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Analysis Study of Shear Strength Beam with UPR Patch using Finite Element Method

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ABSTRACT. Reinforced concrete beam is one of the structural elements of the building, which must be designed to meet the requirements set out in the building design regulations (Building Code of Practices). Handling the degradation of the shear span is very necessary to restore the shear strength of reinforced concrete beams and avoid the occurrence of shear failure which is very dangerous. This study aims to determine the effect of the flexural reinforcement ratio on the shear behaviour of reinforced concrete beams filled with UPR-based patch repair mortar with numerical methods, and to determine the ratio of the flexural reinforcement ratio to the shear behaviour of concrete beams by real testing and numerical testing. This research was conducted numerically with the help of ATENA Engineering 3d Software, to investigate the effect of UPR-mortar and reinforcement ratio on the shear behaviour of reinforced concrete beams. The effect of UPR-mortar was investigated by comparing the results of numerical simulations between normal beams (N) and repair beams (R). Based on the data obtained from the analysis using the finite element method, it was found that the greater the flexural reinforcement ratio, the higher the collapse load given, and the shear capacity of the concrete beams patched using UPR-based patch repair increased compared to reinforced concrete beams in each flexural reinforcement ratio.

Keywords: Patched Beam, Finite Element Method, ATENA, Newton-Raphson.

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1. Introduction

Reinforced concrete beam is one of the structural elements of the building, which must be designed to meet the requirements set out in the building design regulations (Building Code of Practices). These requirements can be categorized into strength requirements and serviceability requirements (Nilson et al, 2010). Strength requirements aim to ensure that the structural elements are capable of bearing any loads that will arise, whether permanent loads (such as gravity loads) or incidental loads (such as earthquakes), without causing the building to collapse. In general, the load borne by beam elements will cause internal stresses, which can be in the form of flexural stresses and shear stresses. The combination of bending-shear stress in the compression zone can trigger shear failure in the beam if the stress exceeds the capacity of the beam (Brown et al, 2006). Therefore, reinforced concrete beams must be designed to be strong enough to bear this combination of flexural-shear stresses.

The strength of a reinforced concrete beam (eg flexural strength), is contributed by the composite action between the compressive forces in the concrete and the tensile forces in the reinforcement. The presence of reinforcement that carries this tensile force allows reinforced concrete beams to be designed to be ductile or ductile. Meanwhile, the ductile properties of reinforced concrete beams tend to be difficult to achieve in the design against shear loads. The results showed that the concrete blocks experienced a sudden collapse under the dominance of shear loading (Hamrat et al, 2010; Slowik, 2014; Said et al, 2016; Kristiawan et al, 2017). This sudden collapse mode is very dangerous for the safety of the building and its occupants compared to the ductile failure mode. On this basis, reinforced concrete beams must be designed to have a shear capacity that is more conservative than the flexural capacity. It is intended that the flexural failure mode can precede the shear failure which maintains the state.

Although reinforced concrete beams have been designed to meet the strength requirements and the mode of flexural failure has been confirmed to precede shear failure, over time reinforced concrete beams may experience damage or degradation. There are various causes of degradation/damage such as corrosion of reinforcement, large earthquake loads, fire, and others. This damage will certainly reduce the strength of reinforced concrete beams (Scott, 2013), which in turn reduces the safety factor of the building. A decrease in the safety factor in the shear span will be more worrying than a decrease in the safety factor in the flexural area (Collalillo and Sheikh, 2012). A decrease in the safety factor in the shear span can occur if the degradation (eg in the form of spalling) is in the shear span area. In this spalled part, the shear strength contributed by the concrete is reduced considering the area of the concrete cross-section in the damaged area (spalling) becomes smaller (Lee et al, 2013). In this case, the shear failure

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mode reverses before the flexural failure. Thus, the building is at risk of sudden collapse without any prior warning to its occupants.

