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DOMINANCE NUMBER OF THE GRAPH RESULTING FROM COMB OPERATION BETWEEN COMPLETE GRAPH AND WHEEL GRAPH

Ajeng Maula Azizin[1] dan Ira Kurniawati[1]
[12)]Mathematics Education, Faculty of Teacher Training and Education, Sebelas Maret University, Surakarta, Indonesia

* Correspondence purposes, email: ajengmaula69@student.uns.ac.id

Abstrak: Tujuan dari penelitian ini adalah untuk mengetahui graf hasil operasi comb antara graf komplit dan graf roda serta rumus umum dari bilangan dominasi graf hasil operasi comb antara graf komplit dan graf roda. Metode penelitian yang digunakan adalah dengan studi literatur dengan mengumpulkan berbagai pustaka yang berkaitan dengan masalah yang diteliti. Dalam teori graf, bilangan dominasi merupakan banyaknya simpul pendominasi dalam suatu graf yang dapat mendominasi simpul-simpul terhubung disekitarnya, dengan simpul pendominasi berjumlah minimal dari simpul pendominasi simpul-simpul terhubung disekitarnya. Berbagai jenis graf pada bilangan dominasi semakin berkembang, salah satunya bilangan dominasi antara graf hasil operasi comb dari graf komplit dan graf roda. Operasi comb pada graf komplit dan graf roda yang dinotasikan sebagai $K_m \rhd W_n$ merupakan operasi yang dilakukan dengan cara mengambil duplikat dari K_m dan $|V(K_m)|$ duplikat dari W_n serta menempelkan simpul u_n pada duplikat ke-i dari W_n dengan simpul ke-i pada graf K_m. Bilangan dominasi dari graf $K_m \rhd W_n$ dibagi menjadi 4 kasus, yaitu: bilangan dominasi graf $K_m \rhd W_n$ dengan m & n ganjil, bilangan dominasi graf $K_m \rhd W_n$ dengan m & n genap, bilangan dominasi graf $K_m \rhd W_n$ dengan m genap & n ganjil dan bilangan dominasi graf $K_m \rhd W_n$ dengan m ganjil & n genap. Rumus umum bilangan dominasi graf hasil operasi comb dari graf komplit dan graf roda $K_m \rhd W_n$ dijelaskan pada artikel ini.

Kata kunci : *Bilangan dominasi, Operasi comb, Graf komplit, Graf roda*

Abstract: The purpose of this research is to know the graph of comb operation result between complete graph and wheel graph and the general formula of domination number of graph of comb operation result between complete graph and wheel graph. The research method used is a literature study by collecting various literature related to the problem under study. In graph theory, the domination number is the number of dominating vertices in a graph that can dominate the surrounding connected vertices, with the minimum number of dominating vertices of the surrounding connected vertices. Various types of graphs in domination numbers are growing, one of which is the domination number between graphs resulting from the comb operation of complete graphs and wheel graphs. The comb operation on complete graphs and wheel graphs denoted as $K_m \rhd W_n$ is an operation performed by taking duplicates of K_m and $|V(K_m)|$ duplicates of W_n and attaching vertex u_n on the i-th duplicate of W_n with the i-th vertex in the graph K_m. The dominance number of the graph $K_m \rhd W_n$ is divided into 4 cases, namely: domination number of graph $K_m \rhd W_n$ with m & n odd, domination number of graph $K_m \rhd W_n$ with m & n even, domination number of graphs $K_m \rhd W_n$ with m even & n odd and the domination number of graphs $K_m \rhd W_n$ with m odd & n even. General formula for the domination number of graphs

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resulting from the comb operation of complete graphs and wheel graphs $K_m \rhd W_n$ is written in this article.
Keywords: *Dominance number, Comb operation, Complete graph, Wheel graph*

INTRODUCTION

Discrete mathematics is one part of mathematics that studies discrete objects. One of the theories in discrete mathematics that has many benefits and is interesting to study is Graph Theory. Leonhard Euler was a Swiss mathematician who introduced graph theory for the first time in 1736 when solving the Konigsberg Bridge case. The problem was to examine the possibility of crossing each of the seven bridges in the city of Konigsberg exactly once and returning to the original place. In graph theory, one of the topics that can be widely developed and interesting to learn more about is dominance numbers.

