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RAYLEIGH RELIABILITY FOR 1 STRENGTH- 4 STRESSES

Ahmed H. Khaleel
Department of Statistics, Faculty of Administration & Economics, University of Sumer, Iraq
ahmedharoon885@gmail.com

Abstrak: Ketika fungsi keandalan ditemukan untuk model tertentu, perhatian harus diberikan pada faktor-faktor yang mempengaruhi model ini, yang sering membagi faktor-faktor ini menjadi faktor daya tahan yang dimiliki oleh komponen-komponen model dan faktor-faktor tegangan yang dimiliki oleh komponen-komponen model tersebut. terbuka. Dalam tulisan ini, ditemukan fungsi reliabilitas untuk salah satu model stress-robustness dimana model tersebut terdiri dari satu komponen yang mempunyai ketahanan yang dinyatakan dengan variabel acak X dan dikenakan empat tegangan yang dinyatakan dengan variabel acak $(Y_1,\ Y_2, Y_3\ , Y_4)$ asumsikan bahwa variabel acak tersebut variabel mengikuti distribusi Rayleigh. Parameter distribusi diestimasi dengan tiga metode estimasi (metode kemungkinan maksimum, metode kuadrat terkecil, dan metode kuadrat terkecil tertimbang), setelah itu fungsi keandalan model diestimasi. Simulasi Monte Carlo juga dilakukan untuk membandingkan hasil yang diperoleh dari estimasi menggunakan kriteria mean squared error. Perbandingan tersebut menunjukkan bahwa ML merupakan estimator terbaik dari fungsi reliabilitas.

Kata kunci:*Komponen, Kemungkinan maksimum, Distribusi Rayleigh, Keandalan, Simulasi*.

Abstract: When a reliability function is found for a particular model, attention should be paid to the factors affecting this model, which often divide these factors into the durability factors possessed by the components of the model and the stress factors to which the components of the model are exposed. In this paper, a reliability function was found for one of the stress-robustness models where the model consists of one component that has robustness expressed by the random variable X and is subjected to four stresses expressed by random variables $(Y_1,\ Y_2, Y_3\ , Y_4)$ assume that the random variables follow the Rayleigh distribution. The distribution parameters were estimated by three methods of estimation (maximum likelihood method, least squares method and weighted least squares method), after which the reliability function of the model was estimated. A Monte Carlo simulation was also performed to compare the results obtained from the estimate using the mean squared error criterion. The comparison showed that ML is the best estimator of the reliability function.

Keywords: *Component, Maximum likelihood, Rayleigh distribution, Reliability, Simulation*.

INTRODUCTION

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Interest in the term stress-strength in industrial systems increased significantly before the second half of the twentieth century. When dealing with the term stress-strength, we show the relationship between two random variables, one represents stress(Khaleel, 2021b), the other represents strength and tries to find the probability that one will overcome the other. The development that has taken place in the world in various branches, such as scientific, medical and construction fields, has led to extension and complexity in stress and strength models (Shang & Yan, 2024), and meanwhile, the dependency is basis for measuring the performance of manufacturing models work ended time (Sarhan & Tolba, 2023), so it has a major effect on refining the performance of this systems and increases their efficiency (Karam, Yousif, Karam, & Abood, 2022; Khan & Jan, 2014). Reliability is defined as the lifetime of a component where the component remains in a working state as long as it can resist the stresses to which it is subjected, expressed by the random variable Y with its strength, and expressed by the random variable X, where $R = P(Y < X)$ and stops working (fails) if the stresses exceed the strength of the component $X < Y$ (Jebur, Kalaf, & Salman, 2020; Patowary, Hazarika, & Sriwastav, 2018).