Handling the degradation of the shear span is very necessary to restore the shear strength of reinforced concrete beams and avoid the occurrence of shear failure which is very dangerous. Repair by patching (patching) is an easy option; however, this patching method requires filling materials that must be compatible so that the shear capacity of reinforced concrete beams is completely restored (Emberson and Mays, 1996; Rio et al, 2005; Pattnaik, 2006; Sahamitmongkol, 2008). One of the concrete filling/repair materials that has been developed by RG SMARTCrete since 2012 is unsaturated polyester resin (UPR) mortar. Previous studies have completed the physical, chemical, mechanical, dimensional, and durability characterization of UPR mortar. This material has also been applied to repair beams, plates and columns (Supriyadi et al, 2015; Kristiawan and Prakoso, 2016; Kristiawan et al, 2017; Kristiawan et al, 2018). The results of previous studies indicate the prospect of this material, which is able to restore the capacity of structural elements that have been damaged. However, there are still many parameters that must be studied further before this material is applied in the field through a full scale test. The number of parameters that must be tested is one of the obstacles in terms of cost aspects, so the approach taken is to develop numerical modeling. On this basis, research on UPR mortar as a patching material carried out by RG SMARTCrete began to be directed at numerical modeling; in accordance with the stages outlined in the research road map.

2. Literature Review

Beam Shear Strength is the strength of a beam structure cross-section that increases the stiffness of the beam and resists lateral forces from self-load and supported loads. According to Chu-Kia Wang and Charles G. Salmon (1991), the shear effects that arise are the result of torsion and the combination of torsion and bending. The condition of the maximum shear stress of a beam cross section is located on the neutral axis of the section. The beam shear force is written with the symbol Vn which has the following variables.

Factors that affect the shear strength of beams that have experienced flexural cracks or flexural-shear cracks so as not to collapse consist of three effects, namely as follows.

1) Cracked concrete interlock

Narrow beam cracks, can still occur through the distribution of forces through the rough surface interlocks that occur in the cracks. This interlock force (Vi) is quite large and can be measured in determining the total shear force.

2) Dowel effect of longitudinal reinforcement

The shear force resisted by this longitudinal reinforcement is quite small. Longitudinal reinforcement which acts as a dowel force (Vd) only has an impact on the area of the concrete cover which can be seen in Figure 1. as follows. This is because the dowel force on the beam can only support the vertical displacement by the concrete blanket. The more forces supported by the reinforcement will reduce the cross-sectional area of the reinforcement so that there is no transfer of energy between the reinforcement and the concrete.

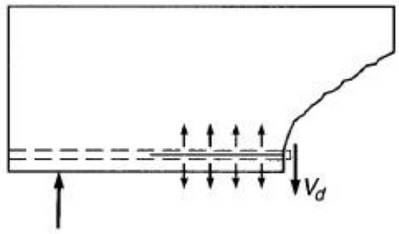

Fig.1 Dowel effect on the shear strength of cracked beams

3) Concrete parts that have not been cracked

From the equilibrium the internal shear force (Vint) must be directly proportional to the external shear force (Vext) that occurs in the concrete so that the following equation can be obtained.

$$Vcz = Vext - Vd - Viy \quad (1)$$

Where:
Vcz = Shear forces in uncracked concrete.
Vext = External shear force.
Vd = Shear force due to dowel.
Viy = Shear force due to interlock concrete cracks.

Spalling is part of the concrete surface that is released in the form of chips or small chunks. Usually caused by the sticking of the material in the formwork, a less homogeneous concrete mixture, or the age factor of the concrete. Based on what has been discussed previously, spalling greatly affects the shear strength of concrete blocks. This is due to the reduced cross-sectional area of the beam which provides a fairly wide distance so that the displacement of the shear force does not occur completely. In addition, spalling can also cause cracks to widen and the shear strength to decrease over time.

With the basis that has been stated by Arthur Nilson, if spalling occurs in the longitudinal reinforcement section, as previously discussed, the reinforcement will experience thinning of the cross-sectional area which has an impact on the shear strength of the beam which decreases over time. This also affects the part that is interlocked in the concrete cracks because the interlock force only increases the value of the shear force on the part of the crack that is still in contact. For spalling that occurs at the top of the beam, it will reduce the value of the shear force, due to the reduced cross-sectional area.

