International Journal of Pedagogy and Teacher Education

Journal Homepage: jurnal.uns.ac.id/ijpte

Discriminant Analysis of Students' Spatial Ability in Understanding Flat-Sided Geometric Shapes

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ARTICLE INFO

Article History

Received : January 15, 2025
1st Revision : February 15, 2025
Accepted : April 17, 2025
Available Online : April 30, 2025

Keywords:

Discriminant Analysis,
Mental Rotation;
Spatial Orientation;
Spatial Perception;
Spatial Relation

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ABSTRACT

Spatial ability is a crucial aspect that supports students in visualizing and understanding abstract mathematical concepts, particularly flat-sided geometric shapes in geometry learning. This study aims to identify the factors that differentiate students with low and high spatial abilities through discriminant analysis. The factors analyzed include Mental Rotation, Spatial Orientation, Visualization, Spatial Relation, and Spatial Perception, measured using a spatial ability test. The test instrument consisted of five questions developed based on the spatial ability framework offered by Maier and validated by mathematics and learning evaluation experts. A total of 34 ninth-grade students from a junior high school in Tangerang Regency were selected through purposive sampling. The analysis results showed that Visualization, Spatial Relation, and Spatial Perception were the main predictors that significantly differentiated the two groups. Visualization supports the ability to imagine geometric objects, Spatial Relation facilitates the understanding of relationships between objects, and Spatial Perception aids in recognizing the position and relationship of geometric elements. The resulting discriminant model had an eigenvalue of 13.967, indicating a strong discriminant power in separating student groups. Understanding these differentiating factors provides a foundation for designing effective learning strategies, such as the use of augmented reality (AR) applications and 3D modeling tools to enhance students' comprehension of spatial figures. Furthermore, interventions using physical or virtual manipulatives can be tailored to students' needs, assisting them in mastering the concept of flat-sided geometric shapes.

How to cite: Lutfi, M. K., Herman, T., & Mulyaning A., E. C. (2025). Discriminant Analysis of Students' Spatial Ability in Understanding Flat-Sided Geometric Shapes. *International Journal of Pedagogy and Teacher Education, 9*(1), 56-71. https://doi.org/10.20961/ijpte.v9i1.98297

1. INTRODUCTION

The concept of flat-sided geometric shapes is a difficult puzzle in learning geometry since the students have diverse spatial abilities (Hidayat et al., 2022; Salsabilah et al., 2023). A lot of students have difficulties constructing a mental representation of 2D and 3D geometric objects, discerning relations between sides, and perceiving geometric properties visually and analytically (Fiantika et al., 2018). These differences are likely to have an impact on students' conceptual knowledge and problem-solving in geometry (Noor Muhammad et al., 2022). Spatial ability—the ability to understand, visualize, and manipulate spatial relations between stimuli—is one of the highest-gifted skills students can have (Chen et al., 2020; Dick, 1976). Enhanced spatial ability promotes understanding and solving of math-oriented exercises (Sorby et al., 2022). The reasonable and effective use of interactive and experiential learning methods can promote such students' study of space and Mathematics well (Cui & Guo, 2022). In geometry, spatial ability helps the students to solve problems involving shapes, understand transformations, and establish other relations between the elements of geometric objects (Palobo & Juniati, 2022; Zakelj & Klančar, 2022). Students who have good spatial skills can understand the concepts of congruence, symmetry, and rotation, while students with low spatial ability have a hard time interpreting visual representations and solving geometric problems (Muhammad et al., 2022; Riastuti et al., 2017).

Proficiency in knowledge of flat-sided geometrical shapes is essential for both academic performance and practical applications (e.g., architecture, design, and engineering) (Lutfi et al., 2023). Thus, the much-needed focus in mathematics education curricula to develop students' spatial ability is necessary to help them with life's

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

real-world challenges necessitating geometric and spatial reasoning (Muhammad et al., 2022). Reinforcement in this area prepares students to achieve the highest academic records and to acquire problem-solving skills for innovation.

Children need to effectively combine two-dimensional and three-dimensional views of a shape, mentally transform their spatial representations, and develop the ability to solve complex problems. Nevertheless, differences in spatial ability influence the level of understanding students have (Ma'rifatin et al., 2019; Ngirishi & Bansilal, 2019). Evidently, there is also a need for teaching methods and strategies that emphasize the use of visuals and interactives to help students understand abstract geometry concepts (Lutfi & Kusumastuti, 2024; Özçakir & Cakiroglu, 2021).

Students with low spatial ability usually face difficulties in visualizing the relationship between two-dimensional drawings and three-dimensional objects (Suprayo et al., 2023). Problems arise when it comes to understanding multiple viewpoints, decoding pictorial representations, and imagining changes such as rotations and reflections (Budiarto et al., 2020; Tadeo & Yoo, 2022). Limited visuospatial skill deters problem-solving in situations that involve computation of area and volume in response to visual cues (Andriani et al., 2022; Juliana et al., 2022). These difficulties emphasize the importance of spatial reasoning ability in geometric proficiency.

Geometry learning is facilitated by multiple dimensions of spatial ability. Maier (1991) identified five primary components: Mental Rotation, Spatial Orientation, Visualization, Spatial Relation, and Spatial Perception. Mental Rotation refers to the mental rotation of an object (Koustriava & Papadopoulos, 2010), an important aspect to perceive symmetry, projection, and orientation in cubes, cuboids, pyramids, prisms, etc. Spatial Orientation concerns understanding an object's orientation with respect to the self (Hazen, 1983; Stuchlík, 2003), which is crucial in recognising relations between sides, angles, and dimensions. It requires visually manipulating representations (Manovich, 2011) so that students can make sense of dimensions such as length, width, and height. Spatial Relation deals with the identification of the location of a side or an angle that makes up a geometric figure (Saad et al., 2015). Finally, Spatial Perception is the aptitude to judge spatial relations between polyshapes despite viewpoint changes (McCarthy, 1990).

These five factors are powerful drivers of successful learning in geometry (Maier, 1991). Achievement in geometry is influenced not only by the mastery of concepts but also by students' readiness to visualize spatial structures (Kusnadi & Barumbun, 2023; Lutfi & Jupri, 2020). While they are all well-theorized, little is known about the role of these dimensions in discriminating among individuals with high versus low spatial abilities, especially within flat-sided geometric shapes. Previous studies have mostly focused on the influence of spatial ability on general geometry learning but have not made any formal statistical determination about which dimensions effectively distinguish among student groups (Muhammad et al., 2022; Sabil et al., 2024). It is important to understand these differences because spatial ability differences are related to differences in how effectively and how much individuals learn. Findings like these not only provide a yardstick of student performance but also can inform the type of instructional responses that teachers need to make. As such, it is necessary to conduct a more thorough examination of the influence of each dimension of space on students' ability differentiation to develop better and more inclusive teaching methods. This article uses statistical discriminant analysis to identify the principal variables that discriminate the high and low spatial ability groups. The discriminant analysis provides detailed classification and emphasizes the dimensions that most contribute to the separation. That is, this research addresses the question of how much the five space-related factors—Mental Rotation, Spatial Orientation, Visualization, Spatial Relation, and Spatial Perception—contribute to the division of students into high and low spatial ability groups. It also describes the role of each aspect in the mastery of flat-sided plane geometric forms.

The purposes of this study are: (1) to determine what 3-D perceptual representations play a role in distinguishing between high and low spatial ability students; (2) to analyze the contribution of each of these 3-D perceptual relations to geometry performance; and (3) to provide a unified approach to three-dimensional and planar reasoning, forming instructional interventions designed to improve spatial skills for low spatial ability students.

