Mass spectra of quarkonium systems in the shifted generalized Cornell–inverse quadratic potential model

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Received 21 August 2025, Revised 12 October 2025, Published 30 September 2025

Abstract: In this study, we present an application of the Shifted Generalized Cornell–Inverse Quadratic Potential (SG-CIQP) to heavy quarkonium systems. By solving the radial Schrödinger equation with the Pekeris-type approximation within the Nikiforov–Uvarov method, we derive closed-form expressions for both the energy eigenvalues and wave functions. This approach is applied to charmonium and bottomonium mesons, yielding mass spectra in excellent agreement with experimental data and established theoretical predictions. Notably, the S-wave states are reproduced with high precision, while the P-wave states are captured with quantitatively reliable accuracy, with minor deviations in the charmonium sector attributable to relativistic and coupled-channel effects. These results not only confirm the robustness of the SG-CIQP framework but also establish its potential as a versatile tool for extending quarkonium studies to spin-dependent interactions, relativistic corrections, and the spectroscopy of exotic hadronic states.

Keywords: Schrödinger equation; Quarkonium spectroscopy; Shifted Generalized Cornell–Inverse Quadratic Potential; Nikiforov–Uvarov method; Pekeris-type approximation

1. Introduction

The study of quarkonium systems—bound states of a heavy quark and its antiquark remain one of the most active and illuminating areas of modern particle physics. These systems provide a crucial platform for testing quantum chromodynamics (QCD), the fundamental theory governing the strong interaction. Heavy quarkonia, particularly charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$), are of special importance because the large masses of the charm and bottom quarks allow their dynamics to be approximated using non-relativistic quantum mechanics [1-5]. This makes them excellent laboratories for probing both the perturbative regime of QCD, dominated by short-distance gluon exchange, and the non-perturbative regime, governed by confinement and other long-range effects [6]. Although significant advances have been achieved, the precise description of quarkonium properties—including their mass spectra—remains incomplete. This ongoing challenge motivates the development and application of improved potential models [7-10]. The challenge has grown with the advent of next-generation facilities such as the Large Hadron Collider (LHC), Belle II, and BESIII, whose precision demands increasingly accurate and predictive theoretical models [11,12]. To meet this challenge, potential models have emerged as a powerful phenomenological framework for describing the quark–antiquark interaction. The Cornell potential, with its short-range Coulomb-like term and long-range linear confining term, has been particularly influential. However, its simplicity necessitates refinements to account for higher-order effects, and excited-state behavior. To address these shortcomings, several extensions of the Cornell potential and related models have been proposed [13,14]. These include the addition of inverse quadratic terms to capture orbital effects, screening modifications to account for quark-gluon plasma dynamics, logarithmic corrections, and relativistic spin-dependent contributions. Recent studies highlight the diversity and vitality of this approach. For example, Kanago et al. [15] investigated heavy quarkonia in a curved space-time background with conical geometry, applying an extended Cornell potential and solving via the bi-confluent Heun function to demonstrate sensitivity to topological defects. Inyang et al. [16] employed the Nikiforov–Uvarov (NU) method with the Cornell potential to explore the influence of these defects on thermal properties and meson mass spectra, achieving better consistency with experimental data. Kumar et al. [17] studied heavy meson spectra with a sextic anharmonic oscillator potential within the NU framework and extended their analysis to thermodynamic functions such as entropy and partition functions. Abu-Shady and Fath-Allah [18] advanced the field by applying a generalized fractional NU method to the fractional Klein–Gordon equation with screened Kratzer and Yukawa potentials to analyze meson mass spectra. Other works have applied alternative approaches. Rani et al. [19] used the asymptotic iteration method for general Cornell-type potentials, reporting excellent agreement with experimental spectra for heavy and heavy-light mesons. Purohit et al. [20] employed the NU method with a linear-plus-modified-Yukawa potential to study heavy-light mesons, accurately reproducing lower-state masses. Horchani et al. [21] investigated mesons in higher-dimensional spaces with a Killingbeck plus inverse quadratic potential, while Atangana Likéné et al. [22] introduced conformable fractional derivatives into the Schrödinger equation with an extended Cornell form. Omugbe et al. [23] applied a Pekeris-type scheme within the WKB approximation to solve the Killingbeck plus inverse quadratic potential, achieving results consistent with both experiment and other analytical methods. Recent numerical and semi-analytical efforts further underscore the variety of techniques in this field. Kaushal and Bhaghyesh [23] used a screened potential with corrections and the Matrix–Numerov method to compute charmed hadron and diquark spectra, reproducing observed charmonium and triply charmed baryon states. Reggab [24] solved the Schrödinger equation with the Cornell potential using the NU method, with modified parameter forms linked to Kratzer and anharmonic potentials, producing results in close agreement with available ED. Collectively, these works demonstrate the wide applicability of both analytical and numerical techniques in refining quarkonium models. Building on this foundation, we propose the Shifted Generalized Cornell–Inverse Quadratic (SGCIQ) potential as a refined framework for heavy quarkonium studies. This potential is expressed as

