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A Learning Trajectory for Statistics Through the Traditional Game of Congklak to Enhance Mathematical Reasoning Skills

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ABSTRACT

The purpose of this study is to design a learning trajectory (develop mathematics reasoning skills) for junior high school statistics material to be used in eighth-grade classrooms. This design research study comprises three stages: preliminary design, experimental design and Retrospective Analysis. The subjects in this study were eighth-grade learners in a public junior-high school in Wonogiri City. Data collection methods included observation, interview and tests. Based on the results of the retrospective analysis, a learning trajectory for statistics material with the traditional Congklak game in application combines both informal and formal activities. In informal activities, through the Congklak game experiments conducted. These helped to form, mode, medium, examination, quartiles and semi-interquartile range interpretation concepts. In formal activities, based on the experimental results learners decided how to define mean (average), mode (the number of times a value appears in given data), medium and quartiles. The statistical results show that there are differences in mathematics reasoning skills between teaching with a learning trajectory assisted by Congklak and the direct method. The marginal mean for learning path is 71.57, while it is 62.66, and this outside the margin of error. This suggests that by application of a realistic mathematics approach using learning path driven Congklak traditional game of so can improve learners' mathematics reasoning ability.

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1. INTRODUCTION

. Mathematical reasoning is crucial in learning mathematics, enabling students to build mathematical knowledge and solve problems. Students engage in mathematical reasoning when they explain their thought processes, justify their methods and conclusions, connect known concepts to unknown ones, transfer knowledge between contexts, and verify the truth or falsity of a proposition. Reasoning allows students to understand and master mathematical concepts, making learning more meaningful beyond mere memorization. The four key components of mathematical reasoning are making conjectures, manipulating numbers, gathering information or proving solutions, and drawing inferences or generalizations (Brodie, 2010). To develop strong mathematical thinking, students must participate in learning activities that nurture these abilities. Various learning experiments, such as demonstration lessons (Vale et al., 2017; 2015), peer team learning (Herbert & Bragg, 2021; 2020), and workshop learning (Hilton et al., 2016), have been conducted to enhance understanding of mathematical reasoning.

Preliminary data collection shows that context-based learning has not been applied to teaching statistics concepts. Students often remain passive in class, focusing only on memorizing formulas to solve problems. This approach is ineffective for developing mathematical reasoning, leading to a deficiency in problem-solving skills. Thus, an innovation in teaching statistics is necessary to improve students' mathematical reasoning abilities.

One of the efforts that can be used to help learners understand the concept of statistics is designing learning media using the Realistic Mathematics Education (RME) approach (Marande & Diana, 2022). RME is a mathematics learning approach that involves learners directly in rediscovering mathematical ideas and concepts through the exploration of real problems. Through these activities, learners get the opportunity to construct

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their knowledge, leading to a deep understanding of the concepts learned. In general, learning with RME emphasizes learner engagement. This is evident in the involvement of learners in informal activities, such as experimental activities to discover mathematical concepts, and formal activities, which connect the results of experiments with mathematical concepts, guiding learners toward understanding definitions, algorithms, or formulas. The application of RME utilizes a learning design that is based on learners' learning trajectory.

A learning trajectory describes the process of thinking and learning in a particular mathematics topic through a series of tasks designed to produce hypothesized mental processes or actions that support the achievement of learning objectives (Clements & Sarama, 2009). It serves as an alternative solution that teachers can use to design lessons aimed at developing learners' mathematical reasoning skills. Learning trajectory-based designs enable teachers to structure learners' thinking and guide them in setting and achieving learning objectives. This approach assists teachers in applying appropriate strategies, materials, and assessments. For learners, it helps in understanding mathematical concepts and reduces learning difficulties by providing alternative strategies. The design of a learning trajectory is based on a Hypothetical Learning Trajectory (HLT), which predicts how learners will progress from an initial, possibly incorrect understanding to a deeper, more accurate one. An HLT consists of three components: specific learning objectives, which define what learners aim to achieve in a topic; learning activities, which help learners understand a topic through discussions, experiments, simulations, practice questions, or other relevant activities; and the hypothetical learning process, which observes how learners think about the topic over time (Clements & Sarama, 2009). These components are interrelated and crucial for designing and implementing an effective learning trajectory.