In principle, this study addresses the gap in knowledge regarding the role of particular spatial skills in relation to geometrical understanding. In a more practical sense, it provides specific advice to teachers to create adaptive learning environments appropriate for different students. Teachers can use focused interventions (e.g., spatial visualization activities or technology-crafted learning environments, such as AR-based applications) (Lutfi et al., 2023) to boost the ability of students to visualize and manipulate 3D structures. Ultimately, this research

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

is expected to contribute to optimizing students' learning outcomes in geometry, ensuring that students across different spatial ability levels are better equipped to master complex spatial concepts necessary for academic and professional success

2. MATERIAL AND METHOD

Research Design

This study employed a quantitative analysis with the discriminant method to outline specific qualities that distinguish low and high spatial ability students. Discriminant analysis was selected as it allows classification of people into group memberships based on predictor variables and helps determine how well spatial ability subtests discriminate between groups. The dimensions of our spatial ability are based on the model of Maier (1991) and can be distinguished into five dimensions (see Figure 1): Mental Rotation, Spatial Orientation, Visualization, Spatial Relation and Spatial Perception.

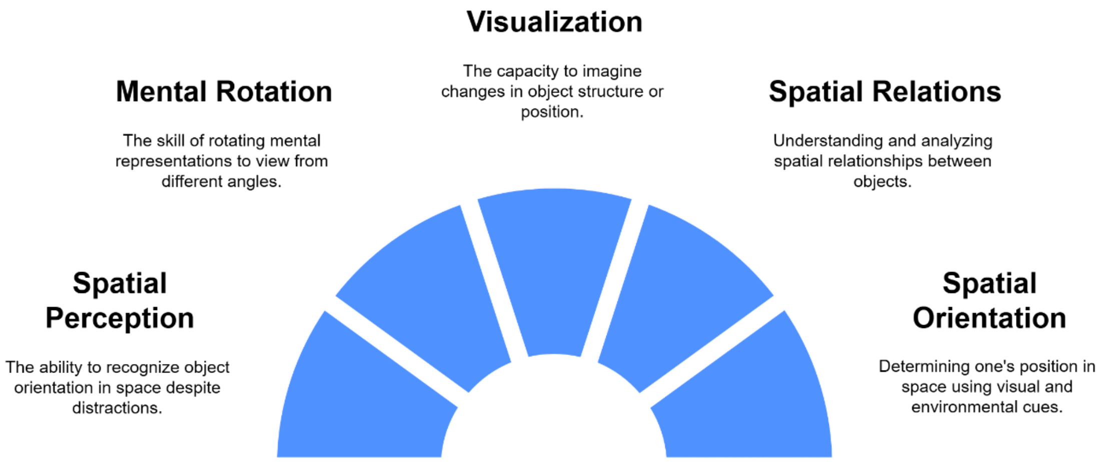

Figure 1. Spatial Ability Aspects (Maier, 1991)

Participants

The respondents of study consisted of 34 ninth-grade students of a junior high school in Tangerang Regency. The purposive sampling was used to select the participants of which their mathematics topic on flat-sided geometric shapes had been covered thoroughly. A high and a low spatial ability group was formed depending on the average mathematics scores of the participants (high spatial ability: ≥ 75 and low spatial ability: < 75). The 75 cut-off score was set based on norms defined by the classroom teacher.

Instrumentation

The study instrument was a spatial ability test focused on five components of spatial ability according to the framework of Maier (1991). All subjects took an examination (providing responses to five items with essay format) on each of the five dimensions, about Mental Rotation, Spatial Orientation, Visualization, Spatial Relation and Spatial Perception. Items were constructed in a principled way to be consistent with the cognitive stress of flat-side geometric shape concepts. Expert validity of the instrument was confirmed by two experts, one in geometry education and another in evaluation of learning. Changes were guided by expert input in the clarity, relevance, and appropriateness of the items for the revised test.

Data Collection

Data collection began with the administration of the validated spatial ability test to all participating students. Additionally, the students' average mathematics scores were collected to facilitate grouping based on spatial ability levels. After data collection, students were classified into the high and low spatial ability groups according to their mathematics scores, following the predetermined cutoff score of 75. An overview of the research stages conducted from beginning to end is summarized in Figure 2.

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

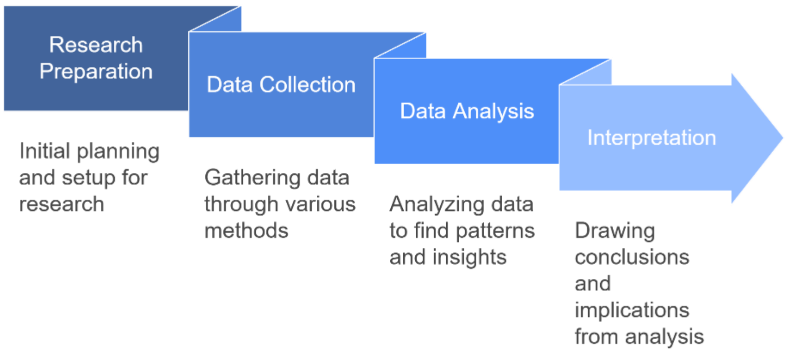

Figure 2. Research Stages

Data Analysis

The collected data was checked to ensure completeness and suitability. Figure 3 fully explains the stages of data analysis using discriminant analysis. Discriminant analysis assumption tests are carried out to ensure the validity of the model, including data normality test, covariance matrix similarity test, and multicollinearity test. After the assumptions are met, discriminant analysis is carried out to build a discriminant function that aims to identify differentiating factors between groups of students with low and high spatial abilities. This discriminant function is evaluated through measuring classification accuracy and statistical significance tests using Wilks' Lambda.

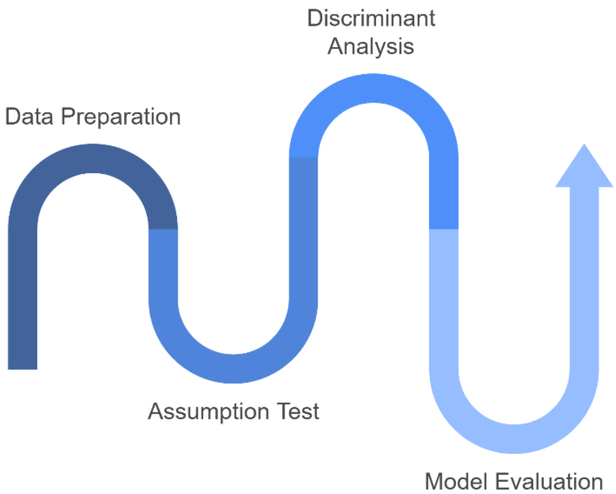

Figure 3. Stages of Discriminant Analysis

3. RESULTS

Assumption Test

Prior to the actual analysis, which is discriminant analysis, the assumption test should be performed first. This assumption check confirms that the data meet the required assumptions for using a statistical procedure. There are several assumptions to be checked in discriminant analysis, such as the Normality Test, Multicollinearity Test, and Covariance Matrix Equality Test. By verifying that these assumptions are met, spurious inferences are avoided, and the main results will be accurate and reliable. It is therefore important to check for assumptions before the application of discriminant analysis, to make sure that the selected technique is consistent with the nature of the available data.

Normality Test

Normality is important in discriminant analysis because departures from this requirement can lead to invalid predictions or improper classification. As a result, verifying the normality of the data is necessary prior to conducting the main analysis to improve the accuracy and reliability of the discriminant analysis model, such that the results in this study are more valid and acceptable.

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
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Table 1. Normality Test Using *One-Sample Kolmogorov-Smirnov Test*

		Unstandardized Residual
N		34
Normal Parameters [a,b]	Mean	0.0000000
	Std. Deviation	0.12533289
Test Statistics		0.092
Asymp. Sig. (2-tailed)		0.200

It can be seen from Table 1 that the number of samples taken is 34 (the sample size is based on the results of the Normality Test using the Kolmogorov-Smirnov test). The mean of the residual data distribution is 0, and the standard deviation is 0.1253. The Kolmogorov-Smirnov test statistic of 0.092 produces a p-value (Asymp. Sig. 2-tailed) of 0.200. At a significance level of $\alpha = 0.05$, a p-value larger than this level implies that there is not enough evidence to reject the null hypothesis (H_0). Therefore, based on the normal probability plots and the Kolmogorov-Smirnov test, it can be assumed that the residual data are normally distributed. Given the satisfaction of the normality assumption, the residualized data can be analyzed accordingly, namely using Discriminant Analysis.

