 below shows that line g_1 and g_2 can't touch these two circles. All three subjects represent that the line g_1 touches both circles (large and small), whereas the line g_1 touches small circles, and cuts big circle. This is not in accordance with the given mathematical problem.
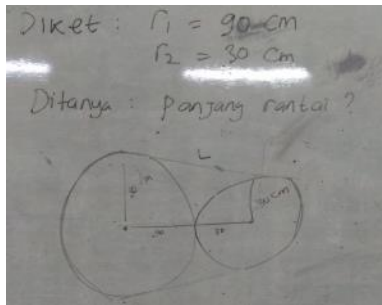
Subject ADC's working
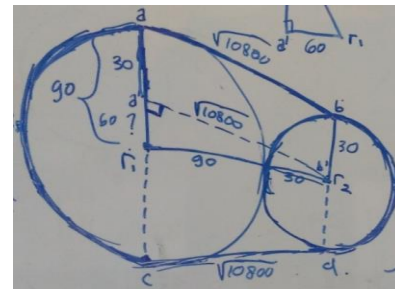
Subject IS's working

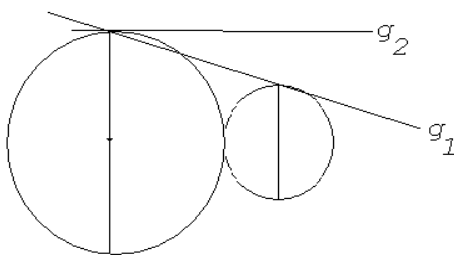Figure 1 Errors of Subject ADC and Subject IS

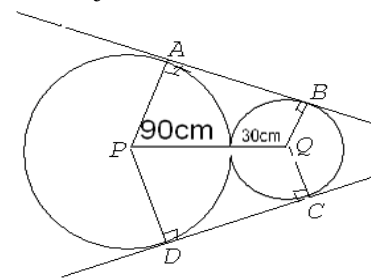
Figure 2. Illustration errors of both subjects

Figure 3. Sketch of the problem given

The subject should draw a tangent touching big circle and a small circle in line. Note that any tangent to the circle is definitely perpendicular to the radius of the circle. This is shown in Figure 3. Errors made by the subject were a conceptual mistake in understanding the concept of a tangent to the circle. It was identified from the conceptual knowledge of the subject about tangent to the circle is still weak. Conceptual knowledge can be considered as a knowledge network, i.e. a network in which the corresponding association is as prominent discrete pieces of information. This correlation includes the individual facts and propositions so that all the pieces of information are connected to multiple networks. Conceptual knowledge is not a stand-alone piece of information (isolated). Student understanding of conceptual knowledge is abstracted in the form of representation. Representation is the process of modeling concrete objects in the real world into abstract concepts or symbols [9]. Furthermore, Izsák, Andrew [11] revealed the representation refers to the object (artifact) produced by humans to think or convey information about some of the different contexts of such works. In this case, the error indicated when subject was not able to represent a problem in the form of a picture of the problem itself.

In this study, subject's beliefs have a negative effect in the problem solving process that failed to make efforts to solve the problem. In addition, the negative effects are also reflected in the lack of effort to avoid failure in the problem solving process. Belief is as internal acceptance which is assumed true by an individual, then determine how one perceives, intends, and acts against any fact, object or event; in this case is the mathematical problem solving. Our results indicated that there was

complex relationship between the student belief system and the implications of that problem solving. If we consider the beliefs about problem solving, it was not possible to establish a causal relationship between particular beliefs and problem-solving activity (or vice versa). This is different to Lerch and Jonassen ([12], [14]) which stated student achievement in solving the problem is directly proportional to their attitudes and beliefs towards solving the problem. In his study [14] four students who became the subject of the research found that the decision made by the students during the search for solutions to problems depend on the material content knowledge and belief system of students to mathematical problems.

The study provided a hint that the lecturers should be able to develop the beliefs of students in problem-solving activities through the provision of correct concepts, providing a problem-based assignments to develop students' ability. Beliefs about mathematics should be studied to make better understanding how students learn to solve problems. Improving students' beliefs that mathematics is beneficial will increase motivation and achievement. Someone who believes the problem should be solved in five minutes or less concluded that one should give up on the problem that cannot be solved in five minutes. Students who do not have the motivation to solve the problem that they cannot solve quickly will have difficulty in college level mathematics. For this reason, it is important to consider the students' beliefs about their ability to solve problems that take more than one or two minutes to complete.

5. Conclusion

Based on research data it could be said that: the subject with low mathematics ability believes that the mathematical problem given has moderate difficultylevel. This is based because the subject does not have a preliminary description of problems resolution, and there are several ways, and there are also a lot steps to be solved, time needed in problem solving is 12 minutes 18 seconds. Further subjects believed the concept of tangent is used to find the length of the chain which not attached to the wheel, while the concept of the circumference will be used to find the length of the chain attached to the wheel. The subject believes that the problem can be solved in two ways but the subject is only able to provide one way, namely the solution of the circumference of a circle and external tangents. It is because they already have an idea how to solve problems the same as the subjects believe to the number of ways of solving problems.

Subject believed phases of problem solving begin with finding the circumference of a half-circle for big circle; then half-circle for a small circle. Next is to search the length of external tangents between circles. Although the subject has convinced the concept, relation between concepts, step-by-step completion; the subject is not sure of the answers that would be obtained from a completion that will be done. Subjects with medium mathematics ability convinced that the given math problems have high difficulty level. It is based because the subject does not have a preliminary description of problems resolution, and there are several ways, and there are many steps to be solved. In the beginning the subject has not been able to determine how long it takes, but eventually the subject believes that in order to resolve the problem it takes between 25 minutes.

Subject IS believed that the concepts needed to solve the problem are a tangent to the circle and outer tangents, and the circumference of the circle. Furthermore, the concept of the tangent line will be used to find the length of chain that does not stick in the wheel, while the circumference of a circle concept will be used to find the length of the chain attached to the wheel. Subject believed that the problem can be solved only in one way that is solving the circumference of a circle and external tangents. It is because they already have an idea about solving problems the same as subject belief's to the number of ways of solving problems. Subject believed that phases of problem solving begin with finding the circumference of a half circle for big circle and half circle for a small circle. Next is to search the length of external tangents between circles. Although the subject has convinced the concept, relation between concepts, step-by-step completion; the subject is not sure of the answers that would be obtained from a solution that will be done.

Based on the conclusions from these results, it is recommended: 1) lecturers should understand the beliefs of candidate teachers in problem solving, in order to provide necessary help for students to improve their ability in problem solving, and 2) the study of students beliefs in this research is still limited, therefor it needs research to study more deeply with the other issues.

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