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Integer 1/0 Knapsack Problem Dynamic Programming Approach in Building Maintenance Optimization

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Abstract: The most common problem in urban areas is the high public demand and the limited provision of housing. In meeting the needs of affordable housing for low income communities, the Government of Indonesia implements Rusunawa Project. Object of this research is Pandanarang Rusunawa. Rusunawa Pandanarang is one of the vertical housing in Cilacap that is facing deterioration issue and needs good maintenance management. This study aims at insetting priority and optimizing maintenance plan due to limited funds (limited budget) and the amount of damage that must be repaired. This study uses one of the optimization methods of Dynamic Programing on the application of Integer 1/0 Knapsack Problem, to determine an schedule the maintenance activities. The Criteria that are used such as: the level of building components damage and the level of occupants participation. In the first criterion, the benefit (p) is the percentage of damage that is fixed with the cost (w). While on the second criterion, the benefit (p) is the percentage of occupant participation rate on the maintenance activities with the cost (w). For the budget of Rp 125.000.000, 00, it was obtained from the simulation that the value of the optimum solution on the first criterion at the 7th stage of 71.88% with total cost Rp 106.000.000, 00. At the second criterion, the value of the optimum solution at the 7th stage of 89.29% with total cost Rp 124.000.000, 00.

Keyword: Mantenance, Optimization, Knapsack Problem, Dynamic Programming

1. Introduction

As one of the essential human need, the increase in housing demand leads into a problem as the resources is limited in urban areas. Furthermore, adequate occupancy qualities is unattainable for urban low-income people as developing a house becomes unaffordable. This situation enforced the Government of Indonesia to tackle the issue by providing flats for low-income people. A simple rental apartment building (Rusunawa) is a multilevel building that is public, has a primary function as a dwelling, and specifically designated for low-income people by paying a rent [2].

During its utilization, the condition of Rusunawa would be deteriorated. Therefore, routine checks, maintenances and rehabilitations are required to maintain its function. Efforts to maintenance the apartment buildings need to be planned and scheduled well in order to be implemented effectively, efficiently and appropriately in terms of cost, time and execution without reducing the occupant comfort.

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The object of this research is Rusunawa Pandanarang (fig 1) which is one of the simple rental apartments in the city of Cilacap. The building consists of 2 (two) twin block, each comprises of 5 (five) floors. Due to the limited budget, current maintenance is carried out only on certain components that are randomly damaged without any assessment of all building components. This study conducts an assessment of the physical condition of the building to determine the type and level of damage and to formulate the cost of repair and the occupants ability to repair damage independently. The Rusunawa building components was evaluated in accordance to the Minister of Public Works Regulation no. 25 Year 2007 regarding Certificate of Feasible Function.





Figure 1. Pandanarang Rusunawa

In the maintenance of the Rusunawa building, a method to determine maintenance activities that will be performed within 1 (one) budget year in good and planned manner, to produce optimal results related to the limited budget available is needed. This research employs Dynamic Programming framework by applying integer 1/0 knapsack problem to determine the schedule of maintenance activities to obtain optimal result within 2 (two) criteria. The first criterion is the level of damage, while the second criterion is the occupant's participation rate. Dynamic Programming Method is one of the decision-making methods with algorithms deterministic based used in solving several optimization cases developed by Richard E. Bellman in 1957 [8]. Integer 1/0 Knapsack Problem is one application of optimization problem solving within Dynamic Programming framework. On the knapsack issue, the stage is the process of inserting the goods into the sack / knapsack while the status states the loading capacity of the sack / backpack left after entering the goods in the previous stage. In this case, the stage at integer 1/0 knapsack is a sub component of the building that will be maintained, while the status are the benefit and the cost of repairing the damage of each subcomponent.

This study is expected to provide solutions/ inputs for decision making in determining the maintenance activities of building components which are better, faster, precise and optimal.