Multicollinearity Test

Multicollinearity is detrimental to discriminant analysis, as it can render discriminant coefficients unstable and uninterpretable. It is noted that a multicollinearity should be detected and solved prior to the main analysis, the discriminant coefficients derived will become more precise and meaningful.

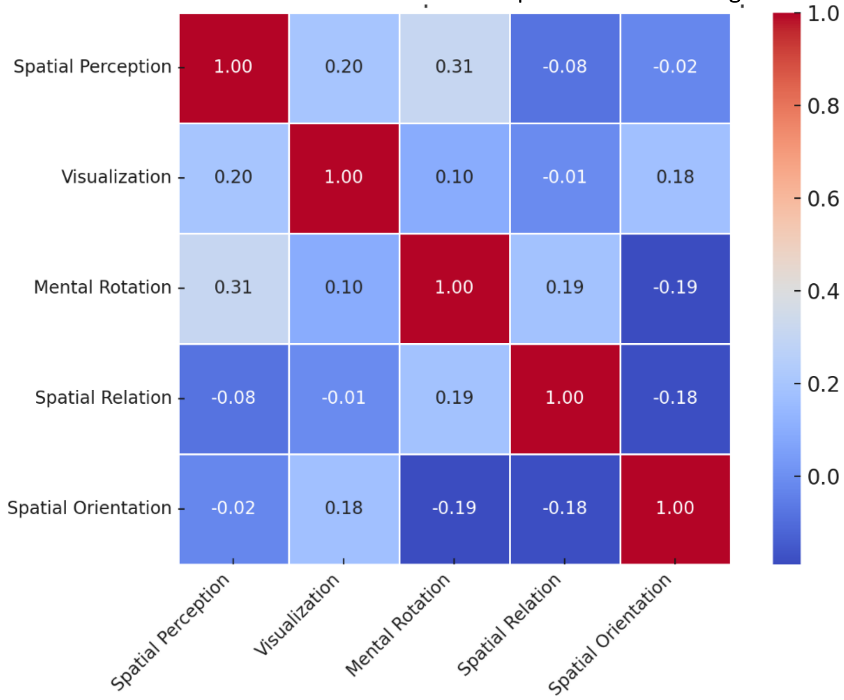

Figure 4. Multicollinearity Test for Discriminant Analysis

In statistical analysis, when the correlation between variables is over 0.7 or 0.8, it is considered to cause multicollinearity (Kim, 2019). We have shown in Figure 4 that none of the pairs of variables have a correlation that is close to or higher than this threshold. For instance, Spatial Perception and Visualization are correlated at 0.20. Spatial Visualization and Mental Rotation are correlated at 0.31. This shows that the variables in the analysis do not have a perfect linear relationship, while there is little possibility of multicollinearity. "On the average," however, relationships between many items are very feeble or near zero, examples being the

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

correlations between Spatial Perception and Spatial Orientation (-0.02) or Visualization and Spatial Relation (-0.01). This increases the separateness of the independent variables in the model.

Covariance Matrix Equality Test

The equality of covariance matrix test is important in discriminant analysis for the equal variance of groups. When this assumption is not met, the resulting analytical results can be seriously affected, such as low classification accuracy, unstable coefficient estimates, and biased hypothesis tests. So, this test should be carried out in advance and other methods should be considered if the assumption can not be met.

Table 2. Covariance-covariance Equality Test

Box's M		7.211
F	Approx.	1,076
	df1	6
	df2	6330.831
	Sig.	0.374

In Table 2, the *p-value* of 0.374 is greater than 0.05; therefore, the *null hypothesis* cannot be rejected. This indicates that there is insufficient evidence to suggest that the population covariance matrices differ between groups. Accordingly, the assumption of equality of covariance matrices holds, and thus, discriminant analysis can be performed without violating this assumption.

Descriptive Analysis

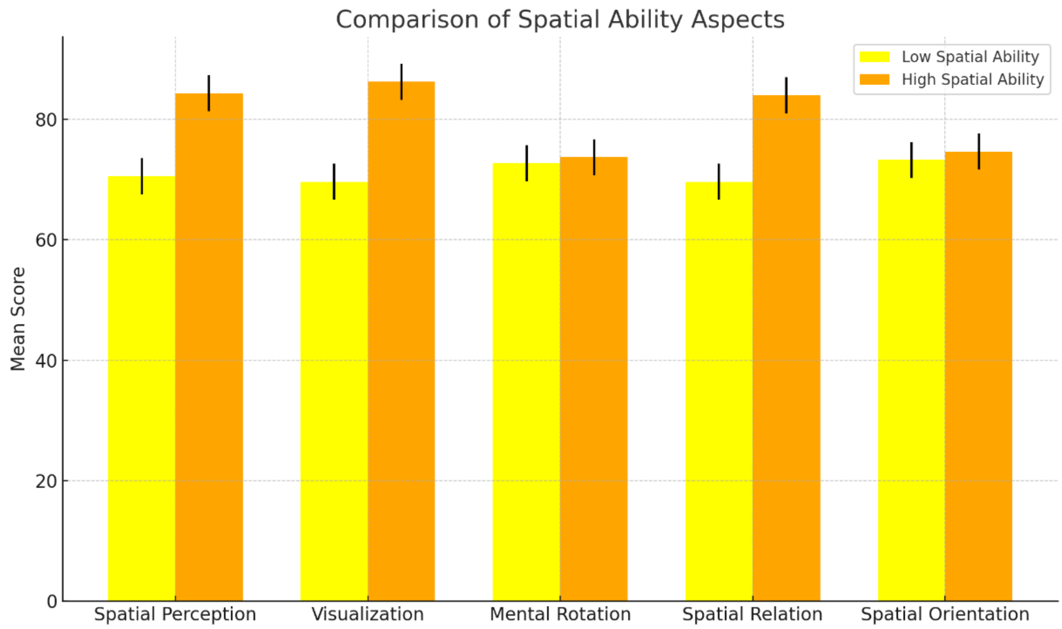

Figure 5. Comparison of Aspects in Spatial Ability Descriptively

Figure 5 reveals a comparison of the average scores of the spatial ability subtests between the *Low Spatial Ability* and *High Spatial Ability* groups across five categories: *Spatial Perception*, *Visualization*, *Mental Rotation*, *Spatial Relation*, and *Spatial Orientation*. Overall, the *High Spatial Ability* group achieved a higher mean score in each factor compared to the *Low Spatial Ability* group. The clearest discrepancies were observed in the aspects of *Visualization* and *Spatial Relation*, where the *High Spatial Ability* group outperformed significantly.

Spatial Relation thinking is considered more difficult than *Mental Rotation* or *Spatial Orientation*, as it involves imagination and, more abstractly, the appreciation of relationships between shapes that may be less evident or less elementary. The graphical error bars represent the *standard deviation* of each group, indicating

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

the extent of score dispersion. A lower *standard deviation* in the *Low Spatial Ability* group suggests that the scores in this group were more clustered compared to the *High Spatial Ability* group. This bar graph effectively illustrates the remarkable spatial distinction between the two groups.

