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# The motion of spinors in the vicinity of spherical star and the magnetic field of non-homogeneous spherical bodies

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**Abstract:** In this article, we developed the spinors connections from Schwarzschild metric and make use of the spinor connections in geodesics in order to obtain the equation of spinor in the vicinity of spherical star. The gravitational scalar potential for non-homogeneous spherical body was use to developed the magnetic field of different regions of some non-homogenous spherical bodies.

**Keyword:**

## 1. Introduction

In 1915, Albert Einstein developed his theory of general relativity, having earlier shown that gravity does influence light's motion. Only a few months later, Karl Schwarzschild found a solution to the Einstein field equations that describes the gravitational field of a point mass and a spherical mass (Schwarzschild, 1999) which shows how gravity affect photon and other mass.

The motion of particle and photon around gravitational field has been a progressive unfolding mystery. The Planetary equations of motion and equations of motion of photons in the vicinity of spheroids have been derived by Chifu and his equations are having additional spheroidal terms not found in Schwarzschild's space-time (Chifu et al., 2008; Chifu & Howasu, 2008; Chifu & Lumbi, 2008). Also Bulus Timothy developed the explicit equations of test particles of non-zero mass; test particles along the equatorial plane and relativistic equation of photon in the vicinity of an ellipsoidal star is differential equation of motion (Bulus, 2022).

In most articles and research work done by many. Those work focus on the motion of either photon or point mass which moves around massive bodies or strong gravitational field. This article is going to provide the equation of motion of spinor in gravitational field.

A planetary magnetic field, or the absence of such a field, informs us about the internal structure of a body and its thermal evolution. Together with the gravitational field, the magnetic field provides a window into the interiors of bodies that we can only probe from a distance. The magnetic fields recorded and preserved in the crusts of planets and satellites also provide a window into the histories of the bodies. (Schubert & Soderlund, 2011). The Earth and most of the planets in the Solar system, as well as the Sun and other stars, all generate magnetic fields through the motion of electrically conducting fluids (Weiss, 2002). The idea given by Weiss, Jordan and Finlay clearly shows that magnetic field varies from one region to another, whereas most researchers model the earth and other planets to be homogenous in order to determine their magnetic field.

This article also developed the equation that is used to determine the magnetic field of non-homogeneous spherical bodies with three regions and also investigate the magnetic of some planets.

## 2. Theoretical Analysis

The Schwarzschild's metric is the solution of Einstein's field equations exterior to a static homogenous spherical body (Gu, 2021) given as:

$$g_{\mu v} = diag\big( -A(r), -r^2, -r^2 sin^2\theta, B(r)\big) \quad (1)$$

**The affine connections can be determine as follows**

The metric tensors are given as

$$g_{44} = B(r),\ \ g_{11} = -A(r),\ \ g_{22} = -r^2,\ \ g_{33} = -r^2 sin^2\theta \quad (2)$$

$$g^{44} = \frac{1}{B(r)},\ \ g^{11} = -\frac{1}{A(r)},\ \ g^{11} = -\frac{1}{r^2},\ \ g^{11} = -\frac{1}{r^2 sin^2\theta} \quad (3)$$

With A and B as yet undetermined functions of r. Note that if A or B is equal to zero at some point, the metric would be singular at that point. On each hyper surface of constant t, constant $\theta$and constant ф (i.e, on each radial line), $g_{11}$ and $g_{44}$ should only depend on r (by spherical symmetry). Hence $g_{11}$ and $g_{44}$ are functions of a single variable.