Learning objectives provide direction, understanding developmental thought processes helps identify challenges, and appropriate activities support learners in achieving these objectives. Hypothetical Learning Trajectories (HLT) hypothesize the learners' understanding process during learning activities (Collins,2004). HLT makes conjectures about how learners learn, allowing teachers to consider not only the material but also the learners' level of understanding (Antonio, 2022). The purpose of HLT is to represent the learners' learning process, guiding them from their existing knowledge toward achieving the learning objectives (Clements & Sarama, 2009).

The Realistic Mathematics Education (RME) learning process uses real-life contexts or situations that learners experience to help them discover and construct informal mathematical concepts, which are then connected to formal mathematics. In this study, a traditional game called Congklak is used as a context for learning statistics. By experimenting with the Congklak game, learners can make conjectures and perform mathematical manipulations to discover concepts. Connecting informal concepts with formal mathematics helps learners compile evidence, find solutions, and make generalizations, all of which are essential parts of mathematical reasoning skills. This study aims to develop a learning trajectory using the traditional Congklak game to enhance mathematical reasoning skills in the context of statistics.

2. MATERIAL AND METHOD

Research Design

This study employs design research as the chosen method, aligning with its objectives. Design research focuses on developing learning activities and understanding how learners think during the learning process. It involves three key stages (**Figure 1**) : Preliminary Design, Design Experiment, and Retrospective Analysis (Akker, 2006). The following outlines the research stages:

Preliminary Design.

During the preliminary design phase, a Hypothetical Learning Trajectory (HLT) for statistics was created. This trajectory was informed by several resources: a reference study on statistical concepts such as mean, mode, median, range, quartiles, and interquartile range; a review of literature focusing on the realistic mathematics approach to enhance learners' mathematical reasoning skills using the context of the Congklak game; and insights gathered from observations, interviews, and classroom discussions related to the teaching of statistics. Additionally, lesson plans and worksheets were developed as part of the learning materials to guide the practical application of the HLT.

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Experimental Design

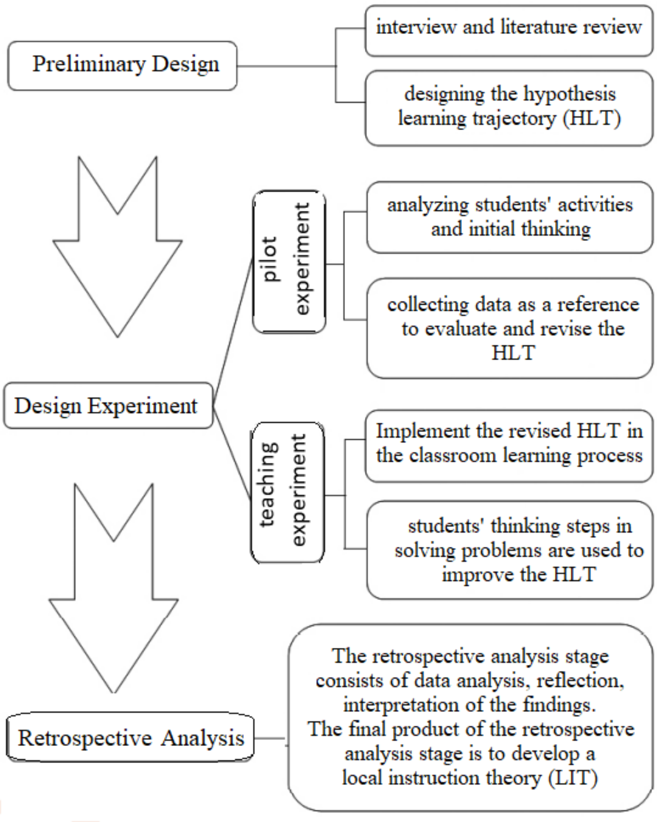

Figure 1. Stages of design research

The experimental design was implemented to evaluate whether the HLT was more effective than previously used strategies. This stage was divided into two cycles: the pilot experiment and the teaching experiment. The pilot experiment, conducted with a small group of students, provided data to assess which aspects of the HLT required revision or further evaluation before progressing to the teaching experiment. Throughout the pilot experiments, learners' challenges and strategies in problem-solving were observed, allowing for improvements to the HLT design before its application in the teaching experiment stage (Gravemeijer & van Eerde, 2009, Van den Akker, 2006)

Retrospective Analysis.