Descriminant Analysis

Table 3. Test of Mean Differences of Variables in Groups

	Wilks' Lambda	F	df1	df2	Sig.
Spatial_Perception	.216	116,398	1	32	.000
Visualization	.140	196,706	1	32	.000
Mental_Rotation	.963	1.218	1	32	.278
Spatial_Relation	.167	159,856	1	32	.000
Spatial_Orientation	.909	3.202	1	32	.083

In Table 3, the *Test of Equality of Group Means* analysis for testing the difference between the means of variables between groups is presented. This test—based on the *Wilks' Lambda* statistic and the *F-test*—indicates whether the groups are significantly different from one another. A smaller *Wilks' Lambda* value suggests that the tested variable is superior for distinguishing individual inter-group differences and, particularly, overall in-group differences, thus improving the performance of the discriminant model classification. In this case, a low *Lambda* value can mean that variables are better at discriminating between groups of students with different spatial abilities.

For *Spatial Perception*, *Wilks' Lambda* = 0.216, $F(3, 243) = 116.398$, and *Sig.* = 0.000, indicating a very significant difference between the groups for this variable. For *Visualization*, the *Wilks' Lambda* value is 0.140, the *F* value is 196.706, and the significance is 0.000, showing a highly significant difference between groups. In the case of *Mental Rotation*, $\Lambda = 0.963$, $F = 1.218$, and $p = 0.278$. Because the *p*-value is greater than 0.05, there is no significant difference for this variable between the groups. For *Spatial Relation*, the *Wilks' Lambda* value is 0.167, the *F* value is 159.856, and it is statistically significant at the 0.000 level, hence there is a significant difference among the groups. Lastly, for *Spatial Orientation*, the *Wilks' Lambda* value is 0.909, the *F* value is 3.202, and the level of significance is 0.083, which means that no statistically significant difference exists between groups for this variable.

Mental Rotation and *Spatial Orientation* were not significant in the discriminant function since they either lack the ability to clearly distinguish groups in terms of spatial ability, have similar variance between groups, or their effect is confounded with other more important variables such as *Spatial Perception* and *Visualization*. In other words, these covariates or statistical methods may have been underestimated or inappropriate for improved group discrimination.

In summary, these findings show that variables such as *Spatial Perception*, *Visualization*, and *Spatial Relation* significantly differentiate groups, whereas *Mental Rotation* and *Spatial Orientation* are not significant. These results may be useful in estimating which features contribute most to group differences in discriminant analysis.

Table 4. Variables Entered/Removeed

Step	Entered	Min. D Squared				
		Statistic	Exact F			
			Statistic	df1	df2	Sig.
1	Visualization	23.467	196.706	1	32.000	3.241E-15
2	Spatial_Relation	43.080	174.913	2	31.000	1.303E-17
3	Spatial_Perception	53.321	139.674	3	30.000	1.023E-17

The discriminant analysis results, showing the steps of identifying significant variables to discriminate between the *Low Spatial Ability* and *High Spatial Ability* groups, are presented in Table 4. This comparison is conducted based on the *Mahalanobis Distance*, which measures how much a variable separates the two groups, and *Sig.*, which tests the significance of the difference.

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
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The *Visualization* variable enters the model at *Step 1*, with a *Mahalanobis Distance* of 23.467 and an *F* value of 196.706 (df_1 = 1, df_2 = 32, *Sig.* = 3.241 × 10^{-15}). This finding indicates that *Visualization* is very important in differentiating the two groups. In *Step 2*, the *Spatial Relation* variable is introduced into the model (*Mahalanobis Distance* = 43.080, *F* = 174.913; df_1 = 2, df_2 = 31, *Sig.* = 1.303 × 10^{-17}), suggesting its major contribution to group discrimination after *Visualization*. In *Step 3*, *Spatial Perception* as a predictor is added, with a *Mahalanobis Distance* of 53.321 and *F* = 139.674 (df_1 = 3, df_2 = 30, *Sig.* = 1.023 × 10^{-17}), helping to complete the model and increasing the group difference.

Visualization stands as the most important of all variables in this discriminant analysis, as it is critical for the mental rotation of three-dimensional figures and serves as a cornerstone of geometric learning, especially with flat-sided geometric figures. Students' visualization of geometric objects and their spatial relationships is also a key factor that acts as a sieve for distinguishing students with high and low spatial abilities.

Overall, these findings suggest a progressive and significant contribution of the three dimensions—*Visualization*, *Spatial Relation*, and *Spatial Perception*—in discriminating between the two groups. *Visualization* has the greatest standardized mean, followed by *Spatial Relation*, and then *Spatial Perception*. Other variables were excluded from the model because their effects were not significant enough to improve group distinction. This analysis is important because it offers insight into the variables that affect differences in spatial ability between groups

Table 5. Variables in the Analysis

Step		Tolerance	F to Remove	Min. D Squared
1	Visualization	1.000	196.706	
2	Visualization	1.000	32.519	19.071
	Spatial_Relation	1.000	22.284	23.467
3	Visualization	.961	13.186	35.875
	Spatial_Relation	.993	18.573	31.473
	Spatial_Perception	.955	6.551	43.080

Table 6. Variables Not in the Analysis

Step		Tolerance	Min. Tolerance	F to Enter	Min. D Squared
0	Spatial_Perception	1,000	1,000	116.398	13.886
	Visualization	1.000	1.000	196.706	23.467
	Mental_Rotation	1.000	1.000	1.218	.145
	Spatial_Relation	1.000	1.000	159.856	19.071
	Spatial_Orientation	1.000	1.000	3.202	.382
1	Spatial_Perception	.961	.961	9.096	31.473
	Mental_Rotation	.990	.990	.010	23.476
	Spatial_Relation	1.000	1.000	22.284	43.080
	Spatial_Orientation	.969	.969	.067	23.526
2	Spatial_Perception	.955	.955	6.551	53.321
	Mental_Rotation	.955	.955	.571	43.973
	Spatial_Orientation	.935	.935	.221	43.426
3	Mental_Rotation	.856	.856	2.207	57,670
	Spatial_Orientation	.930	.927	.355	54,020

To facilitate interpretation, Table 5 presents the contents of the discriminant analysis at each step and the contributions of *High* and *Low Spatial Ability* to the separation of the groups. In *Step 1*, *Visualization* is included in the first term since it has the strongest discriminatory power between the groups. With a value of 196.706 and a *Min. D Squared* value of 19.071, this variable shows good discriminating power. At *Step 2*, *Spatial Relation* is included in the model and makes another significant contribution, with an *F to Remove* value of 22.284 and a *Min. D Squared* value of 23.467. Both *Visualization* and *Spatial Relation* contribute to increasing group separation. In *Step 3*, *Spatial Perception* is added last. This predictor variable has an *F to Remove* value of 6.551 and a *Min. D Squared* value of 43.080, indicating a substantial further contribution to the discriminant model. The high values of tolerance for each independent variable (all above 0.95) suggest the absence of serious

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
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multicollinearity between the variables included in the model. In general, the *Visualization* variable was the most important distinguishing factor between the groups, followed by *Spatial Relation* and *Spatial Perception*. These three factors contributed cumulatively to the optimal discriminant model. The other variables were excluded because their unadjusted associations were not sufficiently strong for inclusion. These findings point out that *Visualization*, *Spatial Relation*, and *Spatial Perception* are the nearly significant factors for distinguishing spatial abilities among groups.