2. Assessment of Components Conditions Rusunawa Building

For the sake of maintenance management, the Rusunawa building is divided into components, for example, architectural, structural, utility, accessibility, building and

environmental components [3; 4]. The architectural components consist of exterior components and interior components of buildings such as interior floor coverings, interior wall coverings, doors / windows, interior ceiling coatings, exterior wall coverings, exterior floor coverings, exterior ceiling coatings. Structural components are parts or members of structures that are strongly bonded to one another and work together as a whole, forming and functioning as a building structure such as foundation, columns, beams, join-column-beams, shear walls, plat) floors and roofs, roof trusses and so on. Utility components include fire prevention, plumbing system (water supply and dirty water) installations, electrical installations and lightning protection installations. Accessibility components include access roads, exits, horizontal connections between spaces, and vertical transportation facilities, and provision of evacuation access. The building and environmental components include conformity with the basic building coefficient, building floor coefficient, and the building border line.

The criteria of physical reliability of the building include aspects of comfort, health, safety, ease and harmony with the environment. Physical observation are carried out to inspects aspects of architecture, structure, utility, accessibility, as well as the structure of the building and environment [5]. The assessment of the condition of the building was performed through visual inspection and measurement of the existing building and damage volume of each component of Rusunawa building to determine the level of damage. After obtaining the data of type and damage volume to each component, then the cost of rehabilitation based on the analysis of unit price (Analisa Harga Satuan Pekerjaan) applicable to the city of Cilacap district is calculated.

3. Utilization and Management of Rusunawa

The stage of Rusunawa are divided into the planning stage, construction stage, and post-construction stage. Post-construction stage is the operationalization / management and utilization of Rusunawa. This stage is very important in the sustainability of Rusunawa. Successful implementation of Rusunawa is mostly determined by the success of operations management [2].

Utilization and occupancy in accordance with regulatory requirements can maintain the quality of the building as a comfortable and healthy residential (dwelling). The environment also contributes to the physical condition of the flats (Rusunawa). The condition of flats (Rusunawa) that deteriorate in quality as a dwelling can be stated is undergoing the process of slaughter. Improvement to socio-economic conditions is expected to occur in residents of Rusunawa. Without an increase in the socio-economic conditions of the occupants, it is feared that the occupancy is not maintained and turned into improper [1]. In terms of utilization and management of Rusunawa, this study will only review the participation of occupants (ability and willingness of occupants) in the repair of damage to sub components of the building independently.

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4. Optimization of Maintenance Activities

Methods of decision making in the maintenance of buildings is really necessary to support the building maintenance in a good condition [6]. One method that can be used in decision making including the determination of optimization of maintenance activities is Dynamic Programming. Dynamic Programming is a mathematical technique used to optimize the decision-making process in phases. The optimal decision in all phases is called optimal policy. The adoption of dynamic programming is said to be capable of solving many problems: allocation, payload (knapsack), capital budgeting, inventory control, and so on [7].

Dynamic Programming is a troubleshooting method by outlining the problem into sub-problems which is a step to obtain a solution (stage) with state/ status inputs to get a decision [8]. The Knapsack issue is a combinatorial optimization problem, each of which has loads and values in a set of items, determines the sum of each item to be included so that the total load is less than or equal to the total value [9; 10]. Mathematically, the knapsack problem formulation is:

$$Maximize \sum_{i=1}^{n} PjXj$$
 (1)

Subject to
$$\sum_{i=1}^{n} WijXi \le Cij$$
, $Xi \in \{0|1\}$ (2)

Pj : Benefit for every item j (Pi>0)

Xi : Decision variables (1 is chosen and 0 is not chosen)

j : Item

Wij : Value for every resource i (Wij≥0) Cij : Capacity for every resource (Ci>0)

i : resource

From stage 1, we enter the 1st object into the sack for each unit of sack capacity to its maximum capacity. The recurrence for this issue is

$$fk(y) = \max\{fk-1(y), pk + fk-1(y - wk)\}, (Recurrence)$$
(3)

k : 1, 2, ..., n

f0(y): 0, y = 0, 1, 2, ..., M (basis); 0 = the value of the empty knapsack problem with the capacity y

fk(y): $-\infty$, y < 0 (basis); $-\infty$ = the value of the knapsack problem for negative capacity

the optimum gain of 0/1 Knapsack problem at stage k for capacity of y

fn(M): the optimum solution of the 0/1 Knapsack problem

5. Research methods

This research uses quantitative descriptive analysis method using studied variables. The data used in the form of primary data include data types and volume of damage to building components obtained through observation by using simple tools meter and camera, and data of the occupants partipacipation rate in the improvement/repair of building components obtained through interviews with respondents sample occupants. Secondary data include as built drawing and building specification, regulation, and analysis of unit price (AHSP) Cilacap District Year 2016/2017. The first step is to