$$V(r)=\frac{A_0}{r^2}+B_0 r-\frac{C_0}{r}+D_0 \tag{1}$$

with A_0, B_0, C_0 *and* D_0 as arbitrary potential parameters. The potential in Eq. (1) reduces to the Cornell potential if we set the constants $(A_0=D_0=0)$ [16]. C_0 is the coupling constant and B_0 is a linear confinement parameter. where the inverse quadratic term enhances modeling of short- to medium-range dynamics, and the constant shift adjusts the overall mass scale while leaving level splittings intact. Solving the Schrödinger equation for this potential is mathematically nontrivial, yet analytic solutions are invaluable for interpreting spectroscopy and computing observables.

In this work, we employ the Nikiforov–Uvarov method to obtain closed-form expressions for the energy eigenvalues and wavefunctions of the SGCIQ potential. A detailed numerical analysis is carried out for charmonium and bottomonium spectra, with results benchmarked against experimental data and existing theoretical models. By doing so, this study not only deepens the theoretical understanding of heavy quarkonium systems but also establishes the SGCIQ potential as a viable and predictive framework consistent with the precision of modern experimental findings.

2. Approximate Solutions for the SG-CIQ Potential

The Schrödinger equation (SE) interacting via potential $V(r)$, is given by [25]

$$\frac{d^2R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[\left(E_{nl}-V(r)\right)-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r)=0 \tag{2}$$

where l, μ, r and $\hbar$ are the angular momentum quantum number, the reduced mass for the quarkonium particle, inter-particle distance and reduced plank constant respectively. Substituting Eq. (1) into Eq. (2), we arrive at the following expression

$$\frac{d^2R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[\left(E_{nl}-\frac{A_0}{r^2}-B_0 r+\frac{C_0}{r}-D_0\right)-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r)=0 \tag{3}$$

Transforming the coordinate of Eq. (3), with $x=\frac{1}{r}$, and after differentiation and simplification, Eq. (4) becomes

$$\frac{d^2R(x_0)}{dx_0{}^2}+\frac{2}{x_0}\frac{dR}{dx_0}+\frac{2\mu}{\hbar^2 x_0{}^4}\left[E-(A_0x_0{}^2+\frac{B_0}{x_0}-C_0x_0+D_0)-\frac{l(l+1)\hbar^2}{2\mu}x_0{}^2\right]R(x_0)=0 \tag{4}$$

To solve Eq. (4), we employ the approximation scheme reported in the literature [26]. The term with $\frac{\alpha_1}{x_0}$ is expanded in a second-order power series around r_0 (equivalently $\delta_0\equiv\frac{1}{r_0}$) in the x-space, which is assumed to represent the characteristic radius of the meson. This approximation transforms the potential into a tractable form suitable for solution via the NU method [27].

With $y_0 = x_0 - \delta_0$, expansion around $y_0 = 0$ yields the power series

$$\frac{\alpha_1}{x_0} = \alpha_1\left(\frac{3}{\delta_0} - \frac{3x_0}{\delta_0^{\ 2}} + \frac{x_0^{\ 2}}{\delta_0^{\ 3}}\right) \tag{5}$$

Substituting Eq. (5) into Eq. (4) yields

$$\frac{d^2R(x_0)}{dx_0^{\ 2}} + \frac{2x_0}{x_0^{\ 2}}\frac{dR(x_0)}{dx_0} + \frac{1}{x_0^{\ 4}}\left[-\varepsilon + \alpha x_0 - \beta x_0^{\ 2}\right]R(x_0) = 0 \tag{6}$$

where

$$\left.-\varepsilon = \frac{2\mu}{\hbar^2}\left(E - \frac{6B_0}{\delta_0} - D_0\right),\ \alpha = \frac{2\mu}{\hbar^2}\left(C_0 + \frac{3B_0}{\delta_0^{\ 2}}\right),\ \beta = \frac{2\mu}{\hbar^2}\left(A_0 + \frac{B_0}{\delta_0^{\ 3}} + \frac{l(l+1)\hbar^2}{2\mu}\right)\right\} \tag{7}$$

Comparison of Eq. (6) with Eq. (A1) in Ref. [1] yields

$$\left.\begin{aligned}&\tilde{\tau}(x_0) = 2x_0,\ \sigma(x_0) = x_0^{\ 2}\\&\tilde{\sigma}(x_0) = -\varepsilon + \alpha x_0 - \beta x_0^{\ 2}\\&\sigma'(x_0) = 2x_0,\ \sigma''(x_0) = 2\end{aligned}\right\} \tag{8}$$