For each step, Table 6 lists the variables along with the contributions that might have been made if they had been included in the discriminant analysis. The table presents *Tolerance*, *F to Enter*, *Min. D Squared*, and between-group differences for each variable. At *Step 0* (before analysis), *Visualization* had the highest *F to Enter* value (196.706) and the largest *Min. D Squared* value (23.467), making this variable the most contributive to the separation of the groups. The factor *Spatial Relation* was also strongly significant (*F to Enter* = 159.856, *Min. D Squared* = 19.071), while *Mental Rotation* and *Spatial Orientation* presented very low scores, indicating that both hardly contributed. At *Step 1*, once *Visualization* was entered, the variable *Spatial Relation* emerged as the next appropriate variable to be entered into the model, with an *F to Enter* value of 22.284 and a *Min. D Squared* of 43.080, thus contributing significantly more to the difference between groups. At *Step 2*, after *Spatial Relation* was inserted, the *Spatial Perception* variable entered as the next greatest predictor, with an *F to Enter* value of 6.551 corresponding to a *Min. D Squared* of 53.321. The other remaining variables, such as *Mental Rotation* and *Spatial Orientation*, continued to contribute little, having low *F to Enter* and *Min. D Squared* values. At *Step 3*, when *Visualization*, *Spatial Relation*, and *Spatial Perception* were entered, additional variables such as *Mental Rotation* and *Spatial Orientation* continued to be non-significant for inclusion, maintaining very low *F to Enter* values. Overall, these findings confirm that the constructs *Visualization*, *Spatial Relation*, and *Spatial Perception* are the most discriminant between the groups, whereas *Mental Rotation* and *Spatial Orientation* do not materially increase the discriminant ability of the model. This suggests that these three main variables must be emphasized when conducting discriminant analysis to differentiate the *Low Spatial Ability* and *High Spatial Ability* groups.

Table 7. Wilks' Lambda

Step	Number of Variables	Lambda	df1	df2	df3	Exact F			
						Statistic	df1	df2	Sig.
1	1	.140	1	1	32	196.706	1	32.000	.000
2	2	.081	2	1	32	174.913	2	31.000	.000
3	3	.067	3	1	32	139.674	3	30.000	.000

The results of the test of how well the variables in the discriminant model distinguish between the *Low Spatial Ability* and *High Spatial Ability* groups are presented in Table 7. The smaller the value of *Wilks' Lambda*, the better the discriminant model is at separating the groups, and the *Sig.* value indicates whether the performance is statistically significant. A decrease in the *Wilks' Lambda* values for the three predictors (*Visualization*, *Spatial Relation*, and *Spatial Perception*) indicates that these predictors add value to the capability of the model to differentiate between groups, making group differentiation more transparent. The lower the *Lambda* value, the better the model separates the groups.

At *Step 1*, only one predictor is introduced into the model, namely *Visualization*. The *Wilks' Lambda* for this variable is 0.140, suggesting that this variable significantly contributes toward the differentiation of the groups. The *F* statistic = 196.706, $df_1 = 1$, $df_2 = 32$, $p = 0.000$, thus the result is extremely significant. At *Step 2*, two predictors are entered, *Visualization* and *Spatial Relation*. The value of *Wilks' Lambda* is gradually reduced to 0.081, implying that the model is improved by the introduction of the *Spatial Relation* variable in its power to discriminate the two groups. The *F* statistic value is 174.913 ($df_1 = 2$, $df_2 = 31$, *Sig.* = 0.000), and the statistical significance remains constant. At *Step 3*, *Visualization*, *Spatial Relation*, and *Spatial Perception* are included in the model. The value of *Wilks' Lambda* decreases further to 0.067, indicating that the model with these three variables has the best ability to differentiate between the groups. The *F* value is 139.674 ($df_1 = 3$, $df_2 = 30$, $p = 0.000$), which is still highly significant. In general conclusion, the discriminant model gains more ability to separate the groups with the inclusion of each variable. All three input predictors—*Visualization*, *Spatial Relation*, and *Spatial Perception*—are significant at $p < 0.05$ in the model, with *Visualization* providing the most original input. The small *Wilks' Lambda* values and consistently very high significance at every step prove that the model

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

is highly effective in differentiating between the two groups. To discriminate among students in terms of spatial variables (*Spatial Perception*, *Visualization*, and *Spatial Relation*), discriminant scores are computed using the discriminant function. These scores are generated with discriminant equations, where the coefficient of each variable indicates how much the variable contributes to the discrimination. Discriminant scores are then calculated, and students are classified into categories (*High Spatial Ability* or *Low Spatial Ability*) depending on their scores.

The discriminant function coefficients for the factors in the model are provided in Table 8. A discriminant function separating the *Low* and *High Spatial Ability* groups was formulated based on these weights. The obtained discriminant function is a linear combination of significant variables, with corresponding coefficients representing the contribution of each variable.

Table 8. Canonical Discriminant Function Coefficients

	Function
	1
Spatial_Perception	.121
Visualization	.170
Spatial_Relation	.195
(Constant)	-37,219
Unstandardized coefficients	

The coefficient for the variable *Spatial Perception* is 0.121, which is positive, meaning that this variable positively contributes to discrimination between the two groups, although its contribution is smaller compared to other independent variables. The *Visualization* variable has a coefficient of 0.170, indicating that it contributes more to distinguishing the groups than *Spatial Perception*. This conclusion is consistent with previous findings, where *Visualization* was identified as an extremely significant variable. Meanwhile, the *Spatial Relation* variable has the largest coefficient, 0.195, which means it makes the greatest contribution to group separation in the discriminant analysis. The constant in the discriminant function is -37.219, indicating that the discriminant function can correctly compute a discriminant score for each individual based on the values of the independent variables.

Thus, the discriminant function is defined as:

$$D = 0.121(\mathit{Spatial\ Perception}) + 0.170(\mathit{Visualization}) + 0.195(\mathit{Spatial\ Relation}) - 37.219$$

The *D* scores produced from this function are subsequently employed to classify each participant into either the *Low Spatial Ability* or *High Spatial Ability* category. Larger *beta* coefficients on the variables suggest that these variables contribute more significantly to separating the two groups; therefore, *Spatial Relation* and *Visualization* are considered the primary variables in the discriminant model.

Table 9. Function at Group Centroids

Spatial_Ability	Function
	1
Low Spatial Ability	-3.222
High Spatial Ability	4,081

Table 9 presents the centroid values of the discriminant function for the *Low Spatial Ability* and *High Spatial Ability* groups. These values represent the mean discriminant scores for each group and are referred to when classifying individuals into a specific group based on their discriminant score. The centroid value is used as a threshold to categorize students according to their proximity to the centroid value. Students are assigned to the group with the closest centroid value (either *Low Spatial Ability* or *High Spatial Ability*) based on their previously obtained discriminant scores.

The centroid value of -3.222 for the *Low Spatial Ability* group means that the group's average discriminant score is located on the negative side of the discriminant function. For the *High Spatial Ability* group, the centroid value is 4.081, indicating that the average discriminant score for this group is located on the positive

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

side of the discriminant function. The discriminant score of an individual determines the classification procedure. If a person's discriminant score is closer to the centroid of the *Low Spatial Ability* group (-3.222) than to that of the *High Spatial Ability* group (4.081), the individual will be classified into the *Low Spatial Ability* group. Conversely, if the discriminant score is closer to the centroid of the *High Spatial Ability* group (4.081), the person will be classified into the *High Spatial Ability* group.

Table 10. Eigenvalues

Function	Eigenvalue	% of Variance	Canonical Correlation
1	*13,967*	*100.0*	*.966*

Table 10 shows the strength of the discriminant function that was applied in the analysis to classify the *Low Spatial Ability* and *High Spatial Ability* groups. A few facts about this table are as follows. The *Eigenvalue* is 13.967, meaning the discriminant function is extremely powerful in discriminating among the groups. A higher eigenvalue is an indicator of a more powerful discriminant function. In terms of *% of Variance* and *Cumulative %*, 100% of the variance in the data is explained by the first discriminant function. This suggests that the function is already well-tuned to differentiate between groups, and there is no need to add more functions to refine the model. The *Canonical Correlation* is 0.966, indicating an extremely strong relationship between the discriminant scores derived from the function and the true group membership. A correlation value near 1 means that the discriminating function is highly effective in separating the groups.