Many papers included strength-stress, (Haddad & Batah, 2021) studied the reliability of the strength-stress model when the factors follow the (Rayl. – Par) distribution. (Karam & Khaleel, 2019; Khaleel & Karam, 2019) derived a special reliability model from the cascade models (2+1), where the model contains two basic components and an excess component with an active standby state. (Khaleel, 2021a) studied a special model of reliability (3+1), which contains three basic components and a component with an active standby state. (Khaleel, 2021c) derived a model of a single component that has strength and is subjected to several stresses when the stress and strength factors follow the Lomax distribution. (Salman & Hamad, 2019) studied the estimation of the reliability function by several different estimation methods when the stress and durability factors trace the Lomax distribution.

This paper aims to find a reliability function for a model consisting of one component, where this component is subjected to four stresses, and these stresses have their own strength, assuming that the stress and strength factors follow the Rayleigh distribution and are independent, as well as estimating the reliability function in three estimation methods "maximum likelihood", "least square" and "weighted least square" and working Monte Carlo simulation to compare the estimation results.

RESEARCH METHODS

Mathmatical Model

It is known that the life of a component is determined in stress - strength models according to its strength X, through which it can resist the stress Y to which the component is exposed (Khaleel & Khlefha, 2021), where it is $(X > Y)$, but if the stresses become greater than the strength of the component $(Y > X)$, the component fails and does not continue to work .The mathematical formula for the reliability of a one-component model can be expressed as follows (Hamad & Salman, 2021):

$$\mathcal{R} = \mathrm{pr}(\mathrm{Y} < \mathrm{X}) = \int_{-\infty}^{\infty} \mathrm{f}(x)\mathrm{F}_{\mathcal{Y}}(x)\mathrm{dx} \qquad \ldots(1)$$

As for the case when the component is subjected to four stresses $(Y_1, Y_2, Y_3$ and $Y_4)$ and resists these stresses with one strength (X), the reliability of this model can be expressed:

$$\mathcal{R} \int_{-\infty}^{\infty} \mathrm{pr}(\mathrm{Y}_1 < \mathrm{X})\mathrm{pr}(\mathrm{Y}_2 < \mathrm{X})\mathrm{pr}(\mathrm{Y}_3 < \mathrm{X})\mathrm{pr}(\mathrm{Y}_4 < \mathrm{X})\mathrm{f}_{x(x)}\,\mathrm{dx}$$

Assume that the random variables of stress and durability are independent and indexical, so the mathematical formula for the reliability of the model is as follows:

$$\mathcal{R} = pr(Max(Y_1, Y_2, Y_3, Y_4) < X)$$

$$= \int_0^{\infty} \int_0^{\mathcal{Y}_1} \int_{\mathcal{Y}_0}^{\mathcal{Y}_2} \int_0^{\mathcal{Y}_3} \int_0^{\mathcal{Y}_4} f(\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \mathcal{Y}_4, x)\, d\mathcal{Y}_4 d\mathcal{Y}_3 d\mathcal{Y}_2 d\mathcal{Y}_1 dx \qquad \ldots(3)$$

Then

$$\mathcal{R} = \int_0^{\infty} \int_0^{\mathcal{Y}_1} \int_0^{\mathcal{Y}_2} \int_0^{\mathcal{Y}_3} \int_0^{\mathcal{Y}_4} f(\mathcal{Y}_1)f(\mathcal{Y}_2)f(\mathcal{Y}_3)f(\mathcal{Y}_4)\, f(x) d\mathcal{Y}_4 d\mathcal{Y}_3 d\mathcal{Y}_2 d\mathcal{Y}_1 dx$$

$$\mathcal{R} = \int_0^{\infty} F_{1\mathcal{Y}_1}(x) F_{2\mathcal{Y}_2} F_{3\mathcal{Y}_3}(x) F_{4\mathcal{Y}_4}(x) f(x) dx \qquad \ldots(4)$$