In 1850 the number of domination appeared in Europe, namely among chess enthusiasts. They faced the problem of determining how many queens should be placed on an 8 x 8 chessboard so that the queen can control all the squares on the chessboard. In this case, the number of queens placed on the chessboard must be minimal, queens are represented as vertices and the movement path between squares on the chessboard is considered as an edge.

The dominance number represents the count of vertices within a graph capable of exerting dominance over adjacent connected vertices, all while minimizing the number of dominating vertices needed among those connected counterparts. According to Haynes (1996), the dominance set (S) of a graph G is a subset of V (G) such that every vertex of G that is not an element of S is connected and one-distance to S. The minimum cardinality among the dominance sets of a graph G is called the dominance number of the graph G and is denoted γ(G). Therefore, the dominance number is closely related to the dominance set.

Here are some theorems about dominance numbers on special graphs, including :

Theorem 1

If the graph K_m is a complete graph with m vertices, then γ (K_m) = 1

Proof :

The definition of graph domination number is the minimum cardinality among the domination sets in a graph G. Whereas in a complete graph, each vertex is an adjacent vertex so that only one vertex can be used as a domination number marked with a different color. Consider the following image

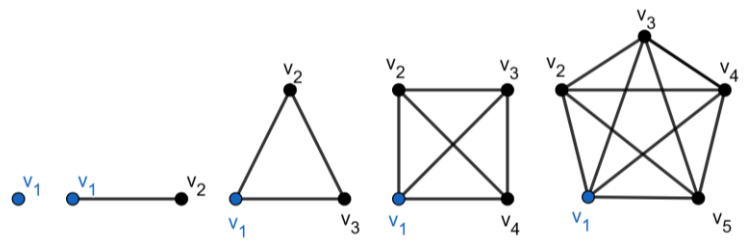


Figure 1. Domination number of complete graphs K_1 to K_5

The figure above shows the number of dominance numbers marked with different colors. Because each vertex is adjacent to another vertex, it forms a pattern so that for each vertex, there is 1 dominance number. K_m we get 1 number of dominance. Going back to the definition of graph dominance number, 1 is the minimum number obtained in the formation of the graph. K_m so that the dominance number of the graph K_m is 1.

Theorem 2

If the graph W_n is a wheel graph with n vertices, then γ (W_n) = 1

Proof :

Suppose u_0 is the center vertex of the wheel graph then every $u_1, u_2, \dots, u_n$ is directly connected to u_0. Hence $u_1, u_2, \dots, u_n$is the point that is dominated by u_0 Consider the following picture.

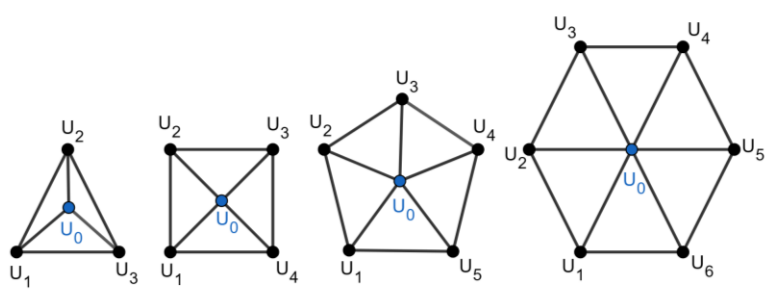


Figure 2. Domination number of complete graphs W_1 to W_5

The figure above shows the dominating vertices marked with different colors. Since each vertex is directly connected to the center vertex, it forms a pattern such that there is only one vertex that dominates all vertices in the Wheel graph. W_n. Going back to the definition of graph dominance number, it is found that 1 is the minimum number obtained in the graph formation W_n so that the dominance number of the graph W_n So, it is proven that If the graph W_n is a wheel graph with n vertices, then γ (W_n) = 1.

In several previous studies, domination numbers on simple graphs have been studied. Various types of graphs are increasingly developing so that further studies can be carried out, in this article the number of domination is presented, including the graph resulting from the comb operation of complete graphs and wheel graphs.

