

A comparative study of the Schwarzschild metric tensor and the Howusu metric tensor using the radial distance parameter as a measuring index

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Abstract: The study investigated Einstein Curvature Tensor $G_{\mu\nu}$ using Schwarzschild Metric Tensor and the Howusu Metric Tensor. It involved the comparison between Einstein Curvature Tensor $G_{\mu\nu}$ derived from the Howusu Metric Tensor and Einstein Curvature Tensor derived from the Schwarzschild Metric Tensor. Results of the analysis indicated that unlike the Schwarzschild $G_{\mu\nu}$, the Howusu $G_{\mu\nu}$ gave a non zero answer. Comparing the Howusu $G_{\mu\nu}$ and the Schwarzschild $G_{\mu\nu}$, they behaved differently as $r \rightarrow 0$; thus, as $r \rightarrow 0$, Howusu $G_{\mu\nu} \rightarrow \infty$ while Schwarzschild $G_{\mu\nu} \rightarrow 0$ but as $r \rightarrow \infty$, the two Metric Tensors were observed to be averagely similar.

Keyword: Einstein Curvature Tensor, Schwarzschild Metric Tensor, Howusu Metric Tensor

1. Introduction

The assumption was that Riemannian geometry was rather more general than the Euclidean geometry after the German mathematician, George Riemann published his geometry for space-time known as Riemannian geometry in 1854 (Howusu, 2009). The belief was that the Riemannian geometry has the potential of providing a more general foundation for theoretical physics (Howusu & Uduh, 2003; Howusu, 2013). However, the problem with the Riemannian geometry was the fact that it was based on an unknown metric tensor, and therefore, its exploitation and possible applications to theoretical physics eluded the world (Howusu 2007). Einstein tried to solve this problem in his contribution to classical mechanics: Einstein's Geometrical Gravitational Field Equations (Einstein, 1905; Einstein, 1915).

The first major breakthrough in developing the Einstein's Geometrical Theory of classical mechanics in the gravitational field known as General Relativity was achieved in 1916 by Schwarzschild (Schwarzschild, 1916) when he introduced a metric tensor called the Schwarzschild metric for all gravitational fields due to static homogeneous spherical distribution of mass. (Schwarzschild, 1916; Heinzle & Steinbauer, 2002) In spite of the great fame since 1915, Einstein's Geometrical Gravitational field equations

cannot be applied to generate any natural metric tensor for the gravitational fields due to any distribution of mass in nature.

In the year 2009, Howusu (Howusu, 2009) came up with a new metric tensor which he claims to be valid for gravitational field which is regular everywhere, continues everywhere including all boundaries, continues normal derivative everywhere including all boundaries and its reciprocal decreases at infinite distance from source in his book entitled: ‘Riemannian Revolution in mathematics and Physics I’ based upon the following criteria (Howusu, 2009):

- 1) It should contain the phenomenon of gravitational space contraction for which there is experimental evidence.
- 2) It should contain the phenomenon of gravitational time dilation for which there is experimental evidence.
- 3) It should contain the phenomenon of singularity in the gravitational field in nature for which there is experimental evidence.
- 4) It should reduce to the pure Euclidean metric tensor in all space-times without gravitational field in all orthogonal curvilinear coordinates.
- 5) It should contain the Schwarzschild metric tensor in the space-times exterior to all static homogenous spherical distributions of mass in all orthogonal curvilinear coordinates.
- 6) It should make the Riemann’s Tensorial Energy for all particles of non-zero rest masses in all gravitational fields to reduce to the corresponding pure Newton’s Lagrangian energy in the limit of c^0 , in the orthogonal curvilinear coordinates.
- 7) It should make the three space parts of the Riemann’s Tensorial Geodesic equation of motion for particles of non-zero rest masses in all gravitational field in nature, in all orthogonal curvilinear coordinates to reduce to the corresponding pure Newton’s equation of motion in limit of c^0 .