The occurrence of spalling is very detrimental because it causes a decrease in the shear strength of the concrete and can cause the concrete to crack easily which is then followed by collapse. Therefore, as for the way to repair the beam, one of them is by patching. But the material used for patching is not arbitrary because there are conditions that need to be met, namely as follows.

a. Strong adhesion

The bond between the repair material and the concrete to be repaired must blend well so that it becomes a unified whole concrete.

b. The modulus of elasticity is able to withstand overstressing

The modulus of elasticity here is the stiffness of a material. The higher the value, the less deformation (deformation) of the repair material when a force is applied. In addition, the higher the modulus of elasticity, the repair material is more resistant to impact and crack resistance.

c. Does not reduce the strength of concrete

The repair material used to repair the concrete is able to withstand the same load on the concrete before it was filled.

d. Thermal expansion coefficient

The repair material used can experience expansion and contraction when there is a change in air temperature in the surrounding environment. It happens depending on the coefficient of thermal expansion used. So the selection of materials must be chosen properly. If the material between the two concretes has a large difference in coefficient of thermal expansion and changes in temperature, it causes damage to the concrete in the form of bonding lines.

e. Have good permeability

The permeability of the repair material is the degree of density possessed by the repair material to be penetrated by liquids (eg water). The repair material is expected to have high permeability so that it is durable and long lasting, besides that the repair material is also able to increase the permeability of the repaired concrete.

f. Deformable on concrete

Repair materials must be able to adjust the shape of the concrete to be repaired.

After determining the conditions that need to be met, the types of materials used for patching concrete blocks can be taken as follows.

a. Cementitious based mortar

Cement-based mortar to bind the beam to be filled in the part where the spalling occurs. This mortar contains water, cement, aggregate.

b. Polymer modified mortar

As with the previous mortar, but to improve the function of the mortar used to fill the spalling, polymer substances were added as follows.

1) Latex polymers.
2) Redispersable dry polymer (example: ethylene vinyl acetate).
3) Water soluble polymers (eg polyvinyl alcohol).

c. Polymer mortar

Mortar with the main material is polymer, this polymer is divided into three types, namely thermoplastic, thermoset, and elastomer.

Thermoplastics include polypropylene (PP), polyethylene (PE), polyvinyl chloride (PVC), polyvinyl acetate (PVAC), polystyrene (PS), polyamide (PA), polyester (PET), polycarbonete (PC) and polyacetate. Thermosets include phenol-formaldehyde (PF), aminoplasts, epoxy resins (ER), usaturated polyester, polyurethane (PU), phenol-aralkyl (xyloks), bissmalleimides (BMI), polymides (PI), polstyryl pyridine (PSP) polyphennylene-quinoxxialine (PPQ). Elastomers are a different type of thermoplastic and thermoset, for example rubber.

The use of the ATENA program in solving the problem of shear strength of normal beams with patched beams is because the variables that affect the shear strength are very large and many tests are needed which at the same time have an impact on increasing costs. Therefore, the ATENA program is very useful in reducing the cost and time to find out any data needed to determine the shear strength of the beam.

The use of the ATENA program in solving the problem of shear strength of normal beams with patched beams is because the variables that affect the shear strength are very large and many tests are needed which at the same time have an impact on increasing costs. Therefore, the ATENA program is very useful in reducing the cost and time to find out any data needed to determine the shear strength of the beam.

In the ATENA program itself, you can determine the material to be used, be it concrete, steel, or the material you want to determine yourself. Furthermore, the yield stress, compressive stress, tensile stress, and modulus of elasticity of the material can be determined by themselves. In addition to determining the properties of the material used, a graph of the strength of the material can be seen.

Furthermore, one of the properties used to determine the mesh (nets), there are 4 types of mesh that can be used, namely tetra which is hexagonal in shape, brick is in the form of bricks, brick and tetra is in the form of bricks and hexagons, and extrude is in the form of emphasis of the drawn shape. This mesh serves as a depiction of cracks in the material after being run.

Based on the testing of normal beams with patched beams, the parameters that need to be known to determine the difference in shear strength of normal beams with patched beams are as follows.

a. Load
b. Deflection
c. Nominal shear force analysis
d. Nominal shear force of normal test object
e. Nominal shear force of patched specimen
f. Normal beam crack pattern
g. Patched beam crack pattern Deformation of a point on a normal beam
h. Deformation of a point in patched beams

The materials used in the ATENA program have various categories which can be determined later. For example: 3D Nonlinear Cementitious 2, Reinforcement, 3D Elastic Isotropic, and so on. In addition to materials already available in ATENA, you can include materials that have been determined by yourself but in a format that can be read by the ATENA program.

