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Determination of the critical exponent for high temperature superconductors using paraconductivity approach

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Abstract: We have developed theoretical ideas to determine the critical exponent, understanding superconductivity for high-temperature superconductors. A detailed analysis is given based on the paraconductivity approach and compared with the experimental results. To develop the theoretical idea, the analysis is performed with the help of well-known paraconductivity expressions. To match the theoretical idea with experimental value, we have reproduced the data points using the 'OriginPro' software. Numerical estimation has been given in favour of the samples $Tl_2Ba_2CaCu_2O_8$ (TBCCO), $Bi_{1.6}Pb_{0.4}Sr_2Ca_2Cu_3O_x$ (BSCCO), $YBa_2Cu_3O_{7-\delta}$ (YBCO) and $SmBa_2Cu_3O_{7-\delta}$ (SmBCO). The theoretical models largely agreed with the experimental results for the mentioned superconducting samples.

Keyword: Paraconductivity, critical exponent, high-temperature superconductor

1. Introduction

The study of superconductivity due to the thermodynamic fluctuations has achieved a lot of interest in high-T_c systems. Fluctuation of these materials become very important because of their physical parameters like, high transition temperature, strong anisotropy and small coherence length. Therefore, it is a very necessary tool for understanding the intrinsic properties of superconductors. These study also provide the dimensionality of the order parameter fluctuation. The critical exponent, denoted by λ is the most important exponent constant determining the dimensionality of the superconducting system. It is a pure (positive) number, also known as dimensional exponent (DE). The creation of Cooper pairs due to thermodynamic fluctuation above the superconducting transition with a finite lifetime will enhance electrical conductivity **[1,2]**, so-called paraconductivity.

As the temperature approaches to the transition temperature Tc, the number of Cooper pairs increase while the normal electron density (NED) decreases. As a result, the resistivity decreases abruptly and the thermal fluctuations induce Cooper pairs give excess conductivity, denoted by $\Delta\sigma$. A major difference between normal conductivity σ and paraconductivity $\Delta\sigma$ is the nature of the charge carriers. The former carries current by a single electron and the latter carries current by a pair of electrons. The study of temperature dependent paraconductivity helps us to determine the critical exponent constant, determining the dimensionality of the superconducting systems.

2. Theoretical Model

The experimental observation of paraconductivity with temperature needs the exponent coefficient λ [which have the values 0.33, 0.5, 1.0, 1.5 etc.] as a fitting parameter. The critical exponent λ, whose values determine the dimensionality of the superconducting samples, can be found in the following way:

2.1. Method-I

First, we have to summarize the paraconductivity expression for isotropic systems:

$$\Delta\sigma(T) \equiv A\varepsilon^{-\lambda} \qquad (1)$$

where

$$A = \frac{e^2}{32\hbar\xi(0)}\,;\ \lambda = \frac{1}{2}\ [\text{3D system}] = \frac{e^2}{16\hbar d};\lambda = 1\ [\text{2D system}] \qquad (2)$$

$$= \frac{\pi e^2\xi(0)}{32\hbar S}\,;\ \lambda = \frac{3}{2}\ [\text{1D system}]$$

and $\varepsilon = \left(T - T_c^{MF}\right)/T_c^{MF}$ is called the reduced temperature.

Again, the temperature dependence of excess conductivity $\Delta\sigma(T)$, which is defined within the Ginzburg-Landau (G-L) mean field approximation **[3]** as

$$\Delta\sigma(T) = \sigma_m(T) - \sigma_n(T) = \frac{1}{\rho_m(T)} - \frac{1}{\rho_n(T)} \qquad (3)$$

where $\sigma_m(T)\ [= 1/\rho_m(T)]$ represents the measured electrical conductivity and $\sigma_n(T)\ [= 1/\rho_n(T)]$ is the linear normal-state conductivity.

Generally, the normal-state resistivity of the sample $\rho_n(T)$ is obtained from the measured resistivity $\rho_m(T)$ at $T \geq 2T_c$ by applying the least square method to the Anderson and Zou relation **[4]**:

$$\rho_n(T) = \alpha T + \rho_0 \ (4)$$

where α and ρ_0 are constants; $\alpha = d\rho/dT$ is the temperature resistivity coefficient which determines the slope of the linear dependence $\rho_n(T)$ and ρ_0 is the residual resistivity cut off by this line on the Y-axis at $T = 0$ **[5-8]**.