The relation for affine connection is given as:

$$\Gamma^{\alpha}_{\mu v} = \frac{1}{2} g^{\alpha\beta}\big[g_{\beta\mu,v} + g_{v\beta,\mu} - g_{\mu v,\beta}\big] \quad (4)$$

Set $\alpha = 1,\ \ \mu = 1,\ \ v = 1,\ \ \beta = 1$

$$\Gamma^{1}_{11} = \frac{1}{2} g^{11}\big[g_{11,1} + g_{11,1} - g_{11,1}\big] \quad (5)$$

Substituted equation (2) and (3) into (4)

$$\Gamma^{1}_{11} = -\frac{1}{2}\frac{1}{A(r)}\big[\partial_r\big(-A(r)\big) + \partial_r\big(-A(r)\big) - \partial_r\big(-A(r)\big)\big]$$

When the above equation is simplified we have

$$\Gamma^{1}_{11} = \frac{A^{'}}{2A} \quad (6)$$

Apply similar approach to that of equation (2) and (3) for other values of $\alpha,\ \ \mu,\ \ v\ and\ \ \beta$ we obtain the following affine connections:

$$\Gamma^{1}_{11} = \frac{A^{'}}{2A} \quad (7)$$

$$\Gamma^{1}_{22} = -\frac{r}{A} \quad (8)$$

$$\Gamma^{1}_{33} = -\frac{r sin^2\theta}{A} \quad (9)$$

$$\Gamma^{1}_{33} = -\frac{B^{'}}{2A} \quad (10)$$

$$\Gamma_{12}^{2} = \Gamma_{21}^{2} = \frac{1}{r} \tag{11}$$

$$\Gamma_{33}^{2} = -sin\theta cos\theta \tag{12}$$

$$\Gamma_{23}^{3} = \Gamma_{32}^{2} = cot\theta \tag{13}$$

$$\Gamma_{14}^{4} = \Gamma_{41}^{4} = \frac{B^{'}}{2B} \tag{14}$$

**Determination of the Spinor connection in curved spacetime**

In other to determine the spinor connection in spacetime. We shall input affine connection and Schuster-Wilson-Backett relation into Spinor connection.

The relation for spinor connection is (Gu, 2021) given as:

$$\Gamma_{\mu v \alpha} = \frac{1}{4}\gamma^{v}\left(\partial_{\mu}\gamma_{v} - \Gamma_{\mu v}^{\alpha}\gamma_{\alpha}\right) \tag{15}$$

Schuster-Wilson-Backett relation:

$$\gamma^{\mu} = \gamma^{v} = \gamma^{\alpha} = \left(\frac{\gamma^{4}}{\sqrt{B}}, \frac{\gamma^{1}}{\sqrt{A}}, \frac{\gamma^{2}}{r}, \frac{\gamma^{3}}{rsin\theta}\right) \tag{16}$$

$$\gamma_{\mu} = \gamma_{v} = \gamma_{\alpha} = \left(\frac{\sqrt{B}}{\gamma^{4}}, \frac{\sqrt{A}}{\gamma^{1}}, \frac{r}{\gamma^{2}}, \frac{rsin\theta}{\gamma^{3}}\right) \tag{17}$$

By substituting (2), (3), (16) and (17) into (15) we obtain the following spinor connections

$$\Gamma_{1_{11}} = \frac{1}{8}\left(\frac{1}{\sqrt{AA^{'}}} - \frac{A^{'}}{A}\right) \tag{18}$$

$$\Gamma_{2_{21}} = \frac{1}{4\sqrt{A}}\frac{\gamma^{2}}{\gamma^{1}} \tag{19}$$

$$\Gamma_{3_{31}} = \frac{1}{4\sqrt{A}}\frac{\gamma^{3}}{\gamma^{1}}sin\theta \tag{20}$$

$$\Gamma_{4_{41}} = \frac{B^{'}\sqrt{A}}{8A\sqrt{B}}\frac{\gamma^{4}}{\gamma^{1}} \tag{21}$$

$$\Gamma_{2_{12}} = -\frac{1}{4\sqrt{A}}\frac{\gamma^{1}}{\gamma^{2}} \tag{22}$$

$$\Gamma_{2_{21}} = \frac{1}{4r}(1-1) = 0 \tag{23}$$

$$\Gamma_{3_{32}} = \frac{1}{4}\frac{\gamma^{3}}{\gamma^{2}}cos\theta \tag{24}$$

$$\Gamma_{3_{32}} = \frac{1}{4}(cot\theta - cot\theta) = 0 \tag{25}$$

$$\Gamma_{3_{23}} = -\frac{1}{4}\frac{\gamma^{2}}{\gamma^{3}}cot\theta sin\theta \tag{26}$$

$$\Gamma_{1_{44}} = \frac{1}{8}\left(\frac{1}{\sqrt{B^{'}}} - \frac{B^{'}}{B}\right) \tag{27}$$

$$\Gamma_{4_{14}} = \frac{1}{8}\frac{\gamma^{1}}{\gamma^{4}}\frac{B^{'}}{\sqrt{AB}} \tag{28}$$