Assuming that the random variables follow a Raleigh distribution where $X \sim R\ (2, \delta)$ and $Y_r \sim R(2, \delta_r)$;r=1,2,3,4 then the pdf and CDF are:

$$f(x) = \frac{2}{\delta} x\, e^{-\frac{x^2}{\delta}} \qquad \ldots(5)$$

$$F(x) = 1 - e^{-\frac{x^2}{\delta}} \qquad \ldots(6)$$

And

$$F_r(\mathcal{Y}_\mathrm{r}) = 1 - e^{-\frac{\mathcal{Y}_r^2}{\delta_r}} \quad ;\mathrm{r}=1,2,3,4 \qquad \ldots(7)$$

Equations 5, 6 and 7 will be used in Equation 4 as follows:

$$\mathcal{R} = \int_0^{\infty} \left[[1 - \mathrm{e}^{-\frac{x^2}{\delta_1}}]\, [1 - \mathrm{e}^{-\frac{x^2}{\delta_2}}]\, [1 - \mathrm{e}^{-\frac{x^2}{\delta_3}}]\, [1 - \mathrm{e}^{-\frac{x^2}{\delta_4}}] \right] \frac{2}{\delta} x\, \mathrm{e}^{-\frac{x^2}{\delta}} dx$$

$$= \int_0^{\infty} \Bigg[1 - e^{-\frac{x^2}{\delta_1}} - e^{-\frac{x^2}{\delta_2}} - e^{-\frac{x^2}{\delta_3}} - e^{-\frac{x^2}{\delta_4}} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}\right)x^2} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_3}\right)x^2} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_4}\right)x^2}$$

$$+ e^{-\left(\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2} + e^{-\left(\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2} + e^{-\left(\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} - e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2} - e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2}$$

$$- e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} - e^{-\left(\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} \Bigg] \frac{2}{\delta} x\, e^{-\frac{x^2}{\delta}} dx$$

$$\mathcal{R}=\int_0^\infty \frac{2}{\delta}x\,e^{-\frac{x^2}{\delta}}dx-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}\right)x^2}dx-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}\right)x^2}dx-\int_0^\infty \frac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_3}\right)x^2}dx$$

$$-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_4}\right)x^2}dx+\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}\right)x^2}dx+\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}\right)x^2}dx+\int_x^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_4}\right)x^2}dx$$

$$+\int_0^\infty \frac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2}dx+\int_0^\infty \frac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2}dx+\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx$$

$$-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2}dx-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2}dx-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx$$

$$-\int_0^\infty \frac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx+\int_0^\infty \frac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx$$

Then

$$\mathcal{R}=1-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_3}\right)}\right]-\left[\frac{1}{\beta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}\right)}\right]$$

$$+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)}\right]$$

$$-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]$$

Finally, the reliability model is:

$$\mathcal{R}=1-\left[\frac{\delta_1}{(\delta+\delta_1)}\right]-\left[\frac{\delta_2}{(\delta+\delta_2)}\right]-\left[\frac{\delta_3}{(\delta+\delta_3)}\right]-\left[\frac{\delta_4}{(\delta+\delta_4)}\right]+\left[\frac{\delta_1\delta_2}{(\delta\delta_1+\delta\delta_2+\delta_1\delta_2)}\right]+\left[\frac{\delta_1\delta_3}{(\delta\delta_1+\delta\delta_3+\delta_1\delta_3)}\right]$$

$$+\left[\frac{\delta_1\delta_4}{(\delta\delta_1+\delta\delta_4+\delta_1\delta_4)}\right]+\left[\frac{\delta_2\delta_3}{(\delta\delta_2+\delta\delta_3+\delta_2\delta_3)}\right]+\left[\frac{\delta_2\delta_4}{(\delta\delta_2+\delta\delta_4+\delta_2\delta_4)}\right]+\left[\frac{\delta_3\delta_4}{(\delta\delta_3+\delta\delta_4+\delta_3\delta_4)}\right]$$

