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Student Misconceptions in Solving Real Analysis Problem Based on Reasoning Framework

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Abstract. Real Analysis is the most important subject in mathematics. Student must have good reasoning ability in Real Analysis learning. This study aims to describe the misconception of students in solving Real Analysis problem based on reasoning framework. This is a qualitative descriptive research with 24 students of Mathematics Education Department, Universitas Muhammadiyah Surakarta who enrolled Real Analysis as participants. The data was collected using test, interview and documentation. The technique of data analysis conducted using the flow-method that consist of data reduction, data display and verification. The result shows that most student make misconceptions in analysis, synthesis, verification and generalization aspects in solving of Real Analysis problem.

Keywords: misconception, student, real analysis

1. Introduction

Real Analysis is an analysis branch course underlying Differential Calculus and Integral Calculus courses. The principles, rules, properties, as well as proof of the truth of the basic concepts that build the structure of mathematical knowledge are learned in this course. Bartle [1] stated that *real analysis is known as the body of mathematics.* In learning of Real Analysis course, each student needed to master the concept, including the definition, properties, theorem, and their proofs since they are each related to another. Hence, the students should have good reasoning skills in studying Real Analysis.

Mullis, et al [5] stated that the reasoning is the highest cognitive domain after knowing and applying. In reasoning, the students are required to think logically and systematically. Reasoning is a means of thinking in constructing mathematical knowledge by using knowledge, ideas, concepts, rules, nature, or mathematical principles. Mullis, et al [5] formulated the reasoning as the ability to analyze, generalize, synthesis, verification, and resolve non-routine problems.

Based on the teaching experience in Real Analysis course for the last three years, the student ability in this course is still low. There are less than 50% of students who get good grades (B) or more. This shows that the student still have a misconception in solving Real Analysis problems. Misconception is a mistaken idea from a misunderstanding of something. There have been several studies about reasoning and student misconception in mathematics problems. Gunha [2] examined the ability of reasoning which includes aspects of analysis, synthesis, generalization, verification and reasoning non routine. The results of this research showed that the reasoning abilities of students in the field of geometry give different results for each aspect. While Syawahid [8] described the formal thinking ability of students in the subject of real analysis is in enough category. This study aims to analyze and describe the student misconceptions in solving Real Analysis problems in Mathematics Education Program, Faculty of Teacher Training and Education, Universitas Muhammadiyah Surakarta based on reasoning framewok. This study used synthesis, analysis, verification and generalization as the aspects of reasoning in student misconseption.

2. Method

This is a qualitative descriptive research with the students fourth semester in Mathematics Education Program, Universitas Muhammadiyah Surakarta who enrolled Analysis Real course in 2012-2013 academic year as the subject of the research . The data collection using documentation and interview. Documentation used to collect the data of student misconception in solving Real Analysis problem, based on the result of middle test examination 2012-2013, while the interview was conducted to clarify student misconception based on reasoning framework. Data validation using method triangulation. The technique of data analysis using the flow-method that consist of data reduction, data display and verification refers to the stage of qualitative data analysis by Miles and Huberman [9].

3. Result and Discussion

The given test consist of 6 problems: the ordered property of R, absolute value inequality, bounded sequence, convergence sequence, Cauchy sequence and Monotone Convergence Theorem. The student test results were analyzed based on reasoning framework that includes analysis, synthesis, verification and generalization. These following paragraphs describe student misconception in solving real analysis based on the reasoning aspects.

3.1. Students' misconceptions Based on Analysis Aspect

Analysis in this study is the students' ability to determine the relationships between variables or objects in mathematical situations. To determine the analytical skills, students was given the test about the ordered properties of R and triangle inequality on question number 1 and 2 respectively. Based on the analysis of the students answer, it found that 29.17% students have been able to do the analysis correctly to the question number 1. The rest, there is no student can analyze and there are students can analyze but they make misconceptions. The misconceptions that occur on student answer to question number 1 is when known $a,b,c,d \in R$ with $a < b \wedge c < d$ (Table 1).

