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ANALYSIS OF METACOGNITIVE ABILITY IN MATHEMATICS PROBLEM SOLVING OF SMA STUDENTS AT NGEMPLAK BOYOLALI

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Abstrak: Kemampuan metakognitif erat kaitannya dengan kemampuan pemecahan masalah. Penelitian ini merupakan penelitian survei yang melibatkan 150 siswa kelas X dari empat SMA di Ngemplak Boyolali. Tujuan penelitian ini adalah mendeskripsikan hasil analisis kemampuan metakognitif dalam menyelesaikan masalah matematika. Data dikumpulkan dengan menggunakan kuesioner *Metacognitive Awareness Inventory* (MAI). Analisis data dilakukan secara kuantitatif dan kualitatif. Hasil penelitian menunjukkan bahwa 23% siswa memiliki kemampuan metakognitif tinggi, 20% siswa memiliki kemampuan metakognitif sedang, dan 57% siswa memiliki kemampuan metakognitif rendah. Sebagian besar siswa yang memiliki kemampuan metakognitif tinggi mampu menggunakan kemampuan metakognitifnya dalam memecahkan masalah matematika. Hampir semua siswa yang memiliki kemampuan metakognitif rendah tidak menggunakan kemampuan metakognitifnya dalam menyelesaikan masalah matematika. Secara keseluruhan, siswa kurang optimal dalam menggunakan kemampuan metakognitif saat menyelesaikan masalah matematika.

Kata kunci : *Metakognitif*, *Matematika*, *Pemecahan masalah*

Abstract: Metacognitive ability is closely related to problem solving ability. This research is a survey research involving 150 students of class X from four high schools in Ngemplak Boyolali. The purpose of this study is to describe the results of the analysis of metacognitive abilities in solving mathematical problems. Data was collected using a Metacognitive Awareness Inventory (MAI) questionnaire. Data analysis was carried out quantitatively and qualitatively. The results showed that 23% of students had high metacognitive abilities, 20% of students had moderate metacognitive abilities, and 57% of students had low metacognitive abilities. Most students who have high metacognitive abilities are able to use their metacognitive abilities in solving mathematical problems. Almost all students who have low metacognitive abilities do not use their metacognitive abilities in solving mathematical problems. Overall, students are less than optimal in using metacognitive abilities when solving mathematical problems.

Keywords: *Metacognitive*, *Mathematics*, *Problem solving*

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INTRODUCTION

Mathematics is a science that cannot be separated from other sciences; this fact is based on the assumption that the development of other sciences will halt if adequate mathematical knowledge is not available. According to the National Council of Teachers of Mathematics (2000), students must have five process standards of mathematical ability: problem solving skills, reasoning and proof abilities, communication skills, connection skills, and presentation/representation skills. The goal of mathematics education in schools is to provide students with all mathematical abilities necessary to achieve optimal learning outcomes, and these abilities can be used to solve problems in everyday life.

Because of the importance of mathematical problem-solving abilities, it has become the primary focus of learning (Novitasari & Wilujeng, 2018). However, it turns out that problem-solving abilities in Indonesia remain relatively low. The average mathematics score obtained by Indonesia in 2018 when participating in PISA was 379, with an average international score of 591. When compared to previous PISA results, Indonesia experienced a decline in 2015, with a PISA score of 386. Problem solving ability is one of the cognitive indicators measured by the PISA survey.

According to Purwaningrum (2019), problem-solving skills help students think analytically when making decisions in daily life and help improve critical thinking skills when dealing with new situations. Polya (1997) identified four stages of problem solving: (1) understanding the problem, (2) developing a problem-solving plan, (3) implementing the problem-solving plan and solving it, and (4) re-examining the results. Problem solving is not only the goal of learning mathematics, but it is also a skill that will help you make the best decisions in life.

According to Baroody and Coslick (1993); Charles et al. (1987), there are three aspects that influence mathematical problem solving: (1) cognitive aspects, which include conceptual knowledge, understanding, and strategies for applying this knowledge; (2) affective aspects, which influence students' tendency to solve problems; and (3) metacognitive aspects, which include the ability to organize their own thoughts. According to Risnanosanti (2008), in order to solve a problem, students must master at least five aspects of ability, namely the ability to understand mathematical concepts, the ability to master mathematical algorithm skills, the ability to process mathematics, the ability to have a positive attitude toward mathematics, and metacognitive abilities.