$$-\left[\frac{\delta_1\delta_2\delta_3}{(\delta\delta_1\delta_2+\delta\delta_1\delta_3+\delta\delta_2\delta_3+\delta_1\delta_2\delta_3)}\right]-\left[\frac{\delta_1\delta_2\delta_4}{(\delta\delta_1\delta_2+\delta\delta_1\delta_4+\delta\delta_2\delta_4+\delta_1\delta_2\delta_4)}\right]-\left[\frac{\delta_1\delta_3\delta_4}{(\delta\delta_1\delta_3+\delta\delta_1\delta_4+\delta\delta_3\delta_4+\delta_1\delta_3\delta_4)}\right]$$

$$-\left[\frac{\delta_2\delta_3\delta_4}{(\delta\delta_2\delta_3+\delta\delta_2\delta_4+\delta\delta_3\delta_4+\delta_2\delta_3\delta_4)}\right]+\left[\frac{\delta_1\delta_2\delta_3\delta_4}{(\delta\delta_1\delta_2\delta_3+\delta\delta_1\delta_2\delta_4+\delta\delta_1\delta_3\delta_4+\delta\delta_2\delta_3\delta_4+\delta_1\delta_2\delta_3\delta_4)}\right] \quad \ldots(8)$$

Maximum likelihood function (ML):

To estimate the parameter δ by the ML method, we can start by using equation (5) as follows :

$$\mathrm{L}(x_1,x_2,\ldots,x_n;2,\delta)=\left(\frac{2}{\delta}\right)^{n}\prod_{i=1}^{n}x_i\,e^{-\frac{\sum_{i=1}^{n}x_i^2}{\delta}} \quad \ldots(9)$$

The logarithm is taken for equation 9:

$$\ln L=n\ln 2-n\ln\delta+\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i^2}{\delta} \quad \ldots(10)$$

Equation 10 is partially derived for the parameter δ:

$$\frac{\partial \ln L}{\partial\delta}=-\frac{n}{\delta}+\frac{\sum_{i=1}^{n}x_i^2}{\delta^2}$$

Then get as $\hat{\beta}_{(ML)}$:

$$\hat{\beta}_{(ML)}=\frac{\sum_{i=1}^{n}x_i^2}{n} \quad \ldots(11)$$

With the same previous steps, $\hat{\delta}_{1(ML)}$, $\hat{\delta}_{2(ML)}$, $\hat{\delta}_{3(ML)}$, $\hat{\delta}_{4(ML)}$ are obtained:

$$\hat{\delta}_{1(ML)} = \frac{\sum_{j_1=1}^{n_1} y_{j1}^2}{n_1} \quad ...(12)$$

$$\hat{\delta}_{2(ML)} = \frac{\sum_{j_2=1}^{n_2} y_{j2}^2}{n_2} \quad ...(13)$$

$$\hat{\delta}_{3(ML)} = \frac{\sum_{j_3=1}^{n_3} y_{j3}^2}{n_3} \quad ...(14)$$

$$\hat{\delta}_{4(ML)} = \frac{\sum_{j_4=1}^{n_4} y_{j4}^2}{n_4} \quad ...(15)$$

Least Square Method (LS) :

By using the minimization equation, it is possible to start estimating the parameters using the least squares method as follows:

$$S = \sum_{i=1}^{n}\left[F(X_{(i)}) - E\left(F(X_{(i)})\right)\right]^2 \quad ...(16)$$

to parameter δ of $X \sim R(2,\delta)$ with size n.

Equal to $E\left(F(X_{(i)})\right)$ with P_i ,where $P_i = \frac{i}{n+1}$; $i = 1,2,\dots,n$

$$(1 - P_i) = e^{-\frac{x_{(i)}^2}{\delta}}$$

Then

$$\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta} = 0 \quad ...(17)$$

Now equation 17 is used in equation 16:

$$S = \sum_{i=1}^{n}\left[\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta}\right]^2 \quad ...(18)$$

Equation 18 is derived for the parameter δ:

$$\frac{\partial S}{\partial \delta} = \sum_{i=1}^{n} 2\left[\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta}\right]\frac{-x_{(i)}^2}{\delta^2}$$

$$\sum_{i=1}^{n} 2\left[\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta}\right]\frac{-x_{(i)}^2}{\delta^2} = 0 \quad ...(19)$$