In this paper, we constructed a solution to Einstein curvature tensor using the Howusu metric tensor and compared it with the already established results from the Schwarzschild metric tensor, if the Howusu Metric Tensor can conveniently replace the Schwarzschild Metric Tensor. However, the scope has been limited to Spherical coordinates. The summarization of the Schwarzschild metric can be:

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2M}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1 - \frac{2M}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (1)$$

where M is the mass of the object and r is the distance away from the object (Kumar, 2009; Obaje, 2022) and the Howusu metric tensor:

$$g_{\mu\nu} = \begin{pmatrix} -\exp\left(\frac{2GM}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \exp\left(-\frac{2GM}{c^2 r}\right) & 0 & 0 \\ 0 & 0 & r^2 \exp\left(-\frac{2GM}{c^2 r}\right) & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \exp\left(-\frac{2GM}{c^2 r}\right) \end{pmatrix} \quad (2)$$

where c is the speed of light; G is the universal constant of gravitation; M is the mass of object and r is the distance away from the object (Howusu, 2012; Obaje, 2022)

2. MATHEMATICAL METHOD

The Einstein curvature tensor used to determine the space-time curvature of objects; the tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \quad (3)$$

Where $G_{\mu\nu}$ is the Einstein Curvature Tensor, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is the Ricci Curvature Tensor and R is the Ricci Scalar. The Christoffel symbols, Riemannian Curvature Tensor, Ricci Curvature tensor, Ricci Scalar, and the Einstein Curvature tensor were all derived from the Howusu metric tensor (Obagboye & Howusu, 2013; Abalaka & Ekpe, 2021).

2.1. Christoffel Symbol

The formula below gave rise to the Christoffel Symbol (Koffa & Omonile, 2016; Obaje, 2020):

$$g_{\alpha\delta}\Gamma_{\beta\gamma}^{\delta} = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}}\right), \quad (4)$$

the non-zero terms are:

$$\Gamma_{10}^0 = \Gamma_{01}^0 = \frac{GM}{c^2r^2} \quad (5)$$

$$\Gamma_{00}^1 = -\frac{GM}{c^2r^2} \quad (6)$$

$$\Gamma_{11}^1 = -\frac{GM}{c^2r^2} \quad (7)$$

$$\Gamma_{22}^1 = \frac{GM}{c^2} - r \quad (8)$$

$$\Gamma_{33}^1 = \frac{GM}{c^2}\sin^2\theta - r\sin^2\theta \quad (9)$$

$$\Gamma_{21}^2 = \frac{1}{r} - \frac{GM}{c^2r^2} \quad (10)$$

$$\Gamma_{33}^2 = -\cos\theta\sin\theta \quad (11)$$

$$\Gamma_{13}^3 = \frac{1}{r} - \frac{GM}{c^2r^2} \quad (12)$$

$$\Gamma_{23}^3 = \frac{\cos\theta}{\sin\theta} = \cot\theta \quad (13)$$

2.2. Riemannian Curvature Tensor

The formula below can give rise to the Riemannian Curvature Tensor (Obaje & Ekpe, 2021):

$$R_{\mu\alpha\nu}^{\alpha} = \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\alpha,\nu}^{\alpha} + \Gamma_{\beta\alpha}^{\alpha}\Gamma_{\mu\nu}^{\beta} - \Gamma_{\beta\nu}^{\alpha}\Gamma_{\mu\alpha}^{\beta} \quad (14)$$

In writing out all the non-zero terms of the Riemann curvature tensor, note that the list involved only half the amount of Riemann tensor because of the following property:

$$R_{\mu\alpha\nu}^{\alpha} = -R_{\mu\nu\alpha}^{\alpha} \quad (15)$$

$$R_{101}^0 = -\frac{2GM}{c^2r^3} \quad (16)$$

$$R_{202}^0 = \frac{GM}{c^2r} - \frac{G^2M^2}{c^4r^2} \quad (17)$$

$$R_{303}^0 = \frac{GM}{c^2 r} \sin^2 \theta - \frac{G^2 M^2}{c^4 r^3} \sin^2 \theta \quad (18)$$

$$R_{010}^1 = \frac{2GM}{c^2 r^3} \quad (19)$$

$$R_{212}^1 = 1 - \frac{GM}{c^2 r} \quad (20)$$

$$R_{313}^1 = -\frac{GM \sin^2 \theta}{c^2 r} \quad (21)$$

$$R_{020}^2 = \frac{GM}{c^4 r^3} - \frac{G^2 M^2}{c^4 r^4} \quad (22)$$

$$R_{121}^2 = -\frac{GM}{c^2 r^3} \quad (23)$$

$$R_{323}^2 = \frac{2GM \sin^2 \theta}{c^2 r} - \frac{G^2 M^2 \sin^2 \theta}{c^4 r^2} - \cos^2 \theta \quad (24)$$

$$R_{030}^3 = \frac{GM}{c^4 r^3} - \frac{G^2 M^2}{c^4 r^4} \quad (25)$$

$$R_{131}^3 = -\frac{GM}{c^2 r^3} \quad (26)$$

$$R_{232}^3 = -\frac{G^2 M^2}{c^4 r^2} \quad (27)$$

2.3. The Ricci Curvature Tensor

To derive the Ricci tensor, which is a contraction of the Riemann tensor, we use (Kumar, 2009; Obaje *et al*, 2022a):

$$R_{\mu\nu} = R_{\mu\alpha\nu}^\alpha \quad (28)$$

Where the explicit expression of $R_{\mu\nu}$ is:

$$R_{\mu\nu} = R_{\mu 0\nu}^0 + R_{\mu 1\nu}^1 + R_{\mu 2\nu}^2 + R_{\mu 3\nu}^3 \quad (29)$$