Table 2. Type of Student Misconception on Analysis Aspect

Known	Student Analysis Result
$a,b,c,d \in R$ dengan $a < b \wedge c < d$.	$a < b \rightarrow b - a \in R$ $c < d \rightarrow d - c \in R$
	$a < b \rightarrow a - b \in P$ $c < d \rightarrow c - d \in P$
	$a < b \rightarrow a - b \in R$ $c < d \rightarrow c - d \in R$
	$a < b \in P \rightarrow a - b \in P$ $c < d \in P \rightarrow c - d \in P$
	$a < b \in P \rightarrow b - a \in P$ $c < d \in P \rightarrow d - c \in P$
	$a < b = a - b$ $c < d = c - d$

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This indicates that students still make misconceptions in determining relationships between variables or objects that in the order properties of R, since the students correct answer should be: $\boldsymbol{a} < \boldsymbol{b} \rightarrow \boldsymbol{b} - \boldsymbol{a} \in \boldsymbol{P}$ **and** $\boldsymbol{c} < \boldsymbol{d} \rightarrow \boldsymbol{d} - \boldsymbol{c} \in \boldsymbol{P}$.

Based on the students answer result, it found that 12.15% students have been able to do the analysis correctly to the question number 2. The rest, students did not perform the analysis at all. To verify $|a+b| \leq |a| + |b|$, students should have conducted an analysis of the known $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{R}$ $\rightarrow -|\boldsymbol{a}| < \boldsymbol{a} < |\boldsymbol{a}|$ dan $-|\boldsymbol{b}| < \boldsymbol{b} < |\boldsymbol{b}|$, but students did not do that. In other words, student still can't determine the relationships between objects or variables on the absolute value topic

3.2. Students' misconceptions Based on Synthesis Aspect

In this study, synthesis is the ability to make a connection between the different elements and to connect mathematical ideas associated, also to combine the facts, concepts and mathematical procedures to determine the results and to combine results to obtain more results. Synthesis ability can also be seen from the student answer in question number 1 and 2. Based on the students answer who can analyze correctly, then the synthesis capability will be seen.

Figure 1 shows that the student first can do analysis $\boldsymbol{a} < \boldsymbol{b} \rightarrow \boldsymbol{b} - \boldsymbol{a} \in \boldsymbol{P}$ **and** $\boldsymbol{c} < \boldsymbol{d} \rightarrow \boldsymbol{d} - \boldsymbol{c} \in \boldsymbol{P}$ so they can make the connection between $\boldsymbol{b} - \boldsymbol{a} \in \boldsymbol{P}$ and $\boldsymbol{d} - \boldsymbol{c} \in \boldsymbol{P}$ becomes $(\boldsymbol{b} - \boldsymbol{a})(\boldsymbol{d} - \boldsymbol{c}) \in \boldsymbol{P}$, and then solve it. While figure 2, students first can do analysis $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{R} \rightarrow -|\boldsymbol{a}| < \boldsymbol{a} < |\boldsymbol{a}|$ and $-|\boldsymbol{b}| < \boldsymbol{b} < |\boldsymbol{b}|$ so they can combine the facts that $-|\boldsymbol{a}| \leq \boldsymbol{a} \leq |\boldsymbol{a}|$ and $-|\boldsymbol{b}| \leq \boldsymbol{b} \leq |\boldsymbol{b}|$ and combining the results obtained with the existing concept (absolute value property) to gain more result that will be proved.