Then $\hat{\delta}_{(LS)}$ is obtained:

$$\hat{\delta}_{(LS)} = \frac{-\sum_{i=1}^{n} x_{(i)}^4}{\sum_{i=1}^{n} x_{(i)}^2 \ln(1-P_i)} \quad ....(20)$$

In the same way, $\hat{\delta}_{1(LS)}$, $\hat{\delta}_{2(LS)}, \hat{\delta}_{3(LS)}$ *and* $\hat{\delta}_{4(LS)}$ are obtained:

$$\hat{\delta}_{1(LS)} = \frac{-\sum_{j_1=1}^{n_1} y_{1(j_1)}^4}{\sum_{j_1=1}^{n_1} y_{1(j_1)}^2 \ln\left(1-P_{j_1}\right)} \quad ...(21)$$

$$\hat{\delta}_{2(LS)} = \frac{-\sum_{j_2=1}^{n_2} y_{2(j_2)}^4}{\sum_{j_2=1}^{n_2} y_{2(j_2)}^2 \ln\left(1-P_{j_2}\right)} \quad ...(22)$$

$$\hat{\delta}_{3(LS)} = \frac{-\sum_{j_3=1}^{n_3} y_{3(j_3)}^4}{\sum_{j_3=1}^{n_3} y_{3(j_3)}^2 \ln\left(1-P_{j_3}\right)} \quad ...(23)$$

$$\hat{\delta}_{4(LS)} = \frac{-\sum_{j_4=1}^{n_4} y_{4(j_4)}^4}{\sum_{j_4=1}^{n_4} y_{4(j_4)}^2 \ln\left(1-P_{j_4}\right)} \quad ...(24)$$

Weighted Least Square Method (WLS) :

By equations 17 and 18 can be used to find the estimate of parameter 1 by the weighted least squares method as follows [9]:

$$\sum_{i=1}^{n} w_i \left[ln(1-P_i) + \frac{x_{(i)}^2}{\delta}\right]^2 = 0 \quad ...(25)$$

Where $w_i = \frac{1}{Var[F(x_{(i)})]}$

By deriving (25) to δ:

$$\sum_{i=1}^{n} w_i x_{(i)}^2 \ln(1-P_i) + \frac{1}{\delta}\sum_{i=1}^{n} w_i x_{(i)}^4 = 0 \quad ...(26)$$

So, $\hat{\delta}_{(WLS)}$ is:

$$\hat{\delta}_{(WLS)} = \frac{-\sum_{i=1}^{n} w_i x_{(i)}^4}{\sum_{i=1}^{n} w_i x_{(i)}^2 \ln(1-P_i)} \quad ...(27)$$

In similar way, $\hat{\delta}_{1(WLS)}$, $\widehat{\delta}_{2(WLS)}$, $\widehat{\delta}_{3(WLS)}$ and $\hat{\delta}_{4(WLS)}$ are:

$$\hat{\delta}_{1(WLS)} = \frac{-\sum_{j_1=1}^{n_1} \mathrm{w}_{j_1} y_{1(j_1)}^4}{\sum_{j_1=1}^{n_1} \mathrm{w}_{j_1} y_{1(j_1)}^2 \ln\left(1-P_{j_1}\right)} \quad ...(28)$$

$$\hat{\delta}_{2(WLS)} = \frac{-\sum_{j_2=1}^{n_2} \mathrm{w}_{j_2} y_{2(j_2)}^4}{\sum_{j_2=1}^{n_2} \mathrm{w}_{j_2} y_{2(j_2)}^2 \ln\left(1-P_{j_2}\right)} \quad ...(29)$$

$$\hat{\delta}_{3(WLS)} = \frac{-\sum_{j_3=1}^{n_3} \mathrm{w}_{j_3} y_{3(j_3)}^4}{\sum_{j_3=1}^{n_3} \mathrm{w}_{j_3} y_{3(j_3)}^2 \ln\left(1-P_{j_3}\right)} \quad ...(30)$$

$$\hat{\delta}_{4(WLS)} = \frac{-\sum_{j_4=1}^{n_4} \mathrm{w}_{j_4} y_{4(j_4)}^4}{\sum_{j_4=1}^{n_4} \mathrm{w}_{j_4} y_{4(j_4)}^2 \ln\left(1-P_{j_4}\right)} \quad ...(31)$$