The larger the eigenvalue, the better the discriminant function in discriminating between groups, reflecting a higher explanation of variance between rather than within groups. A larger distance between groups indicates better performance of the model in the classification of students according to their spatial ability. This is important because the better the classes are separated, the higher the classification accuracy that can be obtained by the discriminant analysis model. Taken together, these findings show that the resulting discriminant function is quite robust and successful in discriminating between the *Low Spatial Ability* and *High Spatial Ability* groups. The high eigenvalue and canonical correlation indicate that the discriminant model is indeed reliable in classifying individuals based on the variation in spatial variables. A synergistic effect in terms of both discrimination power and face validity is achieved by combining these findings.

4. DISCUSSION

According to the discriminant analysis results, some of the variables seemed to be very important in the differentiation between the *High* and *Low Spatial Ability* groups of students. The significance values for *Spatial Perception* (p = 0.000), *Visualization* (p = 0.000), and *Spatial Relation* (p = 0.000) were very high, meaning that all these three variables were highly significant in differentiating between the groups in terms of measured spatial ability. This result implies that the ability to comprehend the position and direction of objects, the ability to form an image of an object, and the ability to orient in space play an important role in understanding flat-sided geometric shapes. The two other predictors, however, *Mental Rotation* (p = 0.278) and *Spatial Orientation* (p = 0.083), did not contribute significantly to this discriminant model.

The findings also suggest that the factors of *Visualization*, *Spatial Relation*, and *Spatial Perception* are able to discriminate the *Low Spatial Ability* and *High Spatial Ability* groups effectively. Of these, the largest model coefficient is assigned to the *Spatial Relation* variable, with *Visualization* and *Spatial Perception* coming next, indicating that mastering the ability to comprehend space relationships and visualize is crucial in determining the order of individual spatial abilities. These findings have important implications for designing effective learning strategies and interventions aimed at enhancing students' spatial abilities, especially in geometry learning.

Visualization as a Differentiating Aspect in Student Groups

Visualization involves the highest level of contribution to the discriminant model obtained in the discriminant analysis, with a Wilks' Lambda value of 0.140 and an F value of 196.706. Visualization is fundamental to geometry instruction in general, and to learning about flat-sided figures in particular. This skill empowers

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

students to operate numbers in their minds, to understand spatial relations, and to perceive geometric transformations, all of which are essential to understanding geometric concepts (Kurnia & Hidayati, 2022; Lutfi et al., 2024). There is abundant evidence that the ability to visually image is highly correlated with success in studying geometry. For example, studies conducted by Prayitno (2020) and Zakelj & Klančar (2022) concluded that students performing visualization activities tend to achieve a greater level of understanding of geometrical properties and relations. These studies also indicated that when students actively focus on mental visualization actions, it facilitates their understanding of concepts such as congruence, similarity, and symmetry. Furthermore, results of the present analysis highlighted the benefits of visualization, in combination with spatial relation and spatial perception, as having the most drastic impact in distinguishing between students with *low* and *high spatial ability*. This suggests that developing visualization abilities may result in better outcomes in geometry instruction (Novita et al., 2018; Zakelj & Klančar, 2022). In addition, research evidence supports the construction of teaching strategies that enhance visualization. For instance, Anwar & Juandi (2020) and Sumarni & Prayitno (2016) argue that students who can visualize geometrical concepts are better prepared to solve problems and associate them with other areas of mathematics.

Spatial Relation as a Differentiating Aspect in Student Groups

Spatial Relation is a critical feature that divides student groups according to spatial abilities. Spatial relation is important in geometry learning for the visual recognition of patterns, the relative positions of elements in a geometric shape, and the relationships between points, lines, and planes in flat-sided geometric objects. For example, for students working out the surface area or volume of a pyramid, it is crucial to understand where shapes are located relative to one another. They must know how the cross-sections of a pyramid are transformed by slicing the figure either horizontally or vertically. Without an intuitive sense of space and shape, students may find it difficult to mentally visualize the new shapes created by these cuts. Spatial relation abilities allow students to recognize how elements of a figure—such as edges, faces, or diagonals—are connected, which is essential for solving a variety of geometric problems, such as computing area, volume, and structural properties of geometrical figures. Furthermore, the ability of spatial relation helps students to imagine changes in the orientation or position of a geometric figure when subjected to manipulations such as rotation, shift, or projection (Benelli et al., 2001; Chen et al., 2020). Skills in this area form the basis for understanding geometric transformations and solving three-dimensional representations from different perspectives. When studying shapes with flat sides, such as cubes, rectangular prisms, and pyramids, spatial relation also facilitates students in visualizing parallelism and perpendicularity between planes and edges.

Several studies emphasize the importance of spatial relation in geometry teaching. Research by Liu et al. (2022) demonstrates that students developing an understanding of spatial relationships can benefit from spatial manipulation using physical models or interactive geometry software. Similarly, Battista et al. (2018) and Handayani (2023) indicated that exercises with a spatial relation theme can enhance students' problem-solving ability on solid figures. Gómez-Tone et al. (2021) stated that spatial relation training activities, such as constructing three-dimensional shapes and drawing their projections, are effective interventions for improving students' understanding of geometric concepts. To foster students' spatial relation skills, teachers can apply several teaching approaches, including the use of physical teaching aids like three-dimensional models that allow tangible exploration of spatial relationships. Additionally, interactive technology-based learning tools such as dynamic geometry software (e.g., GeoGebra) have been found to be effective in helping learners visualize and dynamically manipulate spatial relationships (Girma, 2015).

Spatial Perception as a Differentiating Aspect in Student Groups

Spatial Perception is the last dimension that separates the student groups. Spatial perception is particularly crucial in the learning of geometry, as even when working with flat-sided shapes, students must recognize these shapes not only for what they are, but also as three-dimensional objects with depth. This capability enables students to mentally visualize three-dimensional objects from two-dimensional representations, such as diagrams or illustrations typically found in textbooks. Spatial perception, in conjunction with flat-sided geometry, allows learners to comprehend the shape, size, position, and relationships among the components of a geometric figure. Students with well-developed spatial thinking can envision objects, imagining, for example, how a geometric figure would appear if rotated or partially sliced, thus equipping them to better understand concepts such as volume, surface area, and the interconnectedness of different elements of a figure.

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

The results of this study confirm that strength in spatial perception has a positive influence on geometry learning, as indicated statistically by a Wilks' Lambda value of 0.216, an F-value of 116.398, and a significance level of 0.000. Supporting studies, such as Riastuti et al. (2017), observed that students who engaged in activities manipulating geometric objects exhibited enhanced comprehension of three-dimensional geometric concepts. Similarly, Owens and Clements (1998) emphasized the importance of spatial perception for solving geometric problems involving mental operations such as cutting and fitting, for instance in determining the location of points within prisms or calculating the surface area of complex pyramids. Spatial reasoning can be nurtured through various methods, including the use of visualization-based technologies, direct manipulation using physical models, and activities that involve the construction of three-dimensional structures (Ramesh, 2018). These findings provide direct evidence that *Visualization*, *Spatial Relation*, and *Spatial Perception* are fundamental cognitive components underlying the development of higher-order tasks such as mathematical reasoning and spatial understanding (Hodgkiss, 2019; Judd & Klingberg, 2021; Yu et al., 2022). Further research is warranted to explore and enhance these abilities through innovative instructional designs and practices, as well as to examine their broader impact on students' success in geometry learning. Consequently, it is important to identify and implement strategies that not only increase student engagement but also foster a profound understanding and mastery of geometric concepts, differentiating approaches according to the varying capabilities of different student groups.

Through distinguishing a *Low Spatial Ability* and *High Spatial Ability* group, teachers can provide intervention with learners as needed for the development of spatial skills. Spatial interventions for low ability students might include drawing or manipulating geometric figures of different sizes in the plane (by hand or using digital geometry tools, such as GeoGebra). These experiences help students to develop a deeper understanding of the relationship between the components of an object and to visualize the object mentally. Besides, Augmented Reality learning can also be incorporated in three-dimensional geometry learning where interactive experience would allow students to see, feel, and interact with 3D geometrical objects directly, which improves the spatial relation skills. Online resources and virtual manipulatives that are integrated in online curricula can also create opportunities for students to manipulate and investigate relationships between geometric objects. In addition, through a personalized teaching method, teachers can provide more personal instruction to students with lower spatial skills, such as showing examples of geometry projections and arrangements through stepwise illustrations. On the other hand, teachers may provide students with high spatial ability with more sophisticated geometry problems or with high-level spatial analysis tasks. It is such instruction that creates a more responsive, personalized, and predictive approach to learning that meets the needs of all students.