3. Research Methods

This research was conducted numerically with the help of ATENA Engineering 3D Software, to investigate the effect of UPR-mortar and reinforcement ratio on the shear behavior of reinforced concrete beams. The effect of UPR-mortar was investigated by comparing the results of numerical simulations between normal beams (N) and repaired beams (R) with UPR-mortar. While the effect of the reinforcement ratio is simulated by providing various reinforcements for both normal beams (N) and repair beams (R). The reinforcement ratio is represented by 2 pieces of flexural reinforcement with diameters of 16, 19, 22, and 25 mm. The overall research flow is illustrated in Figure 2. A more detailed explanation of each stage of the research (numerical simulation) is presented in the next sub-chapter.

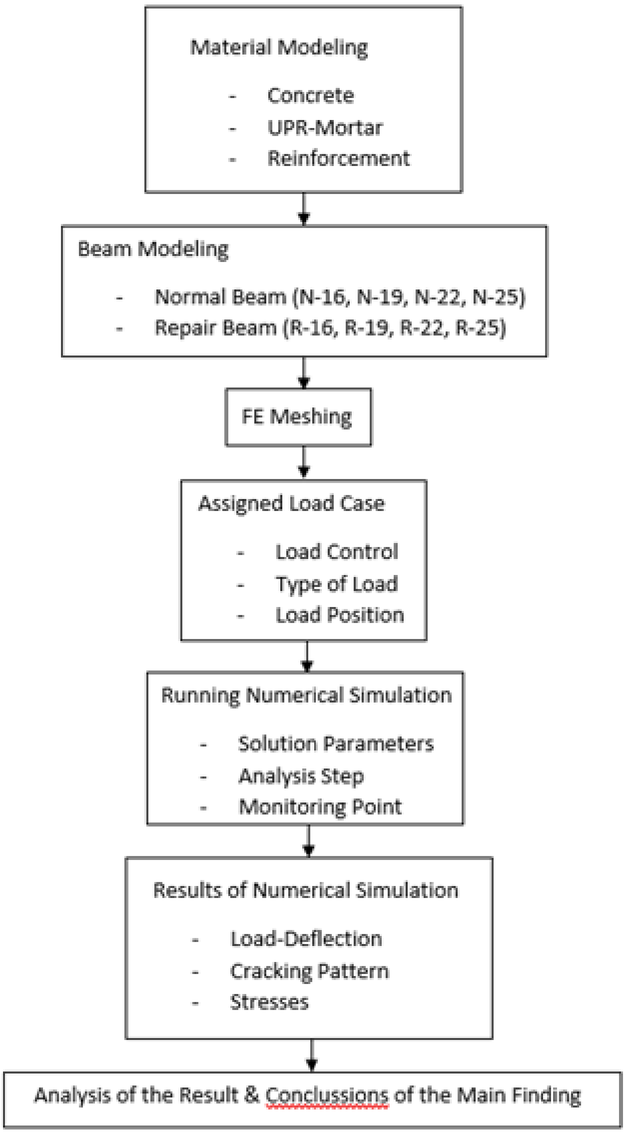

Fig.2 Research Flow

4. Results and Discussion

4.1 Material Modeling

Material modeling (constitutive model) is used to define material properties numerically; which will represent the response of the material when receiving a load or stress. In the ATENA software, the constitutive model for concrete is defined as 3D Nonlinear Cementitious 2. This material has a stress-strain relationship. In the ATENA software, the concrete properties inherent in 3D Nonlinear Cementitious 2 can be generated as a function of the compressive strength of cube concrete. However, the parameters resulting from the "generate" process can be modified according to the real properties of the concrete that will be used in the simulation. UPR-mortar will also be defined as 3D Nonlinear Cementitious 2 material. However, considering the tensile strength of this material is very high compared to concrete, the tensile strength parameter in the default 3D Nonlinear Cementitious 2 will be adjusted to the real value of UPR-mortar tensile strength. Meanwhile, the stress-strain relationship representing the reinforcement material (reinforcement) is defined as a material that has elastic-perfectly plastic (bilinear) properties.