RESULTS AND DISCUSSION

Simulation

A Monte Carlo simulation is performed to compare the results of different estimation methods using MSE. Then the results obtained are deliberated to demonstration which estimation methods are the best, simulations are also made to compare estimation methods for R using different sample sizes (Alshanbari et al., 2022).

The Algorithm:

The MATLAB program is used in the simulation as shown in the steps below**:**

1. random samples $x_1, x_2, \dots, x_n$; $y_{11}, y_{12}, \dots, y_{1n_1}$; $y_{21}, y_{22}, \dots y_{2n_2}$; $y_{31}, y_{32}, \dots y_{3n_3}$ and $y_{41}, y_{42}, \dots y_{4n_4}$ of different sizes $(n, n_1, n_2, n_3, n_4) = (20,20,20,20,20), (40,40,40,40,40), (50,50,50,50,50), (70,70,70,70,70) and (90,90,90,90,90)$ are generated.

2. Let the values of the parameters $\delta, \delta_1, \delta_2, \delta_3, \delta_4$ be the reliability of the five experiments as shown below:

Table 1: The parameter values and reliability

Excrement	$\boldsymbol{\delta}$	$\boldsymbol{\delta_1}$	$\boldsymbol{\delta_2}$	$\boldsymbol{\delta_3}$	$\boldsymbol{\delta_4}$	R
1	0.7	0.7	0.7	0.7	0.7	**0.2000**
2	1.6	0.5	0.9	0.4	0.2	**0.5337**
3	0.9	1.3	1.1	1.1	1.4	**0.1294**
4	1.2	1.1	0.8	0.5	1.2	**0.2996**
5	2.6	0.3	0.6	0.3	0.5	**0.7163**

3. Parameters $(\delta, \delta_1, \delta_2, \delta_3, \delta_4)$ were estimated in equations (11), (12), (13), (14), (15), (20), (21), (22), (23), (24), (27),(28), (29), (30),(31).

4. Calculate the mean with the formula:

$$Mean = \frac{\sum_{i=1}^{L} \hat{\mathcal{R}}_i}{L}$$

5. By using the mean squares error, the results of the estimation methods are compared, the mathematical formula of which is:

$$\text{MSE}(\hat{\mathcal{R}}) = \frac{1}{L}\sum_{i=1}^{L}(\hat{\mathcal{R}}_i - \mathcal{R})^2$$

Results:

The tables below represent the results obtained from the simulation procedure:

Table 2: simulation results of Experiment 1

S.S	MLE	LSE	WLSE	Best
$(\mathbf{20,20,20,20,20})$	0.1662	0.1621	0.1569	MLE
	0.0123	0.0131	0.0138	
$(\mathbf{40,40,40,40,40})$	0.1640	0.1603	0.1527	
	0.0107	0.0114	0.0125	
$(\mathbf{50,50,50,50,50})$	0.1612	0.1587	0.1458	
	0.0086	0.0092	0.0109	
$(\mathbf{70,70,70,70,70})$	0.1585	0.1565	0.1376	
	0.0077	0.0081	0.0105	
$(\mathbf{90,90,90,90,90})$	0.1585	0.1565	0.1355	
	0.0074	0.0077	0.0104	