A complete graph is a straightforward graph in which every two distinct vertices are linked by an edge. A complete graph with m vertices is denoted by K_m, with m ≥ 1 and m natural numbers. Meanwhile, a wheel graph is created by introducing an additional vertex to a spindle graph, ensuring that this new vertex is directly linked to every vertex within the original spindle graph. A wheel graph with m vertices is denoted by W_n, with n ≥ 3 and n natural numbers. A wheel graph is formed from the sum of a cyclic graph and a complete graph.

The comb operation between a complete graph and a wheel graph denoted as $K_m \rhd W_n$ is an operation that is performed by taking one duplicate of the K_m and vertex |V(K_m)| a duplicate of W_n, and attaching vertex u_n on the i-th duplicate of W_n with the i-th vertex in the graph K_m. The comb operation discussed is only limited to the point comb operation. Based on the description above, researchers are interested in performing comb operations on several types of graphs to obtain the form of new graph variations and the graph's domination number resulting from the comb operation. Therefore, the researcher studies more deeply the topic of the graph's domination number resulting from the comb operation between complete graphs and wheel graphs.

RESEARCH METHOD

This research is a literature study, with the steps used are first collecting some literature related to domination numbers, comb operations, complete graphs and wheel graphs, second studying in detail related definitions and theorems from the literature that has been collected, third analyzing the graph of comb operation results between complete graphs and wheel graphs, as well as the general formula of the domination number of the comb operation result graph. The last step is to draw conclusions from this research.

RESULTS AND DISCUSSION

1. Graph resulting from comb operation between complete graph and wheel graph the comb operation on the complete graph and wheel graph produces the following result graph:

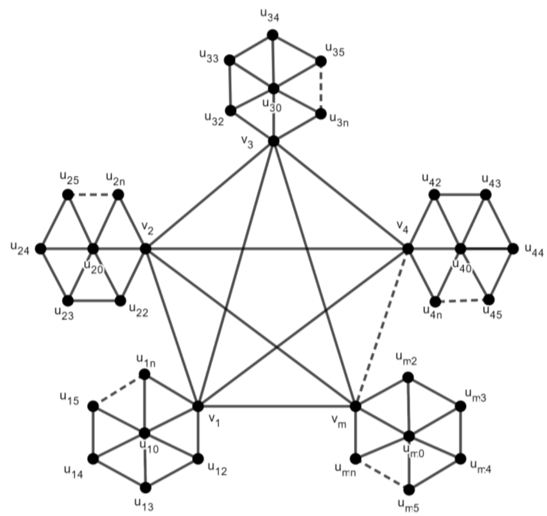


Figure 3. Graphs Resulting from Comb Operation between Complicated Graphs and Wheel Graph ($K_m \rhd W_n$)

2. Domination Numbers of Comb-operated Graphs on Complete Graphs and Wheel Graphs

The domination number of the graph obtained through the comb operation between a complete graph and a wheel graph. $K_m \rhd W_n$ is divided into 4 cases, namely :

a) Domination number of the graph $K_m \rhd W_n$ with m, n odd members, produces the following result graph:

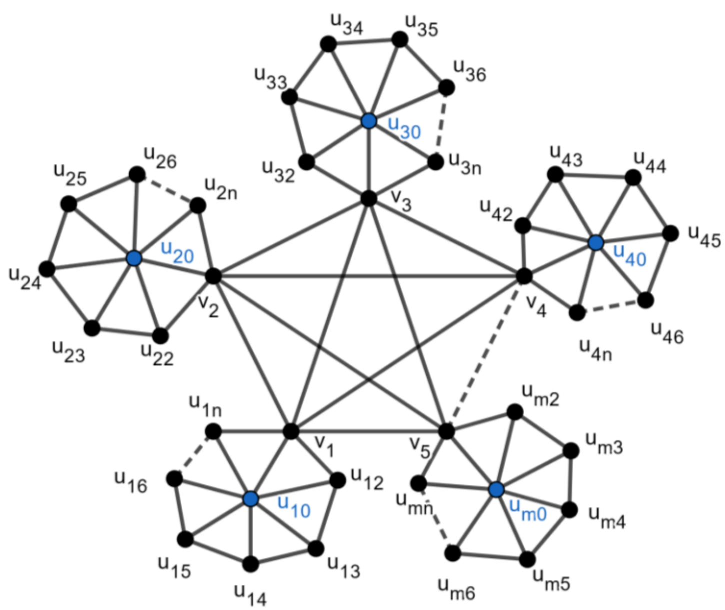


Figure 4. Graph $K_m \rhd W_n$ m, n odd

Based on the graph above, we get the domination number of the graph $K_m \rhd W_n$ with m, n odd number members. as follows.

Table 1. Domination number of the graph $K_m \rhd W_n$ with m, n odd number members

Graph	Dominance number
$K_1 \rhd W_3$	1
$K_1 \rhd W_5$	1
$K_3 \rhd W_3$	3
$K_3 \rhd W_5$	3
$K_5 \rhd W_3$	5
$K_5 \rhd W_5$	5
…	..
$K_m \rhd W_n$	m

In general, the dominance number of a graph $K_m \rhd W_n$ with m, n odd number members, i.e:

$$\gamma(K_m \rhd W_n) = \begin{cases} 1\,; m = 1,\ n \geq 3 \\ 3\,; m = 3,\ n \geq 3 \\ m; m \geq 5, n \geq 3 \end{cases}$$

b) Domination number of the graph $K_m \rhd W_n$ with m, n even number members, produces the following result graph:

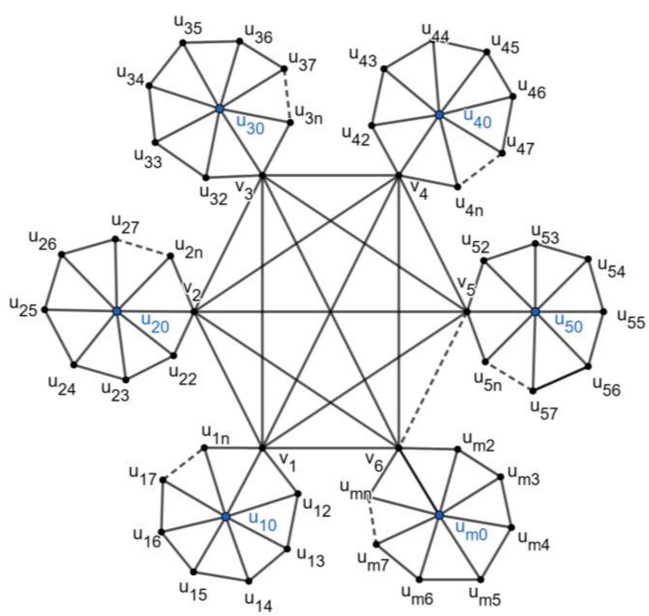


Figure 5. Graph $K_m \rhd W_n$ m, n even

Based on the graph above, we get the domination number of the graph $K_m \rhd W_n$ with m, n even number members is as follows.

Table 2. Domination number of the graph $K_m \rhd W_n$ with m, n even number members

Graph	**Dominance Numbers**
$K_2 \rhd W_4$	2
$K_2 \rhd W_6$	2
$K_4 \rhd W_4$	4
$K_4 \rhd W_6$	4
$K_6 \rhd W_4$	6
$K_6 \rhd W_6$	6
…	…
$K_m \rhd W_n$	m

In general, the dominance number of a graph $K_m \rhd W_n$ with m, n even number members is:

$$\gamma(K_m \rhd W_n) = \begin{cases} 2\,; m = 2, n \geq 4 \\ 4\,; m = 4, n \geq 4 \\ m; m \geq 6, n \geq 4 \end{cases}$$

c) Domination number of the graph $K_m \rhd W_n$ with m even, n odd produces the following result graph:

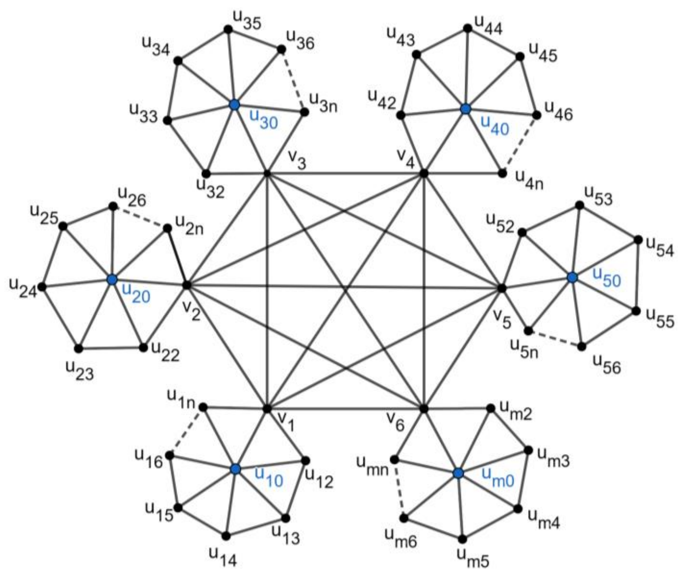


Figure 6. Graph $K_m \rhd W_n$ m even, n odd

Based on the graph above, we get the domination number of the graph $K_m \rhd W_n$ with m even, n odd members is as follows.

Table 3. Domination number of the graph $K_m \rhd W_n$ with m even, n odd

Graph	Dominance Numbers
$K_2 \rhd W_3$	2
$K_2 \rhd W_5$	2
$K_4 \rhd W_3$	4
$K_4 \rhd W_5$	4
$K_6 \rhd W_3$	6
$K_6 \rhd W_5$	6
…	…
$K_m \rhd W_n$	m

In general, the dominance number of a graph $K_m \rhd W_n$ with m even number members, n odd number members is:

$$\gamma(K_m \rhd W_n) = \begin{cases} 2\,; m = 2, n \geq 3 \\ 4\,; m = 4, n \geq 3 \\ m; m \geq 6, n \geq 3 \end{cases}$$

d) Domination number of the graph $K_m \rhd W_n$ with m odd, n even produces the following result graph:

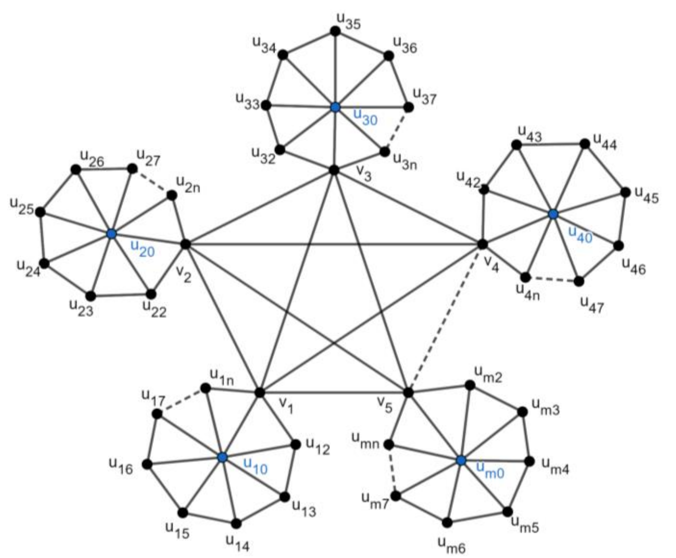


Figure 7. Domination number of the graph $K_m \rhd W_n$, m odd, n even

Based on the graph above, we get the domination number of the graph $K_m \rhd W_n$ with m odd, n even members is as follows.