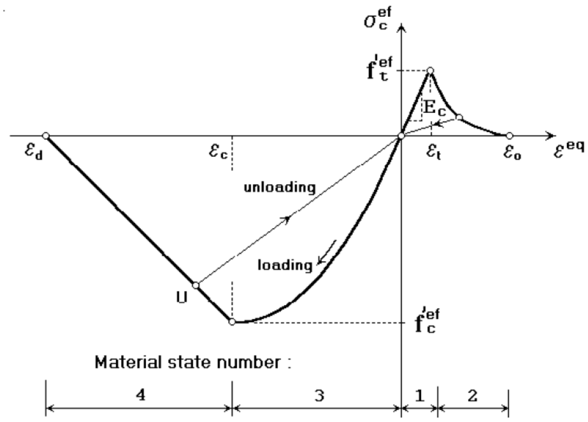

Fig.3 Uniaxial stress-strain relationship for concrete

In the ATENA program, the transfer between point loads (eg external loads, support reactions, etc.) to the concrete elements, must be carried out through intermediate materials that ensure no stress concentration in the concrete. The presence of this stress concentration can cause premature failure in this area. The intermediate material used to help transfer the load to the concrete in this study is a steel plate. The steel plate used is defined as a 3D Elastic Isotropic material, where the constitutive model of this material is linear. The reason for choosing 3D Elastic Isotropic is because it does not affect the beam model under consideration, but eliminates the stress concentration that is loaded.

The material properties used as inputs in material modeling are shown in Table 1.

Table 1
Material Property

Material	Cylinder compressive strength (MPa)	Cube compressive strength (MPa)	Tensile Strength (MPa)	Yield Stress (MPa)	Elasticity Modulus (MPa)	Model Material
Concrete	23.8	28	2.21	NA	29450	3D Nonlinear Cementitious Material 2
UPR-mortar	62.5	73.6	21.5	NA	12500	3D Nonlinear Cementitious Material 2
Reinforcement ϕ 25	NA	NA	NA	515	200000	Reinforcement-Bilinear
Reinforcement ϕ 22	NA	NA	NA	515	200000	Reinforcement-Bilinear
Reinforcement φ 19	NA	NA	NA	515	200000	Reinforcement-Bilinear
Reinforcement φ 16	NA	NA	NA	515	200000	Reinforcement-Bilinear
Reinforcement φ 8	NA	NA	NA	462	200000	Reinforcement-Bilinear
Reinforcement φ 6	NA	NA	NA	395	200000	Reinforcement-Bilinear
Steel Plate	NA	NA	NA	NA	200000	3D Elastic Isotropic

Source: Hapasara (2014)

4.2 Beam Modeling

The geometry of the beam and the reinforcement that will be investigated in this study is illustrated in Figure 4. The figure is an example of a beam with 16 mm diameter flexural reinforcement. The geometry of the other beams is similar to the picture, only the diameter of the flexural reinforcement is changed to 19, 22, and 25 mm. In the ATENA program, solid element geometries are modeled as macro-elements. In this study, there are 5 macro-elements that must be defined, each of which represents a concrete beam element (macro-element 1), a left-supported iron plate element (macro-element 2), a right-hand pedestal (macro-element 3), an iron plate as an intermediary for external load transfer (macro-element 4), and UPR-mortar element (macro-element 5). Meanwhile, the geometry of line elements that represent flexural reinforcement and shear reinforcement is modeled and defined through the “reinforcement bars” menu.

The relationship between solid elements with other solid elements is modeled and defined through 'Contacts' on each interconnected 'surface'. In this study, the contact surface between the concrete beam and the iron plate uses a partial connection. This is to consider that the two solid elements are in fact interconnected without any full bonding. Meanwhile, the contact surface between concrete and UPR-mortar is a full connection relationship. For the connection between reinforcement and concrete, the ATENA program provides a choice between "perfect connection" or using the "bond model" which we have to define ourselves. In this study, the reinforcement is assumed to have a perfect connection with the concrete. The results of the modeling of beams and reinforcement in the ATENA program are shown in Figure 5.