Table 3: simulation results of Experiment 2

S.S	MLE	LSE	WLSE	Best
$(20,20,20,20,20)$	0.4729	0.4674	0.4613	MLE
	0.0241	0.0265	0.0291	
$(40,40,40,40,40)$	0.4734	0.4676	0.4577	
	0.0211	0.0233	0.0270	
$(50,50,50,50,50)$	0.4765	0.4730	0.4577	
	0.0172	0.0186	0.0240	
$(70,70,70,70,70)$	0.4770	0.4745	0.4526	
	0.0159	0.0168	0.0240	
$(90,90,90,90,90)$	0.4803	0.4780	0.4547	
	0.0148	0.0156	0.0230	

Table 4: simulation results of Experiment 3

S.S	MLE	LSE	WLSE	Best
$(20,20,20,20,20)$	0.1103	0.1074	0.1036	MLE
	0.0074	0.0077	0.0080	
$(40,40,40,40,40)$	0.1076	0.1052	0.0994	
	0.0059	0.0062	0.0066	
$(50,50,50,50,50)$	0.1043	0.1028	0.0932	
	0.0045	0.0048	0.0056	
$(70,70,70,70,70)$	0.1027	0.1012	0.0869	
	0.0040	0.0042	0.0053	
$(90,90,90,90,90)$	0.1039	0.1029	0.0874	
	0.0039	0.0040	0.0051	

Table 5: simulation results of Experiment 4

S.S	MLE	LSE	WLSE	Best
$(20,20,20,20,20)$	0.2412	0.2338	0.2249	
	0.0230	0.0242	0.0256	MLE
$(40,40,40,40,40)$	0.2370	0.2312	0.2182	
	0.0213	0.0224	0.0246	
$(50,50,50,50,50)$	0.2381	0.2333	0.2107	
	0.0188	0.0197	0.0233	
$(70,70,70,70,70)$	0.2373	0.2341	0.2024	
	0.0175	0.0181	0.0231	
$(90,90,90,90,90)$	0.2386	0.2357	0.2011	
	0.0172	0.0178	0.0232	

Table 6: simulation results of Experiment 5

S.S	MLE	LSE	WLSE	Best
$(20,20,20,20,20)$	0.6060	0.5962	0.5842	
	0.0470	0.0508	0.0557	MLE
$(40,40,40,40,40)$	0.6048	0.5963	0.5784	
	0.0471	0.0503	0.0576	
$(50,50,50,50,50)$	0.6081	0.6018	0.5714	
	0.0438	0.0461	0.0578	
$(70,70,70,70,70)$	0.6085	0.6038	0.5605	
	0.0426	0.0443	0.0611	

$(\mathbf{90,90,90,90,90})$	0.6079	0.6039	0.5569	
	0.0412	0.0425	0.0600	

CONCLUSIONS

After completing the simulation study, we came to these conclusions:

- The reliability value increases by increasing the value of parameter δ and the reliability value decreases by increasing the values of parameters $(\boldsymbol{\delta_1}, \boldsymbol{\delta_2}, \boldsymbol{\delta_3}, \boldsymbol{\delta_4})$, and this is clear from Table1.
- The performance of the MLE estimator was the best for estimating $\mathcal{R}$.

Table 7: Best estimator of Model Reliability

Values of parameters and sample size	Best
Sizes of Sample $(n, n_1, n_2, n_3, n_4) = (20{,}20{,}20{,}20{,}20), (40{,}40{,}40{,}40{,}40)$, $(50{,}50{,}50{,}50{,}50), (70{,}70{,}70{,}70{,}70) and (90{,}90{,}90{,}90{,}90)$ and values of parameter are $(\delta, \delta_1, \delta_2, \delta_3, \delta_4) = (0.7{,}0.7{,}0.7{,}0.7{,}0.7), (1.6{,}0.5{,}0.9{,}0.4{,}0.2)$, $(0.9{,}1.3{,}1.1{,}1.1{,}1.4), (1.2{,}1.1{,}0.8{,}0.5{,}1.2)\ and\ (2.6{,}0.3{,}0.6{,}0.3{,}0.5)$,	MLE

- There is a convergence between the performance of MLE and LSE estimators.

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