identify the weight of building components based on field conditions in accordance with the weight of Rusunawa building components in the Form Reliability Certificate Function and previous research. Then observation and measurement of damage volume of each component of the building is conducted. From the volume of damage obtained, it can be known the level of damage each component. Then, from the existing types of damage, the plan handling and budget plan repair costs is formulated. The occupant's participation rate is the occupant's ability to repair sub-components damaged independently. This data is obtained through interviews to sample occupant. The lower the occupant participation on independent maintenance, the higher the value of the sub component to be prioritized.

6. Result and discussion

Based on the identification of building components as well as observation and measurement on the existing conditions, it is found some damage, especially in the architectural components.

Table 1. Weight Score and Condition of Building Component

Building	Component		Sub Component		Condit	on
Rusunawa						
Building	Architecture	10	Interior	80		
			Conformity of Function Use		18.75	Good
			Floor cover		12.5	Crack/ Broken
			Floor Plaster		12.5	Exfoliate
			Wall cover		12.5	Faded color
			Wall Plaster		12.5	Exfoliate
			Doors/Windows		18.75	Broken, loose
			Layers of ceiling		12.5	Faded color
			Exterior	20		
			Roof cover		50	Good
			Wall cover		12.5	Faded Color
			Floor cover		15	Faded Color
			Floor Plaster		12.5	Exfoliate
			Layers of ceiling		10	Faded Color
	Structure	30	Sub Structure	25		
			Foundasi		100	Good
			Top Structure	60		
			Joined Column-Beams		16.67	Good
			Column		25.00	Good
			Beams		25.00	Good
			Shear wall		16.67	Good
			Slab Floor		7.50	Good
			Slab roof		0.83	Good
			Roof truss		8.33	Good
			Complementary	15		
			Structure			
			Hanging Ceilings		6.67	Good
			Walls of Brick Pairs / Bricks		13.33	Good
			Canopy		40	Good
			Stairs		40	Good
	Utilities	50	Fire Prevention	33		Good
			Plumbing	25		Good

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Building	Component		Sub Component		Condition	
			Clean water		50	Good
			Dirty water		50	Good
			Electrical Installation	33		
			PLN Resources		50	Good
			Genset Resources		50	Good
			Lightning Protection Installation	8.2		Good
	Accessibility	5	Basic Space Size	17		Good
			Pedestrian Path and RAM	17		Good
			Parking Area	17		Good
			Control Equipment and Equipment	11		Good
			Toilet	22		Good
			Doors	17		Good
	Building Struct.	5	Compability with KGB	40		Good
	& Environment		Compability with KLB	40		Good
	-		Compability with SGB	20		Good

Percentage of damage was obtained through assessment of the condition of building components. Of all building components assessed, there are 7 (seven) sub component of the building that need repair. Each sub component of the building has a weight percentage of the entire building, the value of the damage level and the percentage of sub component improvement on all the required work. Then, cost recovery calculations for any damaged are performed.

Table 2. Level of Damage and Maintenance Costs Required

		Weight (%)	Level	of Damage (%)	
No	Building Sub component	Toward	Value	Toward	Maintenance cost
		Building	v aruc	Occupation	(Rp)
1	Floor cover of interior	2	0.14	12.94	46,976,381.28
2	Wall cover of interior	2	0.44	31.90	41,867,371.07
3	doors/windows	1.5	0.10	9.31	21,416,377.80
4	Layers of ceilings interior	1	0.23	21.47	13,526,235.11
5	Wall cover of exterior	0.25	0.14	12.81	22,900,140.00
6	Floor cover of exterior	0.55	0.09	7.85	59,997,008.70
7	Layers of ceilings exterior	0.2	0.04	3.73	5,997,679.44
Tota	al	7.5	1.06	100.00	212,681,193.40