Thus, the below conveniently expresses the components of the Ricci curvature tensor:

$$R_{00} = \frac{2GM}{c^2 r^3} - \frac{2G^2 M^2}{c^4 r^4} + \frac{2GM}{c^4 r^3} \quad (30)$$

$$R_{11} = -\frac{4GM}{c^2 r^3} \quad (31)$$

$$R_{22} = 1 - \frac{2G^2 M^2}{c^4 r^2} \quad (32)$$

$$R_{33} = \frac{2GM \sin^2 \theta}{c^2 r} - \frac{G^2 M^2 \sin^2 \theta}{c^4 r^3} - \frac{G^2 M^2 \sin^2 \theta}{c^4 r^2} - \cos^2 \theta \quad (33)$$

2.4. The Ricci Scalar

The calculation of the Ricci curvature scalar, R , using the formula below will give the Einstein Tensor (Obaje *et al*, 2022b):

$$R = g^{\mu\nu} R_{\mu\nu} \quad (34)$$

where $g^{\mu\nu}$ is the contravariant metric tensor; and the inverse of the Howusu Metric Tensor is:

$$g^{00} = -\exp\left(\frac{2GM}{c^2 r}\right) \quad (35)$$

$$g^{11} = \exp\left(-\frac{2GM}{c^2 r}\right) \quad (36)$$

$$g^{22} = \frac{1}{r^2} \exp\left(-\frac{2GM}{c^2 r}\right) \quad (37)$$

$$g^{33} = \frac{1}{r^2 \sin^2 \theta} \exp\left(-\frac{2GM}{c^2 r}\right) \quad (38)$$

$$g^{\mu\nu} = 0, \text{ otherwise} \quad (39)$$

Using the summation convention with the Ricci Scalar, one obtains the value of each coefficient in the expansion:

$$R = g^{00}R_{00} + \dots + g^{11}R_{11} + \dots + g^{22}R_{22} + \dots + g^{33}R_{33} \quad (40)$$

$$R = \exp\left(\frac{2GM}{c^2r}\right) \left[\frac{2G^2M^2}{c^4r^4} - \frac{2GM}{c^2r^3} - \frac{2GM}{c^4r^3} \right] + \exp\left(-\frac{2GM}{c^2r}\right) \left[-\frac{2GM}{c^2r^3} + \frac{1}{r^2} - \frac{3G^2M^2}{c^4r^4} - \frac{G^2M^2}{c^4r^5} - \frac{\cot^2\theta}{r^2} \right] \quad (41)$$

2.5. The Einstein Curvature Tensor

Now all the components necessary to solve the Einstein tensor have been determined. The Ricci tensor $R_{\mu\nu}$ is known alongside the Metric tensor $g_{\mu\nu}$ (Howusu Metric Tensor) and the Ricci scalar R . Having obtained all these values, writing, and solving of the Einstein curvature tensor could be as follows:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \quad (42)$$

To obtain:

$$G_{00} = \frac{GM}{c^2r^3} - \frac{G^2M^2}{c^4r^4} + \frac{GM}{c^4r^3} - \exp\left(-\frac{4GM}{c^2r}\right) \left[\frac{GM}{c^2r^3} - \frac{1}{2r^2} + \frac{3G^2M^2}{2c^4r^4} + \frac{G^2M^2}{2c^4r^5} + \frac{\cot^2\theta}{2r^2} \right] \quad (43)$$

$$G_{11} = -\exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^4} - \frac{GM}{c^2r^3} - \frac{GM}{c^4r^3} \right] - \frac{5GM}{c^2r^3} + \frac{1}{2r^2} - \frac{3G^2M^2}{2c^4r^4} - \frac{G^2M^2}{2c^4r^5} - \frac{\cot^2\theta}{2r^2} \quad (44)$$

$$G_{22} = \frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} + \frac{G^2M^2}{2c^4r^3} + \frac{\cot^2\theta}{2} - \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r} \right] \quad (45)$$

$$G_{33} = -\cos^2\theta - \sin^2\theta \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r} \right] - \sin^2\theta \left[\frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} - \frac{3G^2M^2}{2c^4r^3} - \frac{\cot^2\theta}{2} \right] \quad (46)$$

