On Gamma Labeling of Double Cones Graph

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Abstract. Let G be a simple and finite graph with $|V(G)| = p$ and $|E(G)| = q$. A γ – labeling of G is a one-to-one function $f: V(G) \to \{0, 1, 2, \ldots, q\}$ that induces a labeling $f': E(G) \to \{0, 1, 2, \ldots, q\}$ of the edges of G defined by $f' = |f(u) - f(v)|$ for each edge $e = (u, v)$ of G. The value of γ – labeling is $val(f)$ and defined as $val(f) = \sum E_{(e \in E(G))} f'(e)$. The maximum and minimum values of γ – labeling of graph G are $val_{max}(G) = max\{val(f)\}$ and $val_{min}(G) = min\{val(f)\}$ where f is γ -labeling of graph G.
In this paper we find γ – labeling on a double cones graph DC_n.
Keywords : γ – labeling, value of a γ – labeling, a Double Cones graph

1. Introduction

In 2005, Chartrand [1] introduced a γ – labeling as follow. For a graph G of order p and size q, a γ – labeling of G is a one-to-one function $f: V(G) \to \{0, 1, 2, \ldots, q\}$ that induced a labeling $f': E(G) \to \{1, 2, \ldots, q\}$ of the edges of G defined by

$$f' = |f(u) - f(v)| \text{ for each edge } e = (u, v) \text{ of } G.$$

Therefore, a graph G of order p and size q has a γ – labeling if and only if $q \geq p - 1$.

The maximum value of a γ – labeling of graph G is defined as

$$val_{max}(G) = max\{val(f): f\ is\ a\ \gamma - labeling\ of\ G\},$$

while the value of a γ – labeling of graph G is

$$val_{min}(G) = min\{val(f): f\ is\ a\ \gamma - labeling\ of\ G\}\ .$$

Chartrand [2] found a γ – labeling of a path, a cycle, a complete graph, a star and a tree graph. In 2007, Okamoto *et al.*[7] found a γ – labeling of oriented graph, and Roswitha *et al.*[8] work on a γ – labeling of petersen graph. Indriati and Roswitha [4] researched a γ – labeling of a double star graph, a firecracker and sun graph and then Indriati[3] applied a γ – labeling of a wheel graph, fan graph and helm graph for a pattern of cooking oil distribution. The maximum and minimum values of γ – labeling of an umbrella graph, banana tree graph and a friendship graph were researched by Nanang [10] and Rosyida [9]

2. γ - Labeling of a Graph

The notation and terminology on this paper follow Red [] and Chartrand et al [].

Definition 2.1. *Let n (order) and m (size) of a graph G, with* $1 \leq n-1 \leq m \leq \binom{n}{2}$, *let* $S = \{1,2,\cdots,n-1\}$ *and* $\alpha(n,m) = max\{k \in S : \sum_{i=1}^{k}(n-1) \leq m\}$.

Proposition 2.1. *If G is a connected graph of order n and size m with* $\alpha(n,m) = k$, *then* $val_{min}(G) \geq \binom{k+1}{2}\left(n + \frac{k+2}{3}\right) + (m-nk)(k+1)$

3. Main Results

Definition 3.1 *A Double Cones Graph,* DC_n *is the join of cycle* C_n *and complement* K_2 *, and it can be representated as* $DC_n = C_n + \overline{K}_2$, *for* $n \geq 3$.

Teorema 3.1. *For every integer* $n \geq 3$, $val_{max}(DC_n) = \left\lfloor \frac{11n^2}{2} \right\rfloor$

Proof. Let f be a γ – labeling of double cones graph, DC_n , then it has γ maximum labeling. Let u_i , $i = 1,2$ are the label of vertex in the center of DC_n and v_i , $i = 1,2,\cdots,n$ are the label of vertex in the cycle C_n.

We devide the proof into two cases.

Case 1, n is even and $n \geq 3$

$$f(u_i) = i - 1 \qquad i = 1, 2$$

$$f(v_i) = \begin{cases} 3n - \frac{1}{2}(i-1), & i = 1,3,5,\cdots,n-1 \\ \frac{1}{2}(5n - i + 2), & i = 2,4,6,\cdots,n \end{cases} \qquad (1)$$

Based on Definition[3.1] and Equation [1]

$$val_{max}(DC_n) = val\big(f(u_1,v_i)\big) + val\big(f(u_2,v_i)\big) + val\big(f(v_i,v_{i+1})\big) + val\big(f(v_1,v_n)\big)$$

$$= \Big((3n-0) + (3n-1-0) + (3n-2-0) + \cdots + \left(3n - \tfrac{1}{2}n + 1 - 0\right)\Big)$$

$$+ \Big(\left(\tfrac{1}{2}(5n) - 0\right) + \left(\tfrac{1}{2}(5n-2) - 0\right) + \left(\tfrac{1}{2}(5n-4) - 0\right) + \cdots + \left(\tfrac{1}{2}(5n-n+2) - \right.$$