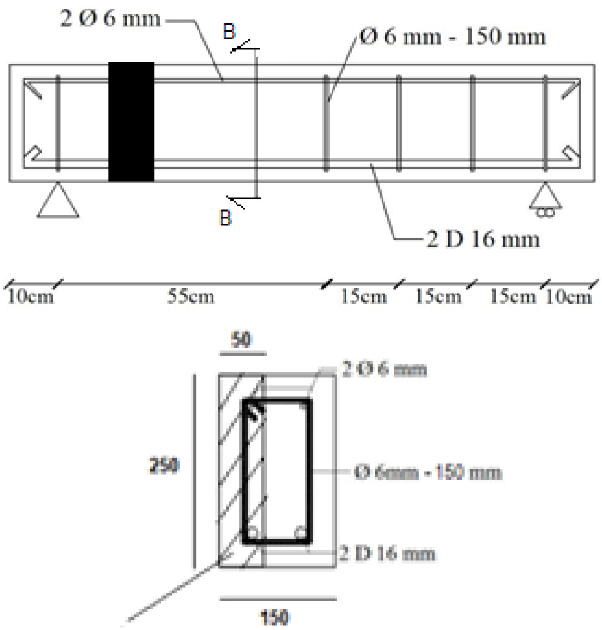

Fig.4 Reinforced concrete beam geometry

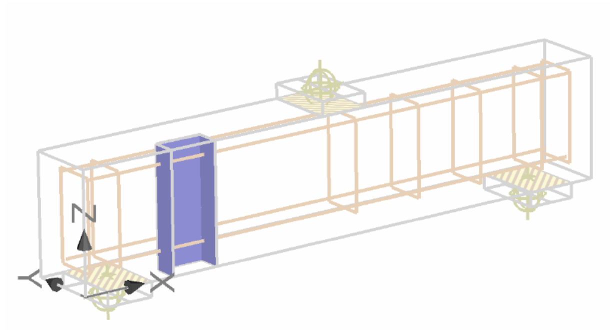

Fig.5 Modelling of beams in ATENA

4.3 Finite Element Meshing

Solid elements that represent blocks are divided (meshing) into small elements (finite elements). This meshing process can be done automatically in the ATENA software. Elements in ATENA are constructed using isoparametric formulations with linear and/or quadratic interpolation functions. The isoparametric formulations of one, two, and three dimensional elements belong to the "classical" element formulations. This is not because of its superior properties, but based on the fact that its versatility and general approach without masking the difficulties and also very important, this element is easy to understand. This is especially important in nonlinear analysis. For example, it is highly undesirable to add problems to elements for problems related to, for example, material modeling.

Although the concept of hierarchical elemental elements is explained for quadrilateral element planes, in ATENA it applies to triangular, 3D Brick, tetrahedral, and wedge element planes as well. There is always a set of basic interpolation functions that can be extended by some higher interpolation function. Part of the interpolation function of the finite element properties is highly dependent on the numerical integration scheme used for the integration of the stiffness element matrices, point element forces, and so on. In ATENA, most elements are integrated by Gaussian integration which ensures n(n-1) order of accuracy, where n is the degree of the polymonial used to estimate the integrated function. The basic elements that can be selected when meshing are brick, tetra, or a combination of brick-tetra. Tetra elements have a higher order analysis compared to bricks. The meshing results are shown in Figure 6.

Fig.6 FE meshing on beam

4.4 Assigned Load Case

In this study, the load to be calculated in the numerical simulation consists of dead load and concentrated load. Dead loads are automatically generated in the ATENA program; while the centralized load must be defined through the options in the "Load Cases" menu. There must be at least 2 types of load cases that need to be defined; first, Load Case 1 (LC-1) which represents the vertical and horizontal reactions on the supports, and the second is Load Case 2 (LC-2) which represents the external load to be applied. Further LC can be added according to the desired simulation.

LC-1 in this study is set through the Editing Supports Line menu in such a way that the left pedestal behaves as a joint (capable of providing both vertical and horizontal reactions), while the right pedestal behaves as a roller (only capable of providing vertical reactions). The load applied to the beam is load control, meaning that the external load applied to the middle span of the beam is carried out with a constant increase in load until the beam collapses. In this study, load control is divided into 2 levels of increase in load (increment). The first load increment level is loading in 200 kg increments which is applied up to 40 load steps or until the estimated flexural reinforcement approaches yielding (LC-2). After this stage, the intensity of the additional load is reduced to 100 kg until the beam collapses (LC-3).