The retrospective analysis involves several stages: data analysis, introspection, interpretation of results, and the formulation of recommendations for further study (Nickerson, & Whitacre,2010). During this stage, all data obtained from the experimental design phase were collected and retrospectively analyzed. This analysis involved comparing the actual learning process observations with the Hypothetical Learning Trajectory (HLT) designed during the experimental preparation stage. The results of the teaching experiment were also considered. Based on the retrospective analysis, elements of the HLT that did not contribute to the achievement of learning objectives were removed, while those that positively impacted students' comprehension were retained.

Participants and Instruments.

The study was conducted with eighth-grade students at a junior high school in Wonogiri City, Central Java. There were six parallel classes, from VIII A to VIII H, each consisting of 30 to 32 learners. The selection of this research site was based on two main factors: the presence of issues related to learners' mathematical reasoning and the availability of necessary information and data at this school. A total of six learners from class VIII F

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participated in the pilot experiment. These learners were selected based on the diversity of their skill levels to gather data on the thinking processes of students across high, medium, and low skill levels. The results of this trial were used as a foundation for refining the initial HLT. Subsequently, the teaching experiment involved all learners in class VIII H.

Instruments and Data Collection

Data collection in this study aimed to gather information related to learners' learning trajectories in the statistics subject. The techniques used for data collection included observation, interviews, and tests. Observation was employed to monitor the stages of the thinking process and strategies used by learners in both the pilot experiment and teaching experiment for the statistics subject. Observation sheets and video recording instruments were utilized during these observations. Semi-structured interviews were conducted to explore the learning flow, process, and strategies employed by learners in relation to the Hypothetical Learning Trajectory (HLT). The test was designed to assess learners' mathematical reasoning skills and to evaluate the impact of implementing the Realistic Mathematics Education (RME) approach with a learning trajectory based on the traditional game of Congklak. The test was administered after the RME implementation using the learning trajectory with the traditional game of Congklak in the context of statistics. It consisted of essay questions aimed at assessing indicators of mathematical reasoning skills, including: the ability to make conjectures; the ability to perform mathematical manipulations; the ability to compile or provide evidence for a solution; and the ability to draw conclusions or make mathematical generalizations.

Data Analysis

Given the research problems and objectives, this study employed a mixed-method design to analyze the data obtained from observations, interviews, and tests. Qualitative data analysis was used to develop the learners' learning trajectories in statistics by utilizing the traditional game of Congklak. Data from observations and interviews were analyzed qualitatively, following three stages: data reduction, data presentation, and conclusion drawing (Sugiyono, 2018). The observation and interview data were transcribed and then reduced to convert all relevant information from oral recordings into written form. The classification method was used to interpret all observations made during the learning activities. The data were presented as a narrative text, arranged systematically for clarity and ease of understanding, and were also displayed as a concept map outlining the flow of learners' thinking processes related to statistical concepts. The test data were analyzed quantitatively using T-test statistics with a significance level of $\alpha = 0.05$ to determine whether there were significant differences in the mathematical reasoning skills of learners as a result of implementing the RME approach with a learning trajectory using the traditional game of Congklak.

3. FINDINGS

Based on the results from the preliminary design stage, the use of the traditional game Congklak effectively engages learners in experiments to understand key statistical concepts such as mean, mode, median, range, quartiles, and interquartile range. The Realistic Mathematics Education (RME) approach promotes significant interaction among learners and between learners and teachers, fostering active dialogue and critical thinking (Goos, 2004). Connections are made as learners link experimental results with general formulas for statistical measures, which enhances their mathematical reasoning skills. For further details, refer to Clements & Sarama (2009) and Goos (2004). The following describes a hypothesis of learning trajectories in statistics using traditional Congklak games. The Hypothetical Learning Trajectory (HLT) is designed to help learners understand the concepts of mean, mode, median, range, quartiles, and interquartile range through the Congklak game, as illustrated in **Figure 2.**

To explore the concept of the mean, learners are asked to rearrange the Congklak seeds so that each hole has the same number of seeds. This task is inspired by findings from Surya et al. (2017), who used the context of a mall in their statistical learning design, representing a clothing store with unit cubes. Learners were instructed to find the mean number of stores per floor, with some groups equalizing the height of the cubes by moving them, while others added all the cubes and divided them by the number of floors. Similarly, in the Congklak game, there are two assumptions for finding the mean: first, learners can move the Congklak seeds between

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holes until each hole contains the same number of seeds; or second, they can count the total number of seeds in all holes, divide this by the number of holes, and then rearrange the seeds in each hole according to the result of this division.