Table 4. Domination number of the graph $K_m \rhd W_n$ with m odd, n even

Graph	Domination Numbers
$K_1 \rhd W_4$	1
$K_1 \rhd W_6$	1
$K_3 \rhd W_4$	3
$K_3 \rhd W_6$	3
$K_5 \rhd W_4$	5
$K_5 \rhd W_6$	5
…	…
$K_m \rhd W_n$	m

In general, the dominance number of a graph $K_m \rhd W_n$ with m odd number members, n even number members is:

$$\gamma(K_m \rhd W_n) = \begin{cases} 1\ ; m = 1, n \geq 4 \\ 3\ ; m = 3, n \geq 4 \\ m; m \geq 5, n \geq 4 \end{cases}$$

After identifying the domination number of the graph $K_m \rhd W_n$ which is divided into 4 cases, namely:

a. Domination number of the graph $K_m \rhd W_n$ with m, n odd

b. Domination number of the graph $K_m \rhd W_n$ with m, n even

c. Domination number of the graph $K_m \rhd W_n$ with m even, n odd

d. Domination number of the graph $K_m \rhd W_n$ with m odd, n even

The overall domination number obtained is as follows.

Table 5. Dominance number $K_m \rhd W_n$

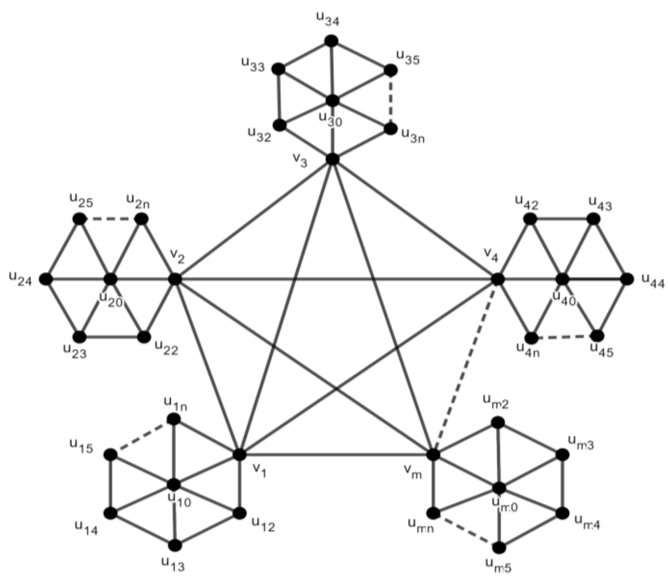
$K_m \rhd W_n$	W_3	W_4	W_5	W_6	…	W_n
K_1	1	1	1	1	1	1
K_2	2	2	2	2	2	2
K_3	3	3	3	3	3	3
K_4	4	4	4	4	4	4
K_5	5	5	5	5	5	5
K_6	6	6	6	6	6	6

$K_m \rhd W_n$	W_3	W_4	W_5	W_6	…	W_n
…	…	…	…	…	…	…
K_m	m	m	m	m	m	m

From the table above it can be noticed that for m =1, the graph's domination number $K_m \rhd W_n$ is the same as the the graph's domination number wheel graph W_n and every domination number of $K_m \rhd W_n = m$. Hence the general formula for the graph's domination number $K_m \rhd W_n$.

CONCLUSIONS AND SUGGESTIONS

The graph resulting from the comb operation of a complete graph and a wheel graph $K_m \rhd W_n$, with $m \geq 1, n \geq 3$ is as follows.



Gambar 8. Graf $K_m \rhd W_n$

The graph's domination number $K_m \rhd W_n$ is divided into 4 cases, namely the domination number of the graph $K_m \rhd W_n$ with m & n odd, domination number of graphs $K_m \rhd W_n$ with m & n even, domination number of graphs $K_m \rhd W_n$ with m even & n odd, and the domination number of graphs $K_m \rhd W_n$ with m odd & n even . After the complete graph and wheel graph are operated by comb operation, then the general formula of graph domination number resulting from comb operation of complete graph and wheel graph is $K_m \rhd W_n$ is : $\gamma(K_m \rhd W_n) = \{ \gamma(W_n)\ ; m = 1, n \geq 3\ m\ ; m > 1, n \geq 3$ This article discusses the domination number of graphs that arise from the combination of complete graphs and wheel graphs through the comb operation. $K_m \rhd W_n$. As a suggestion from the researcher, the reader can study the topic 12 of domination numbers on other graphs. It can also discuss the domination number of the graphs discussed in this paper, but with different operations, for example, the domination number of the result of the corona operation of complete graphs and wheel graphs. $K_m \rhd W_n$.

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