## 3. The Motion of Spinor in the Vicinity of Spherical Star

### Geodesics

The General Relativistic equation of motion for particles of non-zero rest mass in a gravitational field (Chifu et al., 2008; Chifu & Howasu, 2008; Chifu & Lumbi, 2008) is given as;

$$\frac{d^{2}x^{\sigma}}{d\tau^{2}} + \Gamma_{\alpha\lambda}^{\sigma}\frac{dx^{\alpha}}{d\tau}\frac{dx^{\lambda}}{d\tau} = 0 \tag{29}$$

Where $\tau$ is a parameter to be determine

Therefore, the equation of motion are given explicitly as follows;

When $\sigma = 1, x^1 = ct, x^2 = r, x^3 = \theta, x^4 = \phi$
The General relativistic equation of motion for particles of non-zero rest mass in a gravitational field (Chifu et al., 2008; Chifu & Howasu, 2008; Chifu & Lumbi, 2008) is given as (29) can be given another form by replacing affine connection with spinor connection.

$$\frac{d^2x^\sigma}{d\tau^2} + \Gamma_{\sigma_{\alpha\lambda}} \frac{dx^\alpha}{d\tau}\frac{dx^\lambda}{d\tau} = 0 \quad (30)$$

Setting $\sigma = 1$

$$\frac{d^2(ct)}{d\tau^2} + \Gamma_{1_{11}} \frac{dx^1}{d\tau}\frac{dx^1}{d\tau} + \Gamma_{1_{44}} \frac{dx^4}{d\tau}\frac{dx^4}{d\tau} = 0 \quad (31)$$

$$\frac{d^2(ct)}{d\tau^2} + \Gamma_{1_{11}} \frac{d(ct)}{d\tau}\frac{d(ct)}{d\tau} + \Gamma_{1_{44}} \frac{d\phi}{d\tau}\frac{d\phi}{d\tau} = 0 \quad (32)$$

$$c\ddot{t} + \Gamma_{1_{11}} c^2\dot{t}^2 + \Gamma_{1_{44}}\dot{\phi}^2 = 0 \quad (33)$$

Substituting (18) and (27) into (31), we get

$$c\ddot{t} + \frac{1}{8}\left(\frac{1}{\sqrt{AA'}} - \frac{A'}{A}\right)c^2\dot{t}^2 + \frac{1}{8}\left(\frac{1}{\sqrt{B'}} - \frac{B'}{B}\right)\dot{\phi}^2 = 0 \quad (34)$$

Following same as the (34) we get

$$\ddot{r} + c\left(\frac{1}{4\sqrt{A}}\frac{\gamma^2}{\gamma^1}\right)\dot{\phi}\dot{t} - c\left(\frac{1}{4\sqrt{A}}\frac{\gamma^1}{\gamma^2}\right)\dot{\phi}\dot{t} = 0 \quad (35)$$

$$\ddot{\theta} + c\left(\frac{1}{4\sqrt{A}}\frac{\gamma^3}{\gamma^1}sin\theta\right)\dot{\theta}\dot{t} + \left(\frac{1}{4\sqrt{A}}\frac{\gamma^3}{\gamma^2}cos\theta\right)\dot{\theta}\dot{r} - \left(\frac{1}{4\sqrt{A}}\frac{\gamma^2}{\gamma^3}cos\theta\right)\dot{\theta}\dot{r} = 0 \quad (36)$$

$$\ddot{\phi} + \frac{B'\sqrt{A}}{8A\sqrt{B}}\frac{\gamma^4}{\gamma^1}\dot{t}\dot{\phi} + \frac{1}{8}\frac{\gamma^1}{\gamma^4}\frac{B'}{\sqrt{AB}}\dot{t}\dot{\phi} = 0 \quad (37)$$