According to Lestari and Yudhanegara (2015), metacognition is the awareness of one's own cognition, how cognition works, and how one can regulate cognition. According to Lioe et al. (2006), metacognition is one of the main components of mathematical problem solving that emphasizes students' ability to monitor their own thinking. According to Van de Walle (2007), metacognition is the conscious monitoring (being aware of how and why something is done) and regulation (choosing to do something or deciding to make a change) of one's own thought processes. This concept is consistent with the concept of metacognition (Flavell, 1979), which refers to students' awareness of their own cognitive processes and the regulation of these processes to achieve specific goals.

According to Flavell (1979), metacognitive ability consists of four components: (1) metacognitive knowledge, (2) metacognitive experience, (3) goals or tasks, and (4) actions or strategies. According to Larkin (2009), metacognitive knowledge is a person's knowledge about his own state of mind, other states of thought, or thinking in general. Mahmudi (2013) defines metacognitive knowledge as a person's knowledge and beliefs about what to do in certain situations.

Schraw and Dennison (1994) divide the metacognitive component into two parts: (1) knowledge about cognition (declarative knowledge, procedural knowledge, and conditional knowledge) and (2) cognition regulation. Metacognitive control is defined by Ozsoy and Ataman (2017) as the ability to use knowledge to regulate and control thought processes.

Several metacognitive research findings show a positive relationship between metacognitive abilities and mathematical problem solving. Schraw and Dennison (1994) discovered a link between metacognitive awareness and increased use of heuristic problem solving and higher level group responses to a problem in mathematical problem solving strategies. According to Ormord (2009), the more students are aware of their metacognition, the better the learning process and the outcomes they will achieve. OZcan (2014) claims that students with high metacognitive abilities have better problem-solving abilities when learning mathematics than students with low metacognitive abilities. According to Handel et al. (2013), several studies have shown that people who use metacognitive abilities perform better than people who don't use metacognitive abilities.

The stage of formal thinking operations is experienced by children aged 11 and up, according to Piaget's theory of cognitive development (Piaget & Inhelder, 1969). Piaget also proposed that hypothetical deductive thinking is one of the characteristics that distinguishes the emergence of formal operational thinking around the age of 12 years. High school students are in the formal operations stage, according to Piaget's theory. This means that high school students can monitor or think about their own thinking. Based on this, high school students can engage in metacognitive activities.

According to the description above, it is critical for students to have metacognitive abilities, especially when solving mathematical problems. This necessitates teachers providing opportunities for students to practice their metacognitive abilities by posing problem-solving questions. Students are directed through the questions posed by the teacher in the process of learning mathematics with the main focus on problem solving, so that students can finally be aware and optimally use their cognitive strategies. According to Cromley (2000), cognitive strategies are acquired by students through learning, such as asking themselves questions related to the material or questions given by the teacher, so that students can choose appropriate strategies to solve these problems.

RESEARCH METHOD

This study is a survey with a quantitative descriptive approach. The purpose of this study is to gather information about the characteristics of a population based on the results of the sample. This study's population consists of students from class X SMA in Ngemplak Boyolali. International superior private high schools (labeled SMA W), superior public high schools (labeled SMA X), regular private high schools (labeled SMA Y), and regular public high schools comprised the research sample (labeled SMA Z).

Table 1. Research Sample

School	Class	n
SMA W	X IPA 2	32
SMA X	X IPS 1	40
SMA Y	X IPA 1	38
SMA Z	X IPS 3	40

This research is a survey research involving 150 students of class X from four high schools in Ngemplak Boyolali. The purpose of this study is to describe the results of the analysis of metacognitive abilities in solving mathematical problems. Data was collected using a Metacognitive Awareness Inventory (MAI) questionnaire. Data analysis was carried out quantitatively and qualitatively.

A Metacognitive Awareness Inventory (MAI) test and questionnaire were used in the study. The test instrument is a description of five questions. This instrument includes scoring guidelines to make it easier for researchers to provide feedback. The contents of the test instrument were validated by requesting assessments and input from mathematics education evaluation experts.

The Metacognitive Awareness Inventory (MAI) questionnaire was based on one created by Schraw and Dennison (1994). Following the completion of test questions, students are given questionnaires. This questionnaire examines two aspects of metacognitive ability: metacognitive knowledge and metacognitive regulation. This questionnaire consists of 20 positive statements that students must answer with one of two options, true or false.