Tabel 3. Costs and Benefit Repair at Level of Damage

No	Duilding Sub commonant	cost	(W) Donofit (n)
NO	Building Sub component	(x1.000.000)	Benefit (p)
1	Floor cover of interior	47	14.93
2	Wall cover of interior	42	28.94
3	doors/windows	21	8.45
4	Layers of ceilings interior	14	19.49
5	Wall cover of exterior	23	11.62
6	Floor cover of exterior	60	13.19
7	Layers of ceilings exterior	6	3.38
	Total	213	100.00

Tabel 4. Costs and Profit Repair at Occupants Participation Rate

			<u> </u>
	Building Sub component	Cost (x1.000.000)	(w) Benefit (p)
		(X1.000.000)	
1	Floor cover of interior	47	3.57
2	Wall cover of interior	42	7.14
3	doors/windows	21	14.29
4	Layers of ceilings interior	14	10.71
5	Wall cover of exterior	23	21.43
6	Floor cover of exterior	60	17.86
7	Layers of ceilings exterior	6	25.00
	Total	213	100.00

In the first criterion, the application of integer 1/0 knapsack dynamic programming, optimization is based on the cost and benefit of repair at the level of damage of each sub component (fig 2)/ the benefit (p) is the percentage of damage that is fixed with the cost (w). Meanwhile, in the second criterion, optimization is based on cost and level of participation of occupants to repair the damaged (fig 3)/ the benefit (p) is the percentage of occupant participation rate on the maintenance activities with the cost(w).

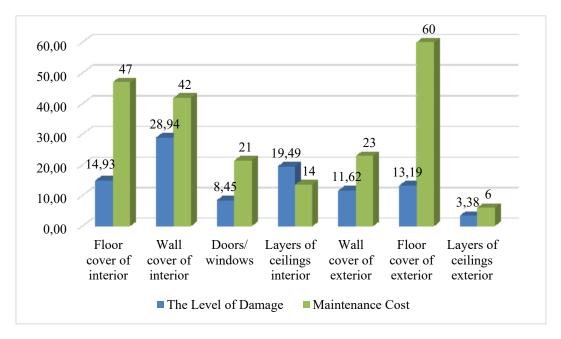


Figure 2. Costs and Benefits of Repair in the Level of Damage

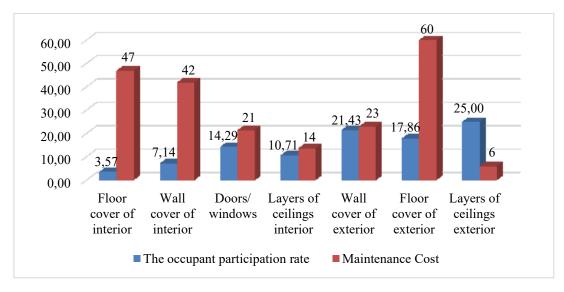


Figure 3. Costs and Benefits of Repair in the Occupants Participation Rate

In applying the knapsack integer 1/0 problem, the stage is an improvement on each sub component, while the status that contains the input/ information is the cost of repair and the level of damage for the first criterion or the level of participation of the occupants for the second criterion. Integer 1/0 on decision variable 1 means yes or in this case the sub component is repaired, and 0 means no or in this case the sub component is not fixed.

Tabel 5. Integer 1/0 Knapsack Problem Dynamic Programming Level of Damage

	0	· 1	-		_
Stage 1:	f1(y)	$= \max \{f0(y), p1 + f0(y - w1)\}$	}		
		$= \max \{f0(y), 14.93 + f0(y-4)\}$	7)}		
у	f0(y)	7.46 + f0(y - 47)	fl(y)	(x1, x2, x3, x4, x5, x6, x7)	
0	0	- ∞	0	(0,0,0,0,0,0,0)	
125	0	14.93	14.93	(1,0,0,0,0,0,0)	
Stage 2:	f2(y)	= $\max \{fl(y), p2 + fl(y - w2)\}$	}		
		$= \max \{fl(y), 28.94 + fl(y - 4)\}$	2)}		
у	fl(y)	28.94 + fl(y - 42)	f2(y)	(x1, x2, x3, x4, x5, x6, x7)	
0	0	- ∞	0	(0,0,0,0,0,0,0)	
125	14.93	43.87	43.87	(1,1,0,0,0,0,0)	
Stage 3:	f3(y)	$= \max \{f2(y), p3 + f2(y - w3)\}$	}		
		$= \max \{f2(y), 8.45 + f2(y-21)\}$)}		
у	f2(y)	8.45 + f1(y-21)	f3(y)	(x1, x2, x3, x4, x5, x6, x7)	
0	0	- ∞	0	(0,0,0,0,0,0,0)	
125	43.87	52.32	52.32	(1,1,1,0,0,0,0)	
Stage 4:	f4(y)	$= \max \{f3(y), p4 + f3(y - w4)\}$	}		
		$= \max \{f3(y), 19.49 + f3(y-1)\}$	4)}		
y	f3(y)	19.49 + f3(y - 14)	f4(y)	(x1, x2, x3, x4, x5, x6, x7)	
0	0	- ∞	0	(0,0,0,0,0,0,0)	
125	52.32	71.81	71.81	(1,1,1,1,0,0,0)	