Using Eq. (1) into Eq. (3), we obtain

$$\frac{(\rho_n - \rho_m)}{\rho_m} = A\rho_n\varepsilon^{-\lambda} \qquad (5)$$

Now, taking derivative with respect to temperature on both sides of Eq. (5) and rearranging, we have

$$\frac{1}{\rho_m^2}\frac{d\rho_m}{dT} - \frac{1}{\rho_n^2}\frac{d\rho_n}{dT} = \frac{\lambda}{T_c^{MF}} A^{-1/\lambda}\left[\frac{\rho_n - \rho_m}{\rho_m\rho_n}\right]^{1+\frac{1}{\lambda}} \qquad (6)$$

The critical exponent λ can be determined from the slope of a logarithmic plot of the left-hand side of Eq. (6) versus $\Delta\sigma = \left(\rho_n - \rho_m\right)/\rho_m\rho_n$.

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2.2. Method-II

In this process, we utilize the Eq. (1) by direct logarithmic operation. Taking logarithm on both sides of Eq. (1), we have

$$\ln\Delta\sigma = \ln A - \lambda\ln\varepsilon$$

$$\text{or, } \ln\Delta\sigma = -\lambda\ln\varepsilon + \text{constant} \quad (7)$$

where the searched value of critical exponent λ is equal to the negative value of the slope of the line fitted to the linear parts of this dependence. Here, the function $\ln\Delta\sigma$ strongly depends on the definition of the critical or transition temperature. The linear parts of Eq. (7) allows us to calculate the values of critical exponent λ .

2.3. Method-III

The third method is very useful to carry out the analysis of the experimental data using Eq. (1). It is convenient to apply the Kouvel-Fisher method **[9]**, also known as the logarithmic derivative method. Thus, we have

$$\frac{d}{dT}(ln\Delta\sigma) = \frac{d}{dT}\left[ln\left(A\varepsilon^{-\lambda}\right)\right]$$

$$= -\frac{\lambda}{T-T_c^{MF}} \quad (8)$$

The inverse of the logarithmic derivative is written as

$$\chi_\sigma^{-1} = \left[-\frac{d}{dT}(ln\Delta\sigma)\right]^{-1} = \frac{1}{\lambda}\left[T - T_c^{MF}\right] \quad (9)$$

This is a linear equation and it depends on the temperature of the superconducting sample only, which enables the direct determination of the critical exponent λ by identifying linear behaviours in curves of χ_σ^{-1} as a function of T. In this formula, the critical exponent is a reversed slope of the temperature dependence within linear regions and it is analogous to Curie-Weiss susceptibility equation **[10]** in ferromagnetism, with the critical exponent λ playing the role of the Curie constant C.

3. Numerical Results

To obatin the critical exponent λ, we take few examples for the methods mentioned before. The exponent λ can be determined from the slope of a logarithmic plot of the left-hand side of Eq. (6) versus $\Delta\sigma = \left(\rho_n - \rho_m\right)/\rho_m\rho_n$. We have reproduced the experimental result **[11]** and fitted with theory, confirming the two-dimensional paraconductivity of the compound Tl-Ba-Ca-Cu-O system.