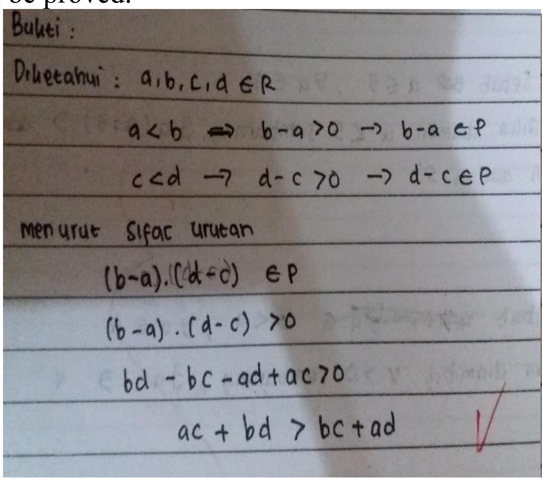

Fig.1. Right Synthesis Number 1

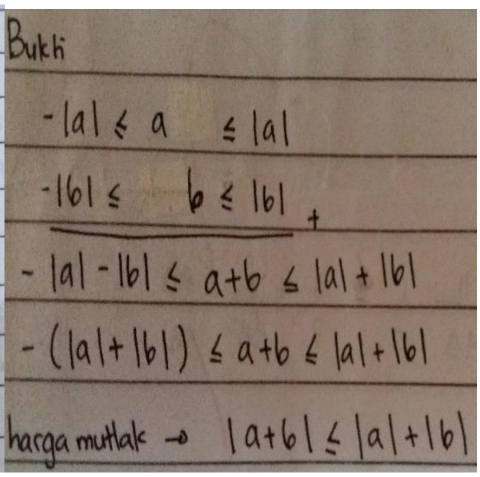

Fig.2. Right Synthesis Number 2

Since students who can perform the synthesis are students who have done the analysis correctly, so there are also 29.17% and 12.5% students do the right synthesis for question number 1 and number 2 respectively. The synthesis ability of the rest students have not been analyzed because their analysis are incorrect.

3.3. Students' misconceptions Based on Verification Aspect

Verification in this study is the ability to prove based on the results or the properties of mathematical concepts that are already known. To determine the verification ability of the students, given a sequence of real number $Y = (y_n), y_n = \frac{3n+2}{7n+1}, n \in N$, students are required to show that the sequence is bounded, convergent and Cauchy in questions number 3,4 and 5 respectively.

Based on the analysis of the student answers, 37.5% of the students can verify question number 3 correctly; 25% of students can verify number 4 and 20.83% of the students can verify number 5. The rest, there are students can't verify and also there are students can verify but they make misconceptions. Misconceptions that occur on student answer to question no 3, 4 and 5 can be seen from how students write down what is to be proved, as like in Table2.

Table 2. Type of Student Misconception in Verification Aspect

Bounded Sequence	Convergence Sequence	Cauchy Sequence
$\exists M > 0 \ni \left\lvert\frac{3n+2}{7n+1}\right\rvert \le M, \forall n \exists N$	$\forall \varepsilon > 0, \exists k = k(\varepsilon) \in N, \forall n \in N$	$\forall \varepsilon > 0, \exists H \in N, \forall n \ge H$
$\exists M > 0 \ni y_n \le M, \forall n \in N$	$\forall \varepsilon > 0, \exists k = k(\varepsilon) \in N, \forall n \ge k$	$\forall \varepsilon > 0, \exists H \in N, \forall n, m \ge k$
$\exists M > 0 \ni \frac{3n+2}{7n+1} \in N, \forall n \in N$	$\forall \varepsilon > 0, \exists k = k(\varepsilon) \in N \ni \forall \varepsilon \ge k$	$\forall \varepsilon > 0, \exists H \in N, \forall m, n \ge H$
$\exists M > 0\ 3 \left\lvert\frac{3n+2}{7n+1}\right\rvert \le M, \forall n \in N$	$\forall \in > 0, \exists k = k(\varepsilon)\, \varepsilon N\ 3\ \forall n \ge k$	$\exists H > 0 \ni \forall m, n \ge H$
$\exists M > 0 \ni \lvert y_n \rvert \le M, \forall n \in N$	$\exists k = k(\varepsilon) \in N, \frac{1}{k} < \frac{49}{11}\varepsilon$	$\forall \varepsilon > 0, \exists H \in N\ 3\ \forall n, m \ge H$
$\forall \varepsilon > 0, \exists M > 0 \ni \left\lvert\frac{3n+2}{7n+1}\right\rvert \le M, \forall n \epsilon N$	$\forall \varepsilon > 0\ \ k = k(\varepsilon) \ni$ $\forall n \ge k: \left\lvert\frac{3n+2}{7n+1} - \frac{3}{7}\right\rvert < \varepsilon$	$\forall \varepsilon > 0, \in H \in M \in \forall n, m \ge M$
$\exists M, \in \left\lvert\frac{3n+2}{7n+1}\right\rvert \le M, \forall n \in N$	$\forall \varepsilon > 0\ \ \exists k = k(\varepsilon) \in N \ni$ $\forall n \in k: \left\lvert\frac{3n+2}{7n+1} - \frac{3}{7}\right\rvert < \varepsilon$	$\forall \varepsilon > 0\ H = H(\varepsilon) \epsilon N \ni \forall \ge H$
$\forall M > 0, \exists \left\lvert\frac{3n+2}{7n+1}\right\rvert \le M, \forall n \in N$		$\forall \varepsilon > 0, \exists H \in N \ni \forall n, m \ge k$
$\ni M > 0 \ni \left(\frac{3n+2}{7n+1}\right) \in M, \forall n \in N$		$\forall \varepsilon > 0, \exists H = H(\varepsilon) \in N \ni \forall n, m \in H: \left\lvert\frac{3n+2}{7n+1} - \frac{3m+2}{7m+1}\right\rvert <$
		$\forall \varepsilon > 0, \exists H = H(\varepsilon) \in N \ni \forall n, m \ni N: \left\lvert\frac{3n+2}{7n+1} - \frac{3m+2}{7m+1}\right\rvert <$