Table 2. Components and Indicators of Metacognitive Ability

Components	Indicators
Declarative knowledge Procedural knowledge	a. Write a problem-solving plan b. Write down some mathematical concepts or ideas used c. Write a mathematical model of the mathematical concept or idea used
Conditional knowledge Planning	Writing reasons using mathematical concepts or ideas Write down known and asked information
Information management Evaluating	Make a sketch or drawing according to the information presented Solve problems according to the resolution plan Check the correctness of the answer

An educational evaluation is then used to validate the completed questionnaire. Language experts offer suggestions for language improvement. The questionnaire was tested for reliability after it had been revised. The reliability estimation yielded a Cronbach's Alpha coefficient of 0.694. Nunnaly (Ghozali & Castellan, 2002) defines a reliable instrument for measuring variables as having a reliability coefficient greater than 0.60. As a result, the questionnaire instrument developed can be said to be reliable.

The collected data was quantitatively and qualitatively analyzed. Students' responses to test questions and the results of questionnaires were used to perform quantitative analysis. Based on the scoring guidelines developed, the results of students' metacognitive ability tests in solving mathematical problems were analyzed. The researcher categorizes students' metacognitive abilities in problem solving based on their acquisition scores, as shown in Table 3.

Table 3. Categories of Students' Metacognitive Ability Based on Score

Interval	Categories
$\boldsymbol{X \geq \bar{X} + 0,2s}$	High
$\boldsymbol{\bar{X} - 0,2s \leq X < \bar{X} + 0,2s}$	Currently
$\boldsymbol{X < \bar{X} - 0,2s}$	Low

Furthermore, the total score for each component of metacognitive ability in each question is calculated, and the percentage is calculated based on the maximum score using the formula.

$P = \frac{\sum B}{\sum T}$ x 100% ……………….(1) Sugiyono (2014),

where is the percentage of achievement, is the score for correct answers, and is the total score. The questionnaire results were analyzed by assigning a score of 1 to the correct answer choice and a score of 0 to the incorrect answer choice. Furthermore, based on Table 3, the score of each student's acquisition in each component of metacognitive ability is calculated and classified.

RESULTS AND DISCUSSION

The questions on the test were designed to assess students' metacognitive abilities in solving mathematical problems using Polya's steps. To determine how far students' metacognitive abilities are used in solving mathematical problems, the score of each stage of problem solving is calculated to the maximum score of each stage. Understanding the problem (understanding the problem), planning a solution (devising a plan), solving the problem according to the plan (carrying out the plan), and checking back are all part of the completion stage (looking back). Metacognitive knowledge and metacognitive regulation were the components of metacognitive ability measured in the study. Metacognitive knowledge (declarative, procedural, and conditional knowledge) and metacognitive regulation are two components of metacognitive ability as measured by tests (planning, monitoring, and evaluating).

Students' ability to write plans to solve problems, write down some mathematical concepts or ideas used, write down mathematical models of mathematical concepts or ideas used, and write down reasons for using mathematical concepts or ideas is an indicator of metacognitive knowledge. Question 4 has the lowest metacognitive knowledge score. This is due to the fact that all students did not create a settlement plan. Students can write down information that is known and asked completely and correctly as planning indicators. The lowest scores are on questions 3 and 4. This is because most students do not completely and correctly write down the information that is known and asked. Students can make sketches or drawings based on the information presented and solve problems using the completion plan as information management indicators. Question number four has the lowest score. This is due to the fact that many students are unable to complete it in accordance with the written plan, and the final results obtained are incorrect. The evaluating indicator is documenting the re-checking procedure. The evaluating score is still very low. This is due to the fact that most students do not record the re-checking process.

The score obtained by each student in solving the problem is then processed in accordance with the established guidelines to determine the final student score. As shown in Table 4, the final score obtained by each student is then divided into three ability categories: high, medium, and low. According to Table 4, 57 percent of students have low metacognitive abilities, while 43 percent of students have metacognitive abilities that are medium or high. This demonstrates that students in the low category have a high level of metacognitive ability.

Table 4. Categories of Metacognitive Ability based on Test Results

Interval	Categories	%
$\boldsymbol{X \geq \bar{X} + 0,2s}$	High	23
$\boldsymbol{\bar{X} - 0,2s \leq X < \bar{X} + 0,2s}$	Currently	20
$\boldsymbol{X < \bar{X} - 0,2s}$	Low	57

A questionnaire was used to assess students' metacognitive abilities in this study, in addition to tests. The MAI questionnaire's metacognitive knowledge component is divided into three sub-components: declarative, procedural, and conditional knowledge. Declarative knowledge receives the highest score. This suggests that when students work on test questions, this knowledge makes a significant contribution. Meanwhile, the MAI questionnaire's metacognitive regulation is divided into five sub-components: planning, information management, monitoring, debugging strategies, and evaluating. Information management receives the highest score. This suggests that when students work on test questions, their information management skills play a significant role.