$$0\Big)\Big) + \Big((3n-1) + (3n-1-1) + (3n-2-1) + \cdots + \Big(3n - \tfrac{1}{2}n + 1 - 1\Big)\Big) + \Big(\Big(\tfrac{1}{2}(5n) - 1\Big) + \Big(\tfrac{1}{2}(5n-2) - 1\Big) + \Big(\tfrac{1}{2}(5n-4) - 1\Big) + \cdots + \Big(\tfrac{1}{2}(5n - n + 2) - 1\Big)\Big) + \Big(\Big(3n - \tfrac{5n}{2}\Big) + \Big(3n - 1 - \tfrac{5n}{2}\Big) + \Big(3n - 1 - \Big(\tfrac{5n-2}{2}\Big)\Big) + \Big(3n - 2 - \Big(\tfrac{5n-2}{2}\Big)\Big) + \Big(3n - 2 - \Big(\tfrac{5n-4}{2}\Big)\Big) + \cdots + \Big(3n - \Big(\tfrac{n-1}{2}\Big) - \Big(\tfrac{4n+2}{2}\Big)\Big)\Big) + \Big(3n - \Big(\tfrac{4n+2}{2}\Big)\Big)$$

$$= \frac{n}{4}\Big(3n + \frac{5n}{2} + 1\Big) + \frac{n}{4}\Big(\frac{5n}{2} + \frac{4n+2}{2}\Big) + \frac{n}{4}\Big(3n - 1 + \frac{5n}{2}\Big) + \frac{n}{4}\Big(\frac{5n-2}{2} + \frac{4n+2}{2} - 1\Big) + \Big(\frac{n^2-n}{2} - \Big(\frac{n-2}{2}\Big)\Big) + (n-1)$$

$$= \Big(\frac{20n^2+4n}{8}\Big) + \Big(\frac{20n^2-4n}{8}\Big) + \Big(\frac{4n^2-8n+8}{8}\Big) + \Big(\frac{8n-8}{8}\Big)$$

$$= \frac{11n^2}{2} \qquad (2)$$

Case 2, n is odd and $n \geq 3$

$$f(u_i) = i - 1, \qquad i = 1, 2$$

$$f(v_i) = \begin{cases} 3n - \frac{1}{2}(i-1), & i = 1, 3, 5, \cdots, n \\ \frac{1}{2}(5n - i + 1), & i = 2, 4, 6, \cdots, n-1 \end{cases} \qquad (3)$$

According to Definition[3.1] and Equation [3]

$$val_{max}(DC_n) = val\big(f(u_1, v_i)\big) + val\big(f(u_2, v_i)\big) + val\big(f(v_i, v_{i+1})\big) + val\big(f(v_1, v_n)\big)$$

$$= \Big(\frac{20n^2+4n}{8}\Big) + \Big(\frac{20n^2-4n}{8}\Big) + \Big(\frac{4n^2-4n}{8}\Big) + \Big(\frac{4n-4}{8}\Big)$$

$$= \frac{11n^2-1}{2} \qquad (4)$$

From $val_{max}(DC_n)$ (2) and $val_{max}(DC_n)$ (4) we found

$$val_{max}(DC_n) = \left\lfloor \frac{11n^2}{2} \right\rfloor$$

Teorema 3. 2. *For every integer $n \geq 3$, then*

$$val_{min}(DC_n) = \begin{cases} 3n+7, & n=3 \\ \left\lfloor \dfrac{n^2+8n+4}{2} \right\rfloor, & n>3 \end{cases}$$

Proof.

Let f is a γ – labeling of double cones graph, DC_n , then it has graph have γ minimum labeling. The minimum labeling of DC_n consists of three cases.

Case 1, $n = 3$.

$$f(u_i) = (n+1)(i-1), \qquad i = 1,2$$

$$f(v_i) = i, \qquad i = 1,2,3 \qquad (5)$$

According to Definition[1] and Equation (5)

$$val_{min}(DC_n) = val(f(u_1,v_i)) + val(f(u_2,v_i)) + val(f(v_i,v_{i+1})) + val(f(v_1,v_n))$$
$$= 3\mathrm{n}+7 \qquad (6)$$

Case 2, n is odd and $n > 3$

$$f(u_i) = \frac{n-1}{2} + 2i - 2, \qquad i = 1,2$$

$$f(v_i) = \begin{cases} i-1, & i = 1,2,3,\cdots,\frac{n-1}{2} \\ i, & i = \frac{n+1}{2} \\ i+1, & i = \frac{n+1}{2}+1, \frac{n+1}{2}+2, \cdots, n \end{cases} \qquad (7)$$

Based on Definition[1] and Equation (7)

$$val_{min}(DC_n) = val(f(u_1,v_i)) + val(f(u_2,v_i)) + val(f(v_i,v_{i+1})) + val(f(v_1,v_n))$$
$$= \frac{n^2+8n+3}{2} \qquad (8)$$

Case 3, n is even $n > 3$

$$f(u_i) = \frac{n}{2} + i - 1, \qquad i = 1,2$$

$$f(v_i) = \begin{cases} i-1, & i = 1,2,3,\cdots,\frac{n}{2} \\ i+1, & i = \frac{n}{2}+1, \frac{n}{2}+2, \quad \frac{n}{2}+3, \cdots, n \end{cases} \qquad (9)$$