Equation (8), when substituted into Eq. (A9) of Ref. [1], yields

$$\pi(x_0) = \pm\sqrt{\varepsilon - \alpha x_0 + (\beta + k)x_0^{\ 2}} \tag{9}$$

To obtain k, the discriminant of the expression beneath the square root is evaluated, giving

$$k = \frac{\alpha^2 - 4\beta\varepsilon}{4\varepsilon} \tag{10}$$

Equation (9), when substituted into Eq. (10), yields

$$\pi(x_0) = \pm\left(\frac{\alpha x_0}{2\sqrt{\varepsilon}} - \frac{\varepsilon}{\sqrt{\varepsilon}}\right) \tag{11}$$

For bound-state problems, the physically acceptable solution corresponds to the negative part of Eq. (11). Differentiation of this expression gives

$$\pi'_-(x_0) = -\frac{\alpha}{2\sqrt{\varepsilon}} \tag{12}$$

Equations (8) and (12), when substituted into Eq. (A7) of Ref. [1], yield

$$\tau(x_0) = 2x_0 - \frac{\alpha x_0}{\sqrt{\varepsilon}} + \frac{2\varepsilon}{\sqrt{\varepsilon}} \tag{13}$$

Differentiation of Eq. (13) yields

$$\tau'(x_0) = 2 - \frac{\alpha}{\sqrt{\varepsilon}} \tag{14}$$

From Eq. (A10) of Ref. [1], we obtain

$$\lambda = \frac{\alpha^2 - 4\beta\varepsilon}{4\varepsilon} - \frac{\alpha}{2\sqrt{\varepsilon}} \tag{15}$$

From Eq. (A11) of Ref. [1], this results in

$$\lambda_n = \frac{n\alpha}{\sqrt{\varepsilon}} - n^2 - n \tag{16}$$

Equating Eqs. (15) and (16), with Eq. (7) substituted, the energy eigenvalues are expressed as

$$E_{nl} = \frac{3D_0}{\delta_0^{\,2}} + D_0 - \frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(C_0 + \frac{3\mu}{\hbar^2\delta_0^{\,2}}\right)}{(2n+1) + \sqrt{1 + \frac{8\mu}{\hbar^2}\left(A_0 + \frac{B_0}{\delta_0^{\,3}} + \frac{l(l+1)\hbar^2}{2\mu}\right)}}\right]^2 \tag{17}$$

The wavefunction can be written as

$$R_{nl}(x) = (x)^{\frac{W}{2\sqrt{T}}} e^{\frac{\sqrt{T}}{x}} B_{nl} \frac{d^n}{dx^n}\left[(x)^{2n} x^{-\frac{W}{\sqrt{T}}} e^{-\frac{2\sqrt{T}}{x}}\right] \tag{18}$$

Here, B_{nl} denotes the normalization constant, obtainable from

$$\int_0^\infty |\mathrm{B}_{nl}(r)|^2 dr = 1 \tag{19}$$

3. Quarkonium Bound-State Interactions

Charmonium and bottomonium mass spectra for radial and angular momentum quantum numbers were evaluated using the meson mass relation [28,29].

$$M = 2m + E_{nl} \tag{20}$$

In this framework, m denotes the bare mass of the quarkonium, while E_{nl} represents the corresponding energy eigenvalues.

Equation (17), when substituted into Eq. (20), provides the expressions for the SG-CIQ potential mass spectra

$$M = 2m + \frac{3D_0}{\delta_0^{\,2}} + D_0 - \frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(C_0 + \frac{3\mu}{\hbar^2\delta_0^{\,2}}\right)}{(2n+1) + \sqrt{1 + \frac{8\mu}{\hbar^2}\left(A_0 + \frac{B_0}{\delta_0^{\,3}} + \frac{l(l+1)\hbar^2}{2\mu}\right)}}\right]^2 \tag{21}$$

4. Discussion of results

The model parameters were determined by fitting Eq. (21) to the experimental data in Tables [1–2]. For charmonium and bottomonium mesons, three algebraic equations were solved simultaneously with the aid of the MAPLE package, using the experimental masses of the 1S, 2S, and 3S states as input. Tables 1 and 2 compare the charmonium and bottomonium spectra calculated with the Shifted Generalized Cornell–Inverse Quadratic Potential (SG-CIQP) to experimental data and other theoretical models. The S-wave states (1S–4S) show excellent agreement with experiment in both systems, demonstrating that the SG-CIQP successfully captures the dominant central potential arising from color-Coulomb attraction at short distances and linear confinement at larger separations. These features are central to quark–antiquark binding in quantum chromodynamics (QCD) and confirm that the model provides a reliable description of the radial excitations of heavy quarkonia.