Even though this research investigated the relationships of the *Visualization*, *Spatial Relation*, and *Spatial Perception* variables for the measure of the participants' spatial ability, other factors that might have affected spatial ability, such as prior knowledge, economic status, and educational background, were not completely controlled. These contingencies could have had a major impact on the results of the study. For example, students who have had early geometry activities are more likely to do well on spatial tasks, even if their inherent spatial ability is relatively low. Likewise, students who are better prepared academically or who have had more opportunities to learn may adjust more easily to geometry materials regardless of their natural spatial ability. The socioeconomic status might influence the accessibility of educational resources (penchants towards visual information contents, such as supplementary reading materials, computer-based learning applications such as geometric software applications, or AR-based applications), and in this regard, promote or impede spatial development of children.

In addition, there were limitations to the sample size and diversity. Small or non-representative samples may limit the generalizability of findings to students with varied educational and socioeconomic backgrounds. This restriction might lead to bias and limit the external validity of the study. To further investigate these issues in future research, control variables including prior knowledge and SES should be entered into the model to differentiate the vestigial influences of *Visualization*, *Spatial Relation*, and *Spatial Perception* on spatial ability. Innovative statistical methods, such as ANCOVA (Analysis of Covariance), could be used to adjust for such confounding variables. Moreover, future research should work with larger, more heterogeneous samples to increase the representativeness of students with different education and socioeconomic backgrounds. Longitudinal designs are also recommended to investigate the development of spatial abilities and how external variables may mediate this development.

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

5. CONCLUSION

In daily learning activities, teachers can promote students' spatial abilities through visualization exercises, three-dimensional models, and interactive technologies, such as geometry software, which can be easily integrated into their teaching practice. These and similar techniques strengthen students' capacity for learning geometric ideas and prepare them to become logical, analytical, and creative thinkers, skills that are critical for solving problems in the real world. Thus, working mathematically has a significant role in how students come to understand flat-sided plane geometric shapes. The results of this study highlight that *spatial visualization*, *spatial relation*, and *spatial perception* are the key skills that discriminate groups of students according to differences in spatial ability. Discriminant analysis found that these variables do significantly distinguish between students deemed as having "*Low Spatial Ability*" and those classified as "*High Spatial Ability*." With the largest coefficient for the spatial relation variable, the result underscores the importance of learning spatial relations to achieve academic success, especially in learning geometry, even though it is the preference of the thirteen students.

6. ACKNOWLEDGMENTS

We would like to express our deepest gratitude to all students who have been willing to participate in this research. In addition, we also express our appreciation to the Ministry of Education and Culture, LPDP, and Puslapdik for providing BPI scholarships.

7. REFERENCES

Andriani, A., Dewi, I., & Manurung, N. (2022). Analysis of student's mathematical spatial abilities in solving geometric problems. *Proceedings of the 4th International Conference on Innovation in Education, Science and Culture, ICIESC 2022, 11 October 2022, Medan, Indonesia*. https://doi.org/10.4108/eai.11-10-2022.2325388

Anwar, & Juandi, D. (2020). *Studies of level visual thinking in geometry*. *1470*(1), 12095. https://doi.org/10.1088/1742-6596/1470/1/012095

Battista, M. T., Frazee, L. M., & Winer, M. L. (2018). *Analyzing the Relation Between Spatial and Geometric Reasoning for Elementary and Middle School Students* (pp. 195–228). https://doi.org/10.1007/978-3-319-98767-5_10

Benelli, G., Caporali, M., Rizzo, A., & Rubegni, E. (2001). *Design concepts for learning spatial relationships*. 22–30. https://doi.org/10.1145/501516.501522

Budiarto, M. T., Khabibah, S., & Firdaus, A. M. (2020). Misconception Of Junior High School Students On Two-Dimentional Figure Materials. *Journal of Database Management*, *8*(1), 1–8. https://doi.org/10.26858/JDS.V8I1.13316

Chen, Y. C., Yang, F. Y., & Chang, C.-C. (2020). Conceptualizing spatial abilities and their relation to science learning from a cognitive perspective. *Journal of Baltic Science Education*, *19*(1), 50–63. https://doi.org/10.33225/JBSE/20.19.50

Cui, X., & Guo, K. (2022). Supporting mathematics learning: a review of spatial abilities from research to practice. *Current Opinion in Behavioral Sciences*, *46*, 101176. https://doi.org/10.1016/j.cobeha.2022.101176

Dick, A. O. (1976). *7 – Spatial Abilities* (pp. 225–268). https://doi.org/10.1016/B978-0-12-746302-5.50014-1

Fiantika, F. R., Maknun, C. L., Budayasa, I. K., & Lukito, A. (2018). Analysis of students' spatial thinking in geometry: 3D object into 2D representation. *Journal of Physics: Conference Series*, *1013*(1). https://doi.org/10.1088/1742-6596/1013/1/012140

Girma, D. D. (2015). *Students' Learning Experiences When Using a Dynamic Geometry Software*.

Gómez-Tone, H. C., Martín-Gutiérrez, J., Bustamante-Escapa, J., & Bustamante-Escapa, P. (2021). Spatial Skills and Perceptions of Space: Representing 2D Drawings as 3D Drawings inside Immersive Virtual Reality. *Applied Sciences*, *11*(4), 1475. https://doi.org/10.3390/APP11041475

Handayani, R. (2023). Analisis kemampuan spasial visualization siswa sekolah dasar dalam pemecahan masalah geometri. *Didaktik: Jurnal Ilmiah PGSD STKIP Subang*, *9*(1), 717–725. https://doi.org/10.36989/didaktik.v9i1.663

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

Hazen, N. (1983). *Spatial Orientation: A Comparative Approach* (pp. 3–37). https://doi.org/10.1007/978-1-4615-9325-6_1

Hidayat, S., Yanti, Y., & Imswatama, A. (2022). Analisis Kesulitan Siswa Dalam Memecahkan Masalah Dan Kemampuan Pemahaman Konsep Bangun Ruang Sisi Datar Pada Pembelajaran Jarak Jauh. *Jurnal PEKA (Pendidikan Matematika)*, *6*(1), 20–28. https://doi.org/10.37150/jp.v6i1.1683

Hodgkiss, A. (2019). *The role of spatial cognition in children's science learning*.