Based on Definition1[1] and Equation (9)

$$val_{min}(DC_n) = val(f(u_1,v_i)) + val(f(u_2,v_i)) + val(f(v_i,v_{i+1})) + val(f(v_1,v_n))$$

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$$= \frac{n^2+8n+4}{2} \quad (10)$$

From $val_{min}(DC_n)$ (6), $val_{min}(DC_n)$ (8) and $val_{min}(DC_n)$ (10), we obtain

$$val_{min}(DC_n) = \begin{cases} 3n+7, & n = 3 \\ \left\lfloor \frac{n^2+8n+4}{2} \right\rfloor, & n > 3 \end{cases} \quad (11)$$

3.1 A Bound on the Minimum Value of γ - Labeling of a Double Cones Graph in Term of Its Order and Size.

Now, by using Definition 2.1 and Proposition 2.1 it is clear that .

$$val_{min}(DC_n) \geq \binom{k+1}{2}\left(n + \frac{k+2}{3}\right) + (m - nk)(k+1)$$

Figure 1. illustrates on gamma labeling of Double Cones, DC_7

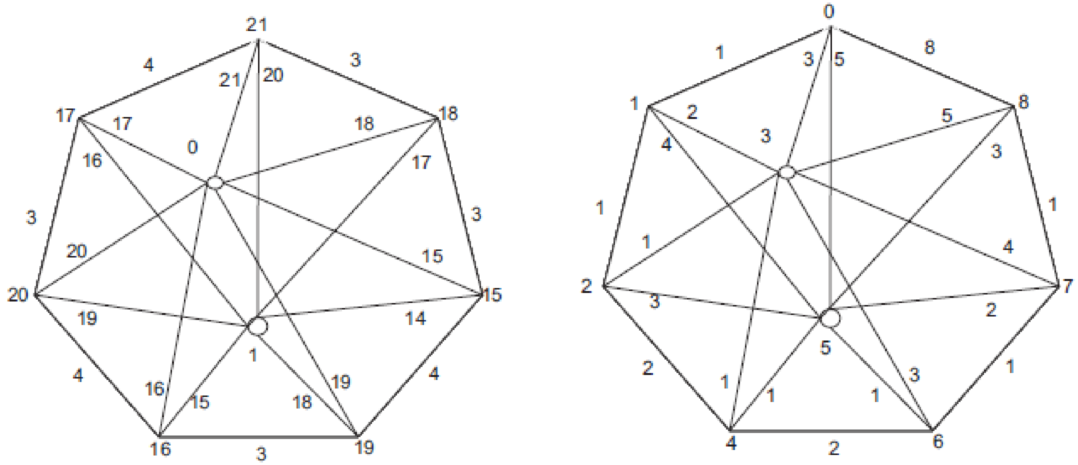

Figure 1. $val_{maks}(DC_7) = 269$, $val_{min}(DC_7) = 54$

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References

[1] G. Chartrand, D. Erwin, D.W. Vander Jagt and P.Zhang, γ-labeling of graph, *Bull. Inst. Combin. Appl.* **(44)** (2005a) pp. 51-68.

[2] G. Chartrand, D. Erwin, D.W. Vander Jagt and P.Zhang, γ-labeling of *Trees*, *Discussiones Mathematicae Graph Theory* **(25)** (2005b) pp. 363-383

[3] D. Indriati, Strategi γ-labeling pada Graf Roda, Graf Kipas dan Graf Helm untuk Pola Distribusi Minyak Goreng, *Penelitian DIPA, FMIPA UNS*, 2000

[4] D. Indriati and M. Roswitha, On γ-labeling of Double Star, Firecracker and n-sun Graph, *Proceeding of ISSTEC*, Universitas Islam Indonesia, 2009

[5] A. Kotzig, A. Rosa, Magic valuation of finite graph, *Cand. Math. Bull.* **13** (1970) 451 – 461

[6] A. Kotzig, A. Rosa, Magic valuations of complete graphs, *Pull CRM*- 175 (1972). A, 2009

[7] F.Okamoto, P.Zhang, and V. Saenpholphat, On γ-labeling of Oriented Graphs, *Mathematica Bohemica*, **132** (2)**, (**2007), pp.185-2003.

[8] M. Roswitha, D. Indriati and T. A. Kusmayadi, On γ-labeling of Petersen Graphs, *Proceeding SEAMS GMU,* **(**2007).

[9] E. M. Rosyida, Nilai Maksimum dan Minimum Pelabelan-γ pada Graf Pohon Pisang dan Persahabatan, Skripsi, UNS, Surakarta, 2009

[10] N. Saputro, Penentuan Nilai Maksimum dan Minimum Pelabelan-γ pada Graf Umbrella,, Skripsi, UNS, Surakarta, 2009.

[11] Redl, Timothy A., *Graceful Graphs and Graceful Labelings : Two Mathematical Programming Formulations and Some Other New Results,* Congressus Numerantium **Vol. 164** (2003), pp. 17-31