The P-wave states also show good agreement, though small deviations emerge. In charmonium, the P-wave levels lie slightly below experiment, which can be understood in terms of relativistic corrections and coupled-channel effects. The lighter charm quark allows stronger mixing with open-charm thresholds, which shifts the levels relative to a purely static potential model. In bottomonium, where the heavier quark mass suppresses relativistic effects and threshold couplings, the P-wave states are reproduced with higher accuracy. This contrast between the two systems reflects a fundamental aspect of hadron physics: charmonium lies in an intermediate regime where both perturbative QCD (at short distances) and nonperturbative effects (confinement and channel mixing) play significant roles, while bottomonium is closer to an ideal nonrelativistic bound system.

From a broader hadron-physics perspective, the agreement of the SG-CIQP with both S- and P-wave states validates its potential structure as a balance of short-range gluon exchange and long-range confinement dynamics. The fact that S-wave levels, which are more sensitive to the short-distance Coulombic part, and P-wave levels, which probe the confining and spin-dependent contributions, are both reasonably well reproduced suggests that the SG-CIQP captures the essential physics across different regimes. This highlights its relevance not only for conventional quarkonium but also as a foundation for studying exotic hadrons, such as tetraquarks and hybrids, where similar interplay between color-Coulomb and confinement forces dictates the spectrum.

Table 1. The Charmonium mass spectrum expressed in GeV

$$\begin{pmatrix} m_c = 1.209GeV, A_0 = 2.032GeV, B_0 = 0.2GeV^2, \\ C_0 = 1.243GeV, D_0 = 0.461GeV, \delta_0 = 0.232GeV \end{pmatrix}$$

States	The present analysis	Ref. [13]	Ref. [14]	Experiment [30-32]
1s	3.096839898	3.0969	3.098	3.097
2s	3.686005578	3.68697	3.688	3.686
3s	4.039049024	4.04143	4.029	4.039
4s	4.267153785	4.27086	-	4.263
1p	3.510571621	3.25581	3.516	3.511
2p	3.912773823	3.77951	3.925	3.927
3p	4.097620019	4.09997	4.301	3.097

Table2. The bottomonium mass spectrum expressed in GeV

$$\begin{pmatrix} m_B = 4.823GeV, A_0 = 1.033GeV, B_0 = 0.2GeV^2, \\ C_0 = 1.553GeV, D_0 = 1.061GeV, \delta_0 = 0.381GeV \end{pmatrix}$$

States	The present analysis	Ref. [13]	Ref. [14]	Experiment [30-31]
1s	9.460424446	9.45851	9.460	9.460
2s	10.02337702	10.0218	10.026	10.023
3s	10.35386764	10.3539	10.354	10.355
4s	10.57826936	10.5661	10.572	10.579
1p	9.898553580	9.61781	9.891	9.899
2p	10.26088755	10.1127	10.258	10.260
3p	10.51273350	10.4106	10.518	10.512

5. Conclusion

In this work, we solved the radial Schrödinger equation for the Shifted Generalized Cornell–Inverse Quadratic Potential (SG-CIQP) and derived analytical expressions for the energy eigenvalues and wave functions using the Nikiforov–Uvarov method. The model was applied to study the mass spectra of charmonium and bottomonium mesons. Results show that the SG-CIQP provides a consistent description of both S- and P-wave states, with predictions in good agreement with experimental data and other theoretical models. This demonstrates its reliability as a framework for heavy quarkonium physics.

The potential's flexibility also makes it suitable for further studies, including the incorporation of spin-dependent interactions, relativistic corrections, and higher excitations. It may additionally be applied to exotic hadrons such as tetraquarks and pentaquarks, or to explore quarkonium properties in the quark–gluon plasma. Thus, the SG-CIQP offers a versatile and accurate approach to modeling quark–antiquark interactions.

Data availability

No external datasets were used in this study. All data were generated numerically from the analytical solutions.

Declaration of Competing Interest

The author declares that there are no financial or personal conflicts of interest that could have influenced the work reported in this manuscript.

Acknowledgements

The author gratefully acknowledges the 2025 TETFund Institutional Based Research (IBR) grant of the National Open University of Nigeria for providing support for this study.

Funding

This research was funded by the 2025 TETFund Institutional Based Research (IBR) grant of the National Open University of Nigeria.

Author Contributions

E. P. Inyang: Conceptualization; Methodology; Formal analysis; Writing – original draft; Writing – review and editing; Final approval of the manuscript.

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