4.5 Running Numerical Meshing

The numerical simulation process is carried out using the Run menu, by first setting the parameters related to the simulation through the solution parameters, load steps and monitoring point sub menus. Solution parameters are an approach to how the numerical solution will be carried out until it reaches the convergence value. There are 2 choices of methods provided by the ATENA program, namely the Standard-Newton Raphson method and the Standard Arch Length method. In this research, the method chosen is Standard-Newton Raphson, which is a form of numerical solution based on linear approximation. The parameters related to the numerical solution process with this method use the default values. The final results of numerical simulations in this study are presented in load-deflection diagrams, stress distribution diagrams, and cracking pattern diagrams.

The results of the modeling that have been carried out prove that the Shear Capacity of the beam has increased. The First Crack value can be shown when the beam is cracked for the first time. It is known that the value of the first crack between the normal beam and the repair beam has almost the same value even though the repair beam has a slightly larger value than the normal beam. Shear capacity is a value at which the beam is unable to increase the load it supports. It is known that the value of the shear strength of the repair beam is higher than that of the normal beam. The Toughness value itself has an influence on the tensile and compressive values of the beam. It is known that the value of the shear strength of the repair beam is higher than that of the normal beam. Stiffness Index is the amount of deflection that occurs due to the load it supports. Repair beam stiffness index is higher than normal beam.

Conclusion

Based on the results of analysis and testing using the finite element method, the following conclusions can be drawn:

a. The ratio of flexural reinforcement of reinforced concrete beams with UPR-based patch repair mortar affects the ability of the beam specimen to withstand its maximum load. The greater the flexural reinforcement ratio, the higher the given collapse load.
b. The value of First Crack reinforced concrete beams patched using UPR has increased compared to normal reinforced concrete beams. First Crack value is directly proportional to the repetition ratio.
c. The shear capacity of reinforced concrete beams patched using UPR-based patch repair mortar has increased compared to normal reinforced concrete beams at each flexural reinforcement ratio. The greater the flexural reinforcement ratio, the greater the capacity value, both for reinforced concrete beams and normal reinforced concrete beams for each type of beam tested. UPR-reinforced beams with a diameter of 16 mm flexural reinforcement have increased shear capacity by 81.61% from normal beams. UPR-reinforced beams with a diameter of 19 mm flexural

reinforcement experienced an increase in shear capacity of 83.33% from normal beams. UPR reinforced beams with a diameter of 22 mm of flexural reinforcement experienced an increase in shear capacity of 84.31% from normal beams. UPR-reinforced beams with a flexural reinforcement diameter of 25 mm experienced an increase in shear capacity of 72.81% from normal beams.

d. The toughness of UPR reinforced concrete beams has increased compared to normal reinforced concrete beams. The effect of reinforcement ratio on Toughness cannot be concluded.
e. The Rigidity Index of UPR reinforced concrete beams has increased compared to normal reinforced concrete beams. The effect of the reinforcement ratio on the Stiffness Index has increased. UPR reinforced beams with a diameter of 16 mm of flexural reinforcement experienced an increase in the stiffness index by 2.75% from normal beams. UPR reinforced beams with a diameter of 19 mm flexural reinforcement experienced an increase in the stiffness index by 0.88% from normal beams. UPR-reinforced beams with a diameter of 22 mm of flexural reinforcement experienced an increase in stiffness index of 1.26% from normal beams. UPR reinforced beams with a diameter of 25 mm of flexural reinforcement experienced an increase in stiffness index of 1.57% from normal beams.
f. The use of UPR-based patch repair mortar as a filling material proves that the contribution of friction between aggregates is large, causing the strain that occurs to be smaller. The effect of flexural reinforcement on normal reinforced concrete beams and UPR is clearly visible on the shear strain of the beam that occurs, this is illustrated by the greater the reinforcement ratio, the smaller the strain that occurs.
g. Crack patterns that appear in reinforced concrete beams repaired using UPR-based patch repair mortar require a higher load for cracks to occur in that section compared to normal reinforced concrete beams. The side of the beam that is patched has fewer cracks than the side of the beam that is not patched and also the first oblique crack that appears is on the side that is not patched and then only appears on the side where the patch is about to collapse. In normal beams oblique cracks appear and increase in length simultaneously on both sides.

The suggestions needed to improve the results of analysis and testing using the finite element method are as follows.

a. It is necessary to conduct a further review of the variation of the thickness of the repair with various diameters of reinforcement, which will provide more optimal results regarding information on the difference in the bond between the reinforcement and the repair material.
b. Further review is needed with variations of finite elements and filling materials.

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