3. Results and Discussion

The mathematical results from the calculation of the Einstein Curvature Tensor using Howusu Metric Tensor is given by:

$$G_{00} = \frac{GM}{c^2r^3} - \frac{G^2M^2}{c^4r^4} + \frac{GM}{c^4r^3} - \exp\left(-\frac{4GM}{c^2r}\right) \left[\frac{GM}{c^2r^3} - \frac{1}{2r^2} + \frac{3G^2M^2}{2c^4r^4} + \frac{G^2M^2}{2c^4r^5} + \frac{\cot^2\theta}{2r^2} \right] \quad (47)$$

$$G_{11} = -\exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^4} - \frac{GM}{c^2r^3} - \frac{GM}{c^4r^3} \right] - \frac{5GM}{c^2r^3} + \frac{1}{2r^2} - \frac{3G^2M^2}{2c^4r^4} - \frac{G^2M^2}{2c^4r^5} - \frac{\cot^2\theta}{2r^2} \quad (48)$$

$$G_{22} = \frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} + \frac{G^2M^2}{2c^4r^3} + \frac{\cot^2\theta}{2} - \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r} \right] \quad (49)$$

$$G_{33} = -\cos^2\theta - \sin^2\theta \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r} \right] - \sin^2\theta \left[\frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} - \frac{3G^2M^2}{2c^4r^3} - \frac{\cot^2\theta}{2} \right] \quad (50)$$

Considering the Einstein Curvature Tensor G_{00} of the Howusu Metric

$$G_{00} = \frac{GM}{c^2r^3} - \frac{G^2M^2}{c^4r^4} + \frac{GM}{c^4r^3} - \exp\left(-\frac{4GM}{c^2r}\right) \left[\frac{GM}{c^2r^3} - \frac{1}{2r^2} + \frac{3G^2M^2}{2c^4r^4} + \frac{G^2M^2}{2c^4r^5} + \frac{\cot^2\theta}{2r^2} \right] \quad (51)$$

It is clear from the above equation that, as $r \rightarrow 0$, G_{00} is $-\infty$ and as $r \rightarrow \infty$, G_{00} tends to zero. Whereas in the case the Schwarzschild Metric Tensor the Einstein Curvature Tensor G_{00} is:

$$G_{00} = 0 \quad (52)$$

Equating the Einstein Curvature Tensor of the Howusu Metric Tensor and the Schwarzschild Metric Tensor, we have

$$\frac{GM}{c^2r^3} - \frac{G^2M^2}{c^4r^4} + \frac{GM}{c^4r^3} - \exp\left(-\frac{4GM}{c^2r}\right) \left[\frac{GM}{c^2r^3} - \frac{1}{2r^2} + \frac{3G^2M^2}{2c^4r^4} + \frac{G^2M^2}{2c^4r^5} + \frac{\cot^2\theta}{2r^2}\right] = 0 \quad (53)$$

This is the condition for the Einstein Curvature Tensor G_{00} to be equal both in the Howusu Metric Tensor and the Schwarzschild Metric Tensor.

Looking at the Einstein Curvature Tensor G_{11} of the Howusu Metric

$$G_{11} = -\exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^4} - \frac{GM}{c^2r^3} - \frac{GM}{c^4r^3}\right] - \frac{5GM}{c^2r^3} + \frac{1}{2r^2} - \frac{3G^2M^2}{2c^4r^4} - \frac{G^2M^2}{2c^4r^5} - \frac{\cot^2\theta}{2r^2} \quad (54)$$

It is clear from the above equation that, as $r \rightarrow 0$, G_{11} is $-\infty$ and as $r \rightarrow \infty$, G_{11} tends to zero. Whereas in the case the Schwarzschild Metric Tensor the Einstein Curvature Tensor G_{11} is:

$$G_{11} = 0 \quad (55)$$

Equating the Einstein Curvature Tensor for both Howusu Metric Tensor and the Schwarzschild Metric Tensor, we have

$$-\exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^4} - \frac{GM}{c^2r^3} - \frac{GM}{c^4r^3}\right] - \frac{5GM}{c^2r^3} + \frac{1}{2r^2} - \frac{3G^2M^2}{2c^4r^4} - \frac{G^2M^2}{2c^4r^5} - \frac{\cot^2\theta}{2r^2} = 0 \quad (56)$$

This is the condition for the Einstein Curvature Tensor G_{11} to be equal both in the Howusu Metric Tensor and the Schwarzschild Metric Tensor.