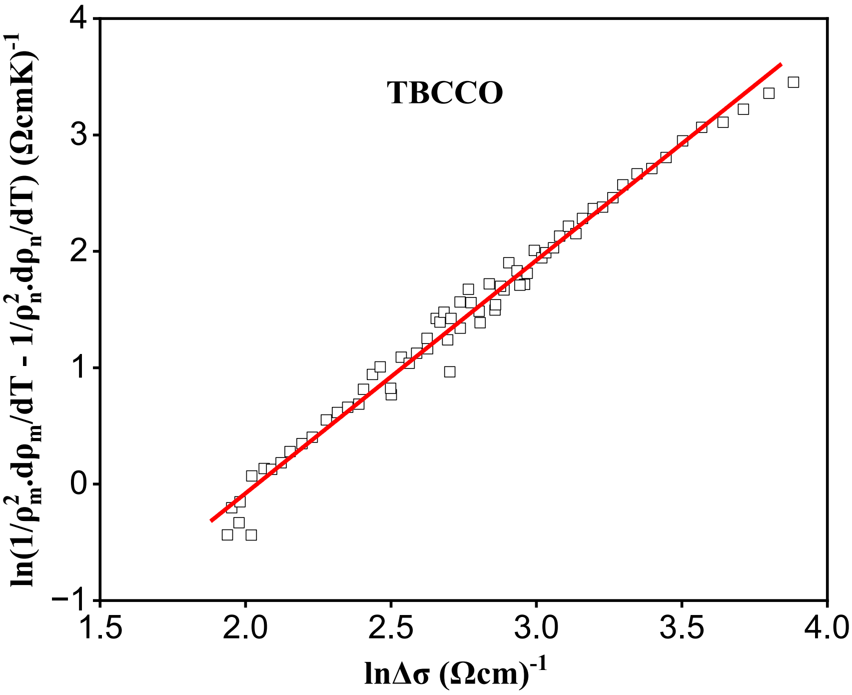

Figure 1. Logarithmic plot of the left-hand side of Eq. (6) as a function of excess conductivity $\Delta\sigma$ for sample $Tl_2Ba_2CaCu_2O_8$ (TBCCO). A slope of 2 is observed over most of the region except for a very narrow region close to T_c.

We take further example for estimating λ described in method-II.

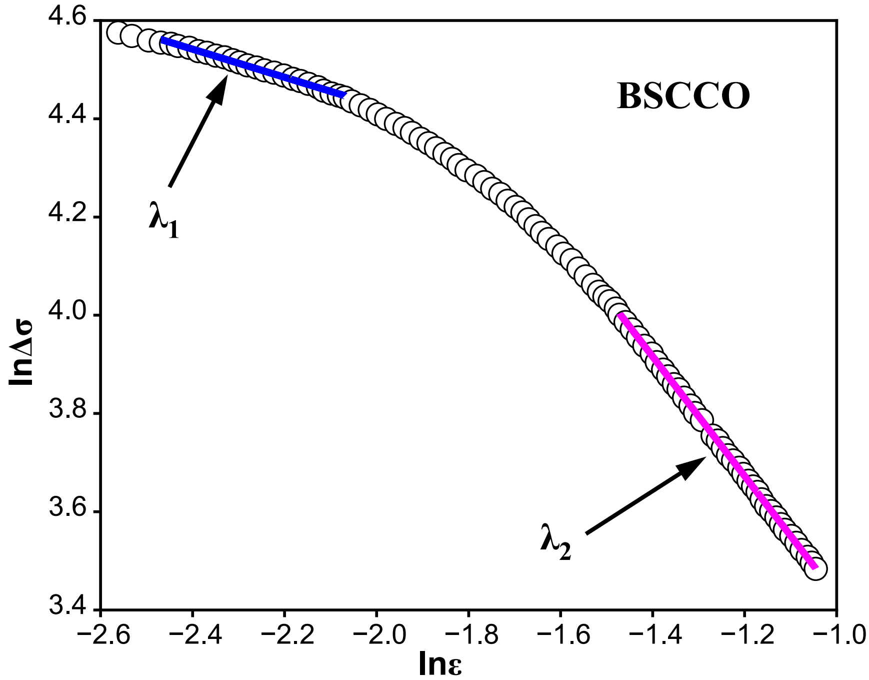

Figure 2. The lnΔσ versus lnε dependence for $Bi_{1.6}Pb_{0.4}Sr_2Ca_2Cu_3O_x$ (BSCCO) tape. The straight lines fitted to linear regions giving λ_1 and λ_2, critical exponents at the zero applied field. The curve is reproduced by 'OriginPro' software and we have obtained $\lambda_1 = -0.286$ and $\lambda_2 = -1.217$ very close to the original values found experimentally **[12]**.

Finally, we take two examples for determining λ, a useful method discussed earlier (method-III).