Stage 5:	f5(y)	$= \max \{f4(y), p5 + f4(y - w5)\}\$
		$= \max \{f4(y), 11.62 + f4(y-23)\}\$

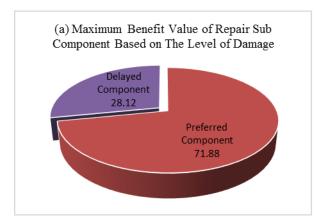
		$- \max \{14(y), 11.02 + 14(y - 2)\}$?)}	
у	f4(y)	11.62 + f4(y - 23)	f5(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	71.81	68.50	71.81	(1,1,1,1,0,0,0)
Stage 6:	f6(y)	$= \max \{f5(y), p6 + f5(y - w6)\}\$		
		$= \max \{f5(y), 13.19 + f5(y - 60)\}$))}	
у	f5(y)	13.19 + f5(y - 60)	f6(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	71.81	61.62	71.81	(1,1,1,1,0,0,0)
Stage 7:	f7(y)	$= \max \{f6(y), p7 + f6(y - w7)\}\$		
		$= \max \{f6(y), 3.38 + f6(y-6)\}\$		
у	f6(y)	3.38 + f6(y-6)	f7(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	71.81	71.88	71.88	(0,1,1,1,1,0,1)

Tabel 6. Integer 1/0 Knapsack Problem Dynamic Programming Level of Occupant Participation

		I		
Stage 1:	fl(y)	$= \max \{f0(y), p1 + f0(y - w1)\}\$		
		$= \max \{f0(y), 3.57 + f0(y-47)\}\$		
у	f0(y)	3.57 + f0(y - 47)	fl(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	0	3.57	3.57	(1,0,0,0,0,0,0)
Stage 2:	f2(y)	$= \max \{f1(y), p2 + f1(y - w2)\}$		
		$= \max \{f1(y), 7.14 + f1(y-42)\}\$		
у	fl(y)	7.14 + fl(y - 42)	f2(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	3.57	10.71	10.71	(1,1,0,0,0,0,0)
Stage 3:	f3(y)	$= \max \{f2(y), p3 + f2(y - w3)\}$		
		$= \max \{f2(y), 14.29 + f2(y-21)\}\$		
у	f2(y)	14.29 + f2(y-21)	f3(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	10.71	25.00	25.00	(1,1,1,0,0,0,0)
Stage 4:	f4(y)	$= \max \{f3(y), p4 + f3(y - w4)\}\$		
		$= \max \{f3(y), 10.71 + f3(y-14)\}\$		
у	f3(y)	10.71 + f3(y - 14)	f4(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	25.00	35.71	35.71	(1,1,1,1,0,0,0)
Stage 5:	f5(y)	$= \max \{f4(y), p5 + f4(y - w5)\}\$		
		$= \max \{f4(y), 21.43 + f4(y-23)\}\$		
у	f4(y)	21.43 + f4(y-23)	f5(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	35.71	53.57	53.57	(0,1,1,1,1,0,0)