Taking the Einstein Curvature Tensor G_{22} of the Howusu Metric

$$G_{22} = \frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} + \frac{G^2M^2}{2c^4r^3} + \frac{\cot^2\theta}{2} - \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r}\right] \quad (57)$$

It is clear from the above equation that, as $r \rightarrow 0$, G_{22} is ∞ and as $r \rightarrow \infty$, G_{22} tends to $\frac{1}{2} + \frac{\cot^2\theta}{2}$. Where as in the case of the Schwarzschild Metric Tensor, the Einstein Curvature Tensor G_{22} is:

$$G_{22} = 0 \quad (58)$$

Equating the Einstein Curvature Tensor for both the Howusu Tensor and the Schwarzschild Metric Tensor, we have

$$\frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} + \frac{G^2M^2}{2c^4r^3} + \frac{\cot^2\theta}{2} - \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r}\right] = 0 \quad (59)$$

This is the condition for the Einstein Curvature Tensor G_{22} to be equal both in the howusu Metric Tensor and the Schwarzschild Metric Tensor.

Considering the Einstein Curvature Tensor G_{33} of the Huwosu Metric

$$G_{33} = -\cos^2\theta - \sin^2\theta \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r}\right] - \sin^2\theta \left[\frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} - \frac{3G^2M^2}{2c^4r^3} - \frac{\cot^2\theta}{2}\right] \quad (60)$$

It is clear from the above equation that, as $r \rightarrow 0$, G_{33} is ∞ and as $r \rightarrow \infty$, G_{33} tends to $-\cos^2\theta - \frac{\sin^2\theta}{2} + \sin^2\theta \frac{\cot^2\theta}{2}$. While in the case of the Schwarzschild Metric Tensor, the Einstein Curvature Tensor G_{33} is:

$$G_{33} = 0 \quad (61)$$

Equating the Einstein Curvature Tensor of the Howusu Metric Tensor and the Schwarzschild Metric Tensor, we have

$$-\cos^2\theta - \sin^2\theta \exp\left(\frac{4GM}{c^2r}\right) \left[\frac{G^2M^2}{c^4r^2} - \frac{GM}{c^2r} - \frac{GM}{c^4r} \right] - \sin^2\theta \left[\frac{1}{2} - \frac{G^2M^2}{2c^4r^2} + \frac{GM}{c^2r} - \frac{3G^2M^2}{2c^4r^3} - \frac{\cot^2\theta}{2} \right] = 0 \quad (62)$$

This is the condition for the Einstein Curvature Tensor G_{33} to be equal both in the Howusu Metric Tensor and the Schwarzschild Metric Tensor.

The present work investigated Schwarzschild Metric Tensor and Howusu Metric Tensor with their comparison using computed values of Einstein Curvature Tensor $G_{\mu\nu}$ derived from both Metric Tensor as the bases for comparison. Results of the analysis indicated that Howusu $G_{\mu\nu}$ and the Schwarzschild $G_{\mu\nu}$ behaved differently as $r \rightarrow 0$; thus, as $r \rightarrow 0$, Howusu $G_{\mu\nu} \rightarrow \infty$ while Schwarzschild $G_{\mu\nu} \rightarrow 0$ but as $r \rightarrow \infty$, the two Metric Tensors appeared averagely similar. Although Howusu $G_{\mu\nu}$ is slightly different from that of the Schwarzschild $G_{\mu\nu}$, it is more generalized because there is no restriction on it unlike the Schwarzschild making it cover more areas.

4. Conclusion

This paper has been able to derive the Einstein curvature tensor $G_{\mu\nu}$ based on the Howusu Metric Tensor that describes the gravitational field which is regular everywhere, continues everywhere including all boundaries, continues normal derivative everywhere including all boundaries and its reciprocal decreases at infinite distance from source (Obaje & Ekpe, 2021b). Comparing these results with the well-known values of the Einstein curvature tensor based on the Schwarzschild Metric Tensor, it is obvious that the two metric tensors were equal at some point. The Einstein Curvature Tensor (43) - (46) derived in this paper has pave way for the comparison of the Howusu $G_{\mu\nu}$ with the $G_{\mu\nu}$ derived from another known metric tensor.

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