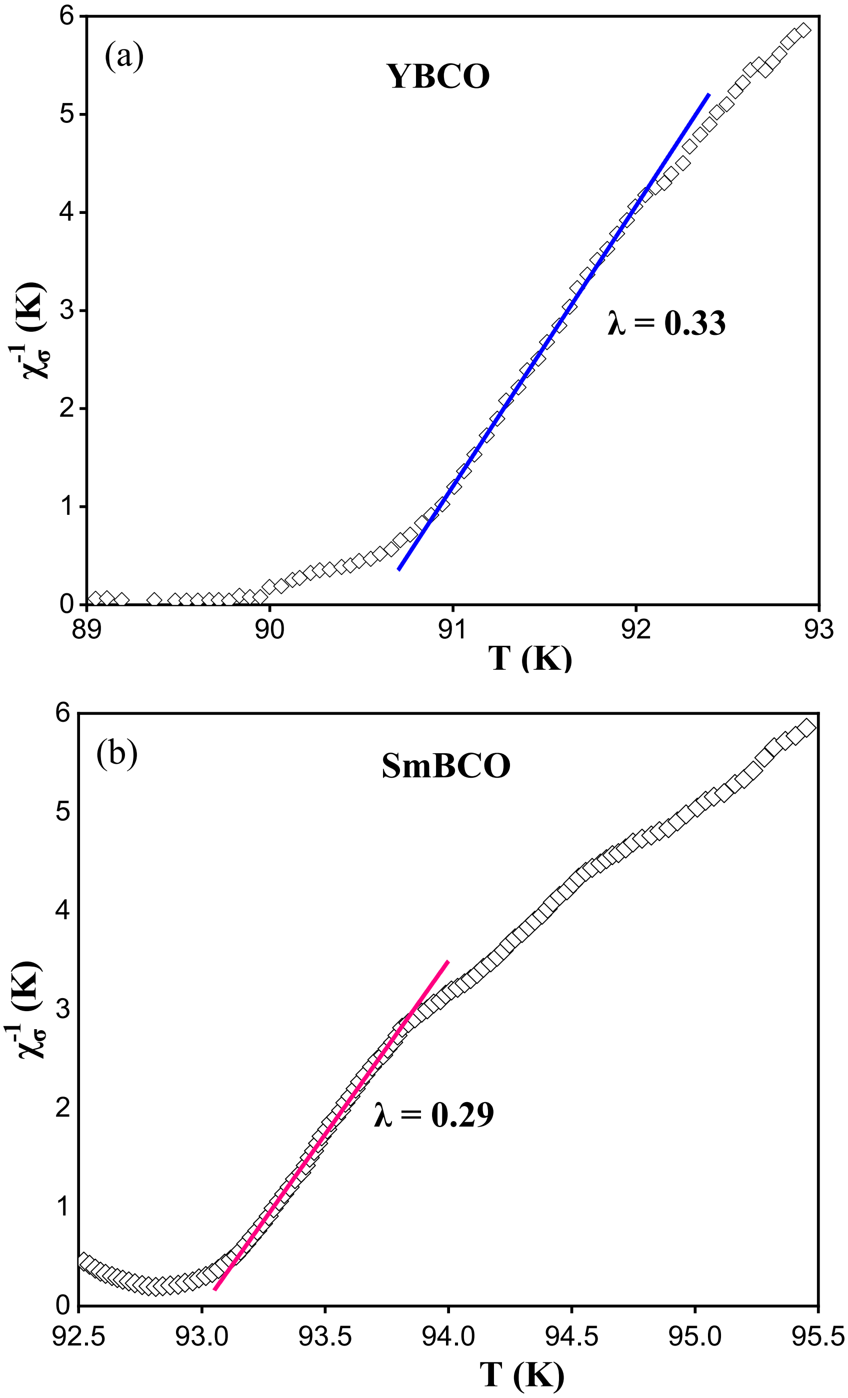

Figure 3: The plot of χ_σ^{-1} versus T **(a)** for sample $YBa_2Cu_3O_{7-\delta}$ (YBCO) **[13]** and **(b)** for sample $SmBa_2Cu_3O_{7-\delta}$ (SmBCO) **[14]**. The straight lines are the theoretical fit for estimating the value of λ.

4. Discussion and Conclusion

In this report, we have presented few theoretical ideas to determine the critical exponents of high-temperature superconductors, using paraconductivity approach. To estimate this parameter, we have used well-known paraconductivity expressions. We obtained the critical exponent constants $\lambda_1 = -0.286$ and $\lambda_2 = -1.217$, respectively for the sample BSCCO tape (method-II). The critical exponents have been obtained for the samples YBCO and SmBCO is $\lambda = 0.33$ and $\lambda = 0.29$, respectively (method-III). For the estimation of these constants, we have used the published values of the related parameters for the mentioned samples. The theoretical methods are well match and comaparable with experimental values. Finally, we may concluded that different theoretical ideas developed here is appropriate for the determination of the critical exponent constant of HTSCs .

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