MARI BERMAIN

Lakukanlah kegiatan memainkan permainan tradisional congklak bersama temanmu. Permainan dilakukan dengan petunjuk sebagai berikut:

1. Pilihlah 2 teman dalam kelompokmu sebagai pemain.
2. 2 orang yang telah dipilih dapat duduk berhadapan dengan papan congklak berada diantaranya.
3. Lubang nomor 1 – 7 merupakan daerah milik pemain 1 dengan lubang induk berada di sebelah kanan dari hadapannya.
4. Lubang nomor 8 – 14 merupakan daerah milik pemain 2 dengan lubang induk berada di sebelah kanan dari hadapannya.
5. Perhatikan tabel di bawah ini!

Pemain 1							
Banyak Biji	7	4	2	6	4	4	8
Pemain 2							
Banyak Biji	8	5	9	6	10	5	-

Susunlah biji congklak dengan urutan biji paling sedikit hingga biji paling banyak sesuai dengan informasi pada tabel di atas.

6. Untuk petunjuk selanjutnya, silahkan perhatikan setiap petunjuk dan pertanyaan pada lembar pekerjaan dan tuliskan jawaban pada tempat yang sudah disediakan.
7. Setiap kegiatan dilakukan secara berkelompok, siswa yang tidak bermain dapat membantu untuk mencatat dan memberikan ide selama percobaan dilakukan.

Figure. 2: Instructions for Experimentation with The Game of Congklak

For the concept of mode, learners identify the number of Congklak seeds that appear most frequently in the holes. This method aligns with Rodríguez-Consuegra (1991), who defines mode as the value with the highest frequency. Learners observe which holes have the same number of seeds, record this in a frequency table, and determine the mode as the value with the highest frequency. As per Fakhmi et al. (2021), the mode can vary: it may not be found if all data points have the same frequency, there may be a single mode, or there may be multiple modes.

The median, being the middle value after sorting the data from smallest to largest, is found by dividing the data into two equal parts, with the middle value representing the median (Rodríguez-Consuegra, 1991). In the Congklak game, learners sort the seeds from the smallest to the largest number and then determine the middle value. The range is calculated by subtracting the smallest value from the largest. In the context of the Congklak game, this involves learners arranging the seeds in ascending order and determining the difference between the seeds in the first and last holes.

Quartiles divide the data into four equal parts after it has been sorted from smallest to largest. To understand quartiles through the Congklak game, learners arrange the seeds sequentially and identify the middle value (Q_2), which divides the data into two equal parts. They then calculate the mean of the data on the left (Q_1) and the mean of the data on the right (Q_3) of Q_2. Since quartiles are closely related to the median, learners use similar methods or formulas to find Q_1, Q_2, and Q_3, ultimately understanding that quartiles consist of these three values that divide the data into four parts. The interquartile range, which is closely associated with Q1 and Q_3, is determined by finding the difference between Q_1 and Q_3. This concept is taught as a continuation of the quartile learning activities.

Experimental Design Stage

The pilot experiment tested the Hypothetical Learning Trajectory (HLT) with six learners of varying abilities, divided into three groups: Group 1 consisted of high-ability learners, Group 2 included medium-ability

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learners, and Group 3 was composed of low-ability learners. In the informal activity, where learners experimented with the traditional game of Congklak to understand the concept of the mean, all groups followed the second conjecture in the HLT. They counted the Congklak seeds and divided them by the number of holes. Interestingly, the first conjecture, which proposed a different approach, did not emerge during this activity.

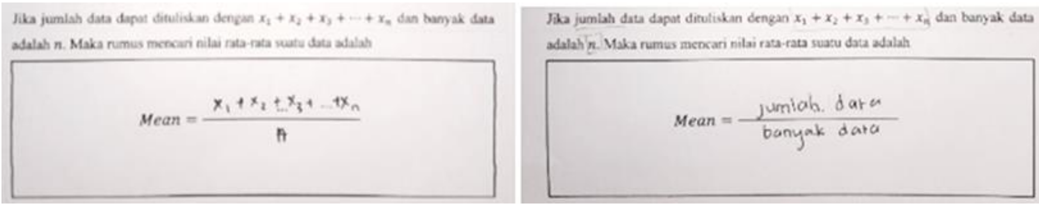

Figure 3: Variation in Learners' Answers to The General Formula for The Mean

In the formal activity, the goal was to derive the formula for the mean based on the results of the informal experiment. As shown in **Figure 3**, learners explained how they determined the mean from their experiments. Groups 1 and 2 correctly articulated that the mean is the sum of all data divided by the number of data points. Group 3 conveyed the correct concept, though they used different wording. When asked to express the mean formula in a general form, Groups 1 and 2 did so accurately, while Group 3 described it in words. The pilot experiment highlighted the need to improve the worksheet instructions to better guide learners in deriving the mathematical formula for the mean.