Most students with strong metacognitive abilities can effectively apply their metacognitive abilities when solving problems. This is evident in the work of students who can write clearly and correctly about the process of problem solving. Students with strong metacognitive abilities write down known information and questions correctly and completely. In addition, students use images to help them understand the problem. According to the written completion plan, students only write it briefly in the form of a formula that will be used to solve the problem, with no detailed steps or reasons for using the concept.

According to the process by which students find answers, it appears that students solve the problem in a systematic and correct manner. Furthermore, students double-check their answers to ensure that they are correct. Not all students with moderate metacognitive abilities can use them effectively when solving problems. This is evident in the results of student work, which are less clear and precise in writing the process of problem solving. Students with metacognitive abilities write down information that is known and correctly and completely asked. However, there are no images to help clarify the information in the questions. According to the written completion plan, students wrote it quickly and clearly. There are no specific steps or reasons for employing the concept.

Based on the process of students finding answers, it appears that students can solve the problem in a systematic and correct manner. However, students did not re-check the answers to ensure that they were correct. Almost all students with low metacognitive abilities do not solve problems using metacognitive abilities. This is evident in the results of student work that is unclear in writing the process of answering questions for students with low metacognitive abilities who do not write down known and asked information. Students also do not create a completion plan. However, based on the process by which students found answers, it appears that students are capable of solving the problem using systematic and correct steps.

The research findings show that students with high metacognitive abilities can use their metacognitive abilities effectively when solving math problems and outperform other students. According to the results of student work and questionnaires, students who can use their metacognitive abilities can solve questions well and achieve higher test scores. Furthermore, students can write down the steps for solving problems in a systematic and correct manner. This is consistent with the findings of Boekaerts et al. (Ozcan, 2014), who claim that students with high metacognitive abilities in mathematic.

Explanation of the material, combined with student experience in solving similar problems, greatly aids students in remembering the problem-solving process. Furthermore, students' awareness of planning, monitoring, evaluating, and improving their thinking processes while solving problems can help to reduce the occurrence of errors. In other words, students continuously monitor and control their thinking processes so that when they encounter difficulty, they will repeat the initial steps to determine the source of the problem and correct it. This demonstrates that knowledge and metacognitive regulation are linked during the problem-solving process.s learning solve problems better than students with low metacognitive abilities. This statement is consistent with Schraw and Dennison's (1994) belief that there is a high correlation between cognitive knowledge and cognitive regulation, implying that knowledge and regulation can aid students in their independent learning.

According to the explanation, the use of metacognitive abilities during problem solving is directly proportional to the performance results obtained, familiarizing students with problem solving problems can help them develop their metacognitive abilities, and using metacognitive abilities can help students become more independent in their learning. This is consistent with Ormord's (2009) assertion that the more students are aware of their metacognition, the better the learning process and the outcomes. Furthermore, Mok et al. (2006) proposed that the metacognitive approach will aid student learning and self-evaluation. Furthermore, students' understanding abilities will improve as a result of metacognitive strategies (Iftikhar, 2015). According to Schneider and Artelt (2010), metacognitive aspects such as metacognitive knowledge and self-regulation have a significant impact on mathematical performance. According to Anggo (2011), involving students in mathematical problem solving can help them develop metacognitive abilities. Furthermore, getting students used to problem solving can encourage them to use metacognitive skills in writing while problem solving. The use of metacognition that occurs continuously as students solve problems causes students to repeat the initial steps to find and fix the error.

CONCLUSIONS AND SUGGESTIONS

Based on the findings and discussions, it is possible to conclude that the metacognitive ability of X grade high school students in Ngemplak Boyolali in solving math problems still needs to be improved. Students continue to underutilize their metacognitive abilities when solving mathematical problems. The findings revealed that 23% of students possessed high metacognitive abilities, 20% possessed moderate metacognitive abilities, and 57% possessed low metacognitive abilities. Most students with strong metacognitive abilities can apply their metacognitive abilities to solve mathematical problems. Almost all students with low metacognitive abilities do not use their metacognitive abilities in problem solving. Overall, students use metacognitive abilities less than optimally when solving mathematical problems.

Based on the findings of this study, several recommendations can be made: (1) teachers should familiarize students with problem-solving questions to improve students' metacognitive abilities; (2) the teacher should provide a more detailed explanation of the basic concepts of a material and its application; and (3) more research on metacognitive abilities in solving mathematical problems, particularly methods to improve these abilities, is required.

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