The student misconceptions in proving of the sequence that is bounded, convergent and Cauchy occur when students write down what must be proved. Judging from the way students write mathematical symbols, it appears that students still have not mastered the concept of a bounded, convergent and Cauchy sequence. Based on interviews with students, in general, students still have difficulty writing down the symbol because it is not familiar with the concept of making it less able to interpret the symbols. As a result, in the writing of the symbol is still much upside.

3.4. Students' misconceptions Based on Generalization Aspect

Generalization in this study is the ability to expand the domain so that the results of mathematical thinking or problem solving can be applied in a more general or wider. To determine the ability of students in generalization, the student given a sequence $X = (x_n)$ with $x_1 = 0,\ x_{n+1} = \frac{1}{4}(2x_n + 8), \forall n \in N$. They should be able to verify that this sequence is convergent by using monotone convergence theorem.

Based on the results of the analysis of the answers to student work, 15.28% of the students have been able to generalize correctly. The misconceptions in generalization aspect occurs when students prove a bounded and monotonic increasing sequence with mathematical induction steps is still not right, can't make the conclusion that if the sequence is bounded and monotone then it is convergent, and not understand that the tail-1 of the sequence, $X = X_1 = (x_{1+n} : n \in N)$ is convergent if and only if the sequence convergent. There are many the students who wrote the symbol tail-1 of sequence X with the first element of X that is x_1 . This indicates that the student still have misconceptions in the monotone convergence theorem. Based on the interviews with the students, besides due to less familiar with symbols, the students also argued for less scrupulous when do.

The results of this study supported with the results of Sanapiah research [6], Syawahid [8] and Sari [7]. Gunha [2] in his study also concluded that the reasoning abilities of students on aspects of analysis, synthesis, generalization, verification and non routine reasoning in geometry give different results for each aspect. While Muzangwa [4] stated the results from tests show that some learners had a weak background of mathematics such as a low pass at “A” level and yet most pre-calculus concepts such as algebra, limit, basic differentiation and integration have a strong link with High School mathematics. Performance in the first test on basic calculus concept also reflect the same problem, and some errors on limit of functions of a single variable were observed again in limits of functions of several variables. Learners also seem to have difficulties with analytic concepts. Zakaria,et.al [3] also conclude that the students'error in solving quadratic equation was due to their weakness in mastering topics such as algebra, fractions, negative numbers and algebraic expansions.

4. Conclusion

The result shows that based on reasoning framework, most student make misconceptions in analysis, synthesis, verification and generalization aspects in Real Analysis problem solving. Based on the analysis of the students answer, it found that 29.17% students can analyze correctly in the ordered property of R, 12.15% students can analyze correctly in absolute value problem. The misconception in analysis aspect occurs when student can't determine the relationship between elements of R that known in the ordered property and the absolute value problems. Since students who can perform the synthesis are students who have done the analysis correctly, so there are also 29.17% and 12.5% students do the right synthesis. The misconceptions in synthesis aspect occurs when students are not able to make connections between different elements and connect mathematical ideas associated, also combine facts, concepts and mathematical procedures to determine the results and combine results to

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obtain more results. The number of student who can verify a bounded, convergence and Cauchy sequence are 37.5%, 25% and 20.83% respectively. The student misconceptions in verification aspect occurs when students can write down the statement what must be proved in bounded, convergent and Cauchy sequence by using the definitions correctly. Finally, there are 15.28% of the students have been able to generalize correctly. The misconceptions in generalization aspect occurs when students prove a bounded and monotonic increasing sequence with mathematical induction steps is still not right, can't make the conclusion that if the sequence is bounded and monotone then it is convergent, and not understand that the tail-1 of the sequence, $X = X_1 = (x_{1+n}: n \in N)$ is convergent if and only if the sequence convergent.

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