Judd, N., & Klingberg, T. (2021). Training spatial cognition enhances mathematical learning in a randomized study of 17,000 children. *Nature Human Behaviour*, *5*(11), 1548–1554. https://doi.org/10.1038/S41562-021-01118-4

Juliana, A., Nurjanah, & Usdiyana, D. (2022). The learning obstacles in solving geometry problems based on spatial ability in term of Van Hiele level. *Nucleation and Atmospheric Aerosols*. https://doi.org/10.1063/5.0117208

Kim, J. H. (2019). Multicollinearity and misleading statistical results. *Korean Journal of Anesthesiology*, *72*(6), 558–569. https://doi.org/10.4097/KJA.19087

Koustriava, E., & Papadopoulos, K. (2010). Mental Rotation Ability of Individuals with Visual Impairments. *Journal of Visual Impairment & Blindness*, *104*(9), 570–575. https://doi.org/10.1177/0145482X1010400910

Kurnia, A. N., & Hidayati, N. (2022). Analisis kemampuan berpikir geometri berdasarkan tahap berpikir van hiele pada pembelajaran matematika siswa smp. *EduMatSains : Jurnal Pendidikan, Matematika Dan Sains*, *6*(2), 419–430. https://doi.org/10.33541/edumatsains.v6i2.3618

Kusnadi, D., & Barumbun, M. (2023). Analisis kemampuan spasial siswa melalui teori belajar van hiele pada pembelajaran matematika di sekolah dasar. *Jurnal Mathematic Paedagogic*, *7*(2), 146–157. https://doi.org/10.36294/jmp.v7i2.3100

Liu, Q.-T., Ma, J., Yu, S., & Wu, L. (2022). *Geometry Wall: An Embodied Gesture-based Game for Supporting Spatial Ability*. 258–263. https://doi.org/10.1109/TALE54877.2022.00050

Lutfi, M. K., Cahya Mulyaning, E., & Annisa Kusumastuti, F. (2024). Analysis of Students' Geometrical Thinking from Geometry Task Related to HOTS from PISA. *KnE Social Sciences*, *2024*, 943–952. https://doi.org/10.18502/kss.v9i13.16020

Lutfi, M. K., & Jupri, A. (2020). Analysis of junior high school students' spatial ability based on Van Hiele's level of geometrical thinking for the topic of triangle similarity. *Journal of Physics: Conference Series*, *1521*(3). https://doi.org/10.1088/1742-6596/1521/3/032026

Lutfi, M. K., & Kusumastuti, F. A. (2024). Integrasi Augmented Reality berbantuan Geogebra sebagai Media Pembelajaran Interaktif dalam Pembelajaran Materi Bangun Ruang. *Social, Humanities, and Educational Studies (SHES)*, *7*(3).

Lutfi, M. K., Kusumastuti, F. A., Akib, I., & Rohmawati, A. (2023). Media E-Learning Bangun Ruang Sisi Datar: Kelayakan pada Pembelajaran Daring. *Edu Komputika Journal*, *9*(2), 78–87. https://doi.org/10.15294/edukomputika.v9i2.54743

Ma'rifatin, S., Amin, S. M., & Siswono, T. Y. E. (2019). *Students' mathematical ability and spatial reasoning in solving geometric problem*. *1157*(4), 42062. https://doi.org/10.1088/1742-6596/1157/4/042062

Maier, P. H. (1991). *Spatial Geometry and Spatial Ability - How to Make Solid Geometry Solid*. 69–81. http://webdoc.sub.gwdg.de/ebook/e/gdm/1996/maier.pdf

Manovich, L. (2011). What is visualisation. *Visual Studies*, *26*(1), 36–49. https://doi.org/10.1080/1472586X.2011.548488

McCarthy, R. A. (1990). *4 – Spatial Perception* (pp. 73–97). https://doi.org/10.1016/B978-0-12-481845-3.50007-2

Muhammad, N., Rehman, D. S., & Naeemullah, D. M. (2022). An Investigation into Spatial Ability in Geometry among Secondary School Students. *Sir Syed Journal of Education & Social Research*, *5*(3), 22–28. https://doi.org/10.36902/sjesr-vol5-iss3-2022(22-28)

Ngirishi, H., & Bansilal, S. (2019). *An exploration of high school learners' understanding of geometric concepts*. *77*(1), 82–96. https://doi.org/10.33225/PEC/19.77.82

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297

Noor Muhammad, Dr. Sajid Rehman, & Dr. Muhammad Naeemullah. (2022). An Investigation into Spatial Ability in Geometry among Secondary School Students. *SJESR*, *5*(3), 22–28. https://doi.org/10.36902/sjesr-vol5-iss3-2022(22-28)

Novita, R., Putra, M., Rosayanti, E., & Fitriati, F. (2018). *Design learning in mathematics education: Engaging early childhood students in geometrical activities to enhance geometry and spatial reasoning*. *1088*(1), 12016. https://doi.org/10.1088/1742-6596/1088/1/012016

Owens, K., & Clements, M. A. (Ken. (1998). Representations in Spatial Problem Solving in the Classroom. *The Journal of Mathematical Behavior*, *17*(2), 197–218. https://doi.org/10.1016/S0364-0213(99)80059-7

Özçakir, B., & Cakiroglu, E. (2021). Fostering spatial abilities of middle school students through augmented reality: Spatial strategies. *Education and Information Technologies*, 1–34. https://doi.org/10.1007/S10639-021-10729-3

Palobo, M., & Juniati, D. (2022). The geometric thinking process of students in constructing the concept of area. *International Journal of Research - Granthaalayah*, *10*(5), 75–87. https://doi.org/10.29121/granthaalayah.v10.i5.2022.4613

Prayitno, L. L., Purwanto, Subanji, S., Susiswo, S., & As'ari, A. R. (2020). *Exploring Student's Representation Process in Solving Ill-Structured Problems Geometry. 7*(2), 183–202. https://doi.org/10.17275/PER.20.28.7.2

Ramesh, V. M. (2018). *Spatial abilities in early childhood*. 1–7. https://doi.org/10.1109/FIE.2018.8658655

Riastuti, N., Mardiyana, & Pramudya, I. (2017). Analysis of students geometry skills viewed from spatial intelligence. *AIP Conference Proceedings*, *1913*. https://doi.org/10.1063/1.5016658

Saad, M. N., Muda, Z., Ashaari, N. S., Hamid, H. A., & Hasan, N. H. binti A. (2015). *The spatial relation features for describing objects relationships within image*. 126–131. https://doi.org/10.1109/ICEEI.2015.7352482

Sabil, H., Simanjuntak, S. M. O. U., Iriani, D., & Junita, R. (2024). *Analysis of Students ' Spatial Ability in Geometry Material*. *13*(3), 436–448.

Salsabilah, A. S., Nur Afifah, N. P., & Putri Herdiansyah, R. F. (2023). Analisis Kesulitan dalam Menyelesaikan Soal Luas Bangun Datar Gabungan Siswa Kelas IV SD. *Journal on Education*, *6*(1), 2601–2608. https://doi.org/10.31004/joe.v6i1.3290

Sorby, S. A., Duffy, G. A., & Yoon, S. Y. (2022). Math Instrument Development for Examining the Relationship between Spatial and Mathematical Problem-Solving Skills. *Education Sciences*, *12*(11), 828. https://doi.org/10.3390/educsci12110828

Stuchlík, A. (2003). Space and spatial orientation. *Chekhoslovatskaia Fiziologiia*, *52*(1), 22–33.

Sumarni, S., & Prayitno, A. T. (2016). *Kemampuan visual-spatial thinking dalam geometri ruang mahasiswa universitas kuningan*. *2*(2). https://doi.org/10.25134/JES-MAT.V2I2.349

Suprayo, T., Sugiman, S., Pujiastuti, E., Setiyani, S., & Oktoviani, V. (2023). Analisis kesulitan siswa smp dalam menyelesaikan soal bangun ruang sisi datar. *Jurnal Lebesgue*, *4*(1), 352–363. https://doi.org/10.46306/lb.v4i1.203

Tadeo, D., & Yoo, J. (2022). Students' Recognition of Concepts of Reflection and Refraction in Multiple Representational Formats. *Jurnal Pendidikan Fisika*, *10*(2), 75–92. https://doi.org/10.26618/jpf.v10i2.7639

Yu, M., Cui, J., Wang, L., Gao, X., Cui, Z., & Zhou, X. (2022). Spatial processing rather than logical reasoning was found to be critical for mathematical problem-solving. *Learning and Individual Differences*, *100*, 102230. https://doi.org/10.1016/j.lindif.2022.102230

Zakelj, A., & Klančar, A. (2022). The Role of Visual Representations in Geometry Learning. *European Journal of Educational Research*, *11*(3), 1393–1411. https://doi.org/10.12973/eu-jer.11.3.1393

p-ISSN: 2597-7792 / e-ISSN: 2549-8525
DOI: https://doi.org/10.20961/ijpte.v9i1.98297