Hence, the equations (34), (35), (36) and (37) are the explicit equations of motion of spinors in the vicinity of spherical stars

## 4. The Equation of Magnetic Field of Non-Homogeneous Spherical Bodies of Three Regions

The Schwarszchild's Metric

$$ds_0^2 = dt^2 - r^2d\theta^2 - r^2sin^2\theta ds^2 - dr^2 \quad (38)$$

The constitutive relations can be written as

$$H^\theta = \frac{\Psi}{\mu_0 r^2}B_\theta,\ \ H^\varphi = \frac{1}{\Psi\mu_0}\frac{B_\varphi}{r^2sin^2\theta},\ H^r = \frac{B_r}{\mu_0 r^4 sin^4\theta},\ H^{x^0} = \frac{1}{\Psi\mu_0}B_{x^0} \quad (39)$$

$$B_\theta = \frac{1}{\Psi}\mu_0 r^2 H^\theta, B_\varphi = \Psi\mu_0 r^2 sin^2\theta H^\varphi, B_r = \mu_0 r^4 sin^4\theta H^r, B_{x^0} = \Psi\mu_0 H^{x^0}, \quad (40)$$

By substituting the Vector potential F with the electromagnetic field tensor $G_{\mu\beta}$ (Gu, 2021)

$$\underline{\nabla}.G_{\mu\beta} = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}B_r\right\} + \frac{1}{rsin\theta}\frac{\partial}{\partial\theta}B_\theta + \frac{1}{r^2sin^2\theta}\frac{\partial}{\partial\varphi}B_\varphi + \frac{\partial}{\partial x^0}\left\{\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}B_{x^0}\right\} = \frac{4\pi}{c}j^{\boldsymbol{\beta}} \quad (41)$$

Substituting equation (39) into (41) and simplifying we get

$$\left\{\left(1+\frac{2}{C^2}f\right)^{-\mathbf{1}}(20\mu_0 r^2 sin^4\theta H_r)\right\} = \frac{4\pi}{c}j^\beta \quad (42)$$

Putting $B_r = \mu_0 r^4 sin^4\theta H^r$ into (41) and simplifying we will obtain $B_r$ as

$$B_r = \frac{2\pi r^2}{5c^3} f(r) \quad (43)$$

**The Newton's Dynamical Gravitational Scalar Potential for non- homogeneous bodies**

Newton's dynamical gravitational scalar potential for the three regions of non static homogeneous spherical body is given as (Minister, 2015):

$$f_1(r) = 2\pi G\left[R_1^{}(\rho_1 - \rho_2) - \rho_2 R_2^2\right] + \frac{2}{3}\pi G \rho_1 r^2 \; ; \; r < R_1 \quad (44)$$

$$f_2(r) = \frac{4}{3}\pi G R_1^2(\rho_1 - \rho_2)\frac{1}{r} + \frac{2}{3}\pi G \rho_2 r^2 - 2\pi G \rho_2 R_2^2 \; ; \; R_1 < r < R_2 \quad (45)$$

$$f_3(r) = \frac{4}{3}\pi G\left[\rho_2\left(R_1^2 - R_2^{}\right) - \rho_1 R_1^{}\right]\frac{1}{r} \; ; \; r > R_2 \quad (46)$$

Substituting equation (44) into (43) we get

$$B_r = \frac{2G\pi^2 r^2}{5c^3}\left\{\left[R_1^{}(\rho_1 - \rho_2) - \rho_2 R_2^{}\right] + \frac{2}{3}\rho_1 r^2\right\} \quad r < R_1 \quad (47)$$

Substituting equation (45) into (43) we get

$$B_r = \frac{2G\pi^2 r^2}{5c^3}\left\{\frac{2}{3}R_1^2(\rho_1 - \rho_2)\frac{1}{r} + \frac{\rho_2}{3}r^2 - \rho_2 R_2^{}\right\} R_1 < r < R_2 \quad (48)$$

Substituting equation (46) into (43) we get

$$B_r = \frac{2G\pi^2 r}{5c^3}\left\{\frac{2}{3}\left[\rho_2\left(R_1^2 - R_2^{}\right) - \rho_1 R_1^{}\right]\right\} r > R_2 \quad (49)$$

Equations obtained as (47), (48) and (49) are the equations for the magnetic fields of three regions of planets.

$R_1$ is the radius of the core
$R_2$ is the radius of the upper layer
$\rho_1$ is the density of the core
$\rho_2$ is the density of the upper
$\rho_g$ is mass density of planets with, SI unit kg.m$^{-3}$
$c$ is the speed of light given as $c = 2.9979 \times 10^8 ms^{-2}$
$r$ is the radius of planets
$f(r)$ is the gravitational scalar potential that depend of r
G is the universal gravitational constant $G = 6.67 \times 10^{-11} NKg^2 m^{-2}$

**The magnetic field of planet Earth**

$R_1$= 3486km, $R_2$= 2885km, $\rho_1$= 12g/cm$^3$ $\rho_2$= 3.816g/cm$^3$
Radius of the earth $r = 6.371 \times 10^6$m
G is the universal gravitational constant $G = 6.67 \times 10^{-11} NKg^2 m^{-2}$
$c$ is the speed of light given as $c = 2.9979 \times 10^8 ms^{-2}$
1Gauss = $10^{-4}$Tesla

By proper substitution of all parameters in (47), (48) and (49) will give the values found in the below table

Table 1. The magnetic field of the planets

| **Planets** | Magnetic field for region 1 ($B_{r_1}$)nT | Magnetic field for region 2 ($B_{r_2}$)nT | Magnetic field for region 3 ($B_{r_3}$)nT |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Earth | 128.8449 | 20.4863 | $1.9254 \times 10^{-6}$ |
| Jupiter | 1191266.3760 | 131037.2995 | $7.4556 \times 10^{-4}$ |
| Moon | 2.3807 | 0.0576 | $5.7995 \times 10^{-8}$ |

## 5. Conclusion

The Equation (47), (48) and (49) gave the magnetic field of the three regions of planets. The equation of magnetic field obtained from this research work contained some addition terms such as $\rho_1$, $\rho_2$, $R_2, R_1$ which account for densities and radius of the regions of planets.

The Earth and most of the planets in the Solar system, as well as the Sun and other stars, all generate magnetic fields through the motion of electrically conducting fluids (Schwarzschild, 1999). Its field originates in its core. This is a region of iron alloys extending to about 34000km (Jordan, 1979). The Earth magnetic field at the surface ranges from $25\mu T$ to $65\mu T$ (Finlay & Constable, 2010). Also magnetic field of the massive fields is generally associated with small-scale features and often varies markedly over distances of a few kilometres.

The computer simulation of Earth's field in a period of normal polarity between reversal. The lines represent magnetic field lines, blue when the field points towards the center and yellow when away. The rotational axis of Earth's is centered and vertical. Then clusters of lines are within Earth's core.

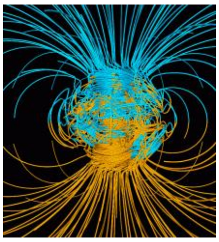

Figure 1 (Glatzmaier, 2014). The Earth's field in a period of normal polarity

The variation in the magnetic field at different regions of planets as suggested by Weiss, Finlay, Gary and others. And couple with clusters nature of the magnetic field at the core as shown in figure 1 clearly suggest that the equation obtained from this article as (47), (48) and (49) is the appropriate equation when dealing with non-homogeneous spherical bodies.

The results obtained in this paper have paved the way for the study of motion of spinors in the vicinity of spherical bodies and also this paper can be used to determine the magnetic field of the different regions of other planetary bodies that are non-homogenoeus in nature. The immediate consequences of the results obtained in this paper are as follows:

1) The spinors connections for other planetary bodies like elliptical and oblade spheroidal can be determine. And afterward the equation of motion of spinors around such body can be determined.

2) The gravitational scalar potential for non-homogeneous elliptical bodies can also be obtained and thereby the magnetic field around such planet or bodies could be obtained.

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