Stage 6:	f6(y)	$= \max \{f5(y), p6 + f5(y - w6)\}\$		
		$= \max \{f5(y), 17.86 + f5(y-60)\}\$		
у	f5(y)	17.86 + f5(y - 60)	f6(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	53.57	64.29	64.29	(1,1,1,1,0,0,0)
Stage 7:	f7(y)	$= \max \{f6(y), p7 + f6(y - w7)\}$		
		$= \max \{f6(y), 25 + f6(y-6)\}\$		
у	f6(y)	25 + f6(y - 6)	f7(y)	(x1, x2, x3, x4, x5, x6, x7)
0	0	- ∞	0	(0,0,0,0,0,0,0)
125	64.29	89.29	89.29	(0,0,1,1,1,1,1)

The result of the study may be described as follows. In the first criterion, for the budget of Rp 125.000.000,00, it was obtained from the simulation that the value of the optimum solution at the 7th stage of 71.88% with total cost Rp 106.000.000,00 as seen at figure 4(a). While on the second criterion, for the same budget, it was obtained from the simulation that the value of the optimum solution at the 7th stage of 89.29% with total cost Rp 124.000.000,00 as seen at figure 4(b).



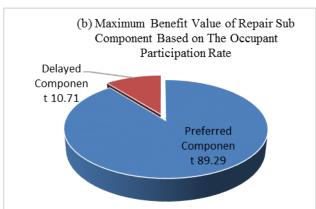


Figure 4. Maximum Benefit Value of Total Repair

7. Result and discussion

In this study, the determination of the optimization of building maintenance is based on the criteria of damage level and the level of participation of occupants on the handling of damage. From the results of observations it is obtained 7 (seven) sub components of the building that have been damaged and need to be repaired. Based on the results of the study approach and the analysis that has been done with Dynamic Programming method on the application of knapsack problem integer 1/0 problem, it is obtained the optimum solution and maximum benefit value for each criterion. In the criterion of damage level, the optimum solution is decision variable X (0,1,1,1,1,0,1) which means improvement is done in sub component to 2,3,4,5 and 7, that is wall cover of interior, door/ windows, layers of ceilings interior, wall cover of exterior and layers of ceilings exterior. The maximum score is 71.81% and maintenance costs Rp 106.000.000, 00. Meanwhile, the criteria of occupant participation level, the optimum solution is decision variable X

(0,0,1,1,1,1,1) which means repair is done in sub component to 3,4,5,6 and 7 that is door/ window, layers of ceilings interior, wall cover of exterior, floor cover of exterior and layers of ceilings exterior. The maximum score is 89.29 and maintenance costs Rp 124.000.000, 00.

From this research, it is expected to give input specially to decision maker about method which can be used in decision making quickly, accurately and optimally in building maintenance at each criterion or by combining it.

References

- [1] Hendaryono, S.M., 2010, Evaluation Management of Rusun Pekunden and Bandarharjo, Thesis, Universitas Diponegoro, Semarang.
- [2] State Minister for Public Housing, 2007. Permen Negara Pera. No.14/PRT/M/2007 Pengelolaan Rumah Susun Sederhana Sewa.
- [3] Ministry of Public Work, 2007. Permen PU. No.25/PRT/M/2007 Sertifikat Laik Fungsi.
- [4] Ministry of Public Work, 2007, 2008. Permen PU. No.24/PRT/M/2008 Pedoman Teknis Pembangunan Bangunan Gedung Negara.
- [5] Priyo, M dan Wijatmiko I.H., 2011. Evaluation of the physical reliability of buildings. Jurnal Semesta Teknika, Yogyakarta, https://scholar.google.co.id/.
- [6] Hartono, Widi dkk., 2015, Application of AHP method for prioritizing maintenance of building of assisted health center in sukoharjo district, e-Jurnal Matriks Teknik Sipil, Surakarta, https://scholar.google.co.id/.
- [7] Nurhidayati, F.U., 2010. The use of dynamic programming to determine the minimum total cost of production planning and inventory control, Skripsi, Universitas Islam Negeri Maulana Malik Ibrahim. Malang.
- [8] Imam, K., Dynamic Programming. (serial on line), https://id.scribd.com/doc/14580611/ Dynamic-Programming
- [9] Prasetyowati, M.I. and Wicaksana, A., 2013. Implementation of Dynamic Programming Algorithm for Multiple Constraints Knapsack Problem. SNATI, 2013, ISSN: 1907-5022.
- [10] Nurhayati, O.D., Dynamic Programming. (serial on line), https://eprints.undip.ac.id/18675/