For the activity focused on finding the mode, learners were provided with a Congklak board and a worksheet and worked in pairs according to the worksheet's instructions. Both the experimental and formal activities aligned with the conjectures in the HLT. In the activity to discover the median, each group had two participants: one worked with an odd number of Congklak holes, and the other with an even number. Learners arranged the Congklak seeds from the smallest to the largest number and determined the middle value.

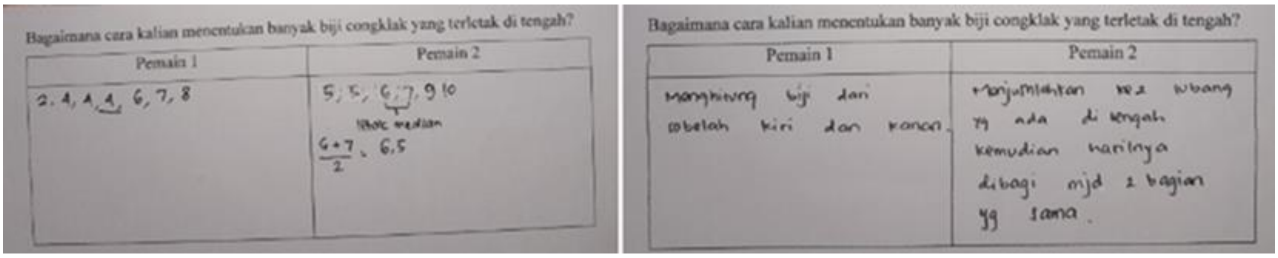

Figure 4: Variation in Learners' Answers on How to Find the Median

Based on their experiments, learners found different ways to determine the middle value. As illustrated in **Figure 4**, Group 3 identified the middle value as expected in the HLT. Groups 1 and 2 found the middle value for an odd number of data points by counting the Congklak holes from both ends toward the center, noting the number of seeds in the center hole. For an even number of data points, they used a similar method but averaged the seeds from the two middle holes. This approach differed from the HLT's conjecture, indicating a need for refinement in the HLT. In the formal activity, where learners connected their experiment to the concept of the median, all groups concluded that the median represents the middle value. However, they overlooked the critical condition that the data must be sorted from smallest to largest. When asked to formulate the general expression for the median, learners were guided to understand that the data can be represented using symbols n and the n^{th}- order data as x_n.

Group 2's response involved adding the n^{th} data point and dividing by two, which did not align with the HLT conjecture. **Figure 5** discrepancy suggested that learners encountered difficulties in finding the general

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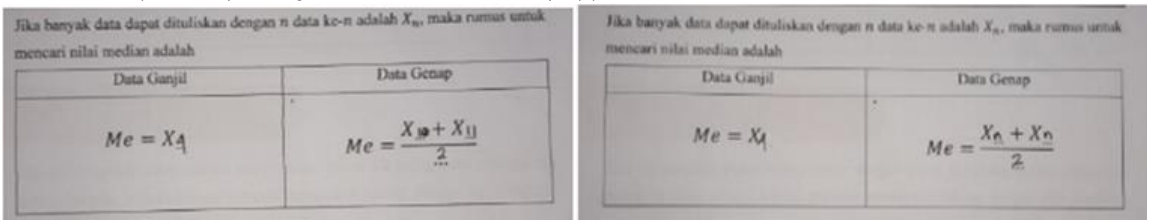
formula for the median. The pilot experiment highlighted the need to improve the instructions to help learners correctly identify medians for both odd and even data sets.

The next activity focused on understanding the concept of data range. Using the same seed arrangement, learners were asked to calculate the difference between the Congklak seeds in the last and first holes. All groups followed the HLT conjecture and concluded that the range is the difference between the highest and lowest values. They correctly used x_{min} and x_{mak}. For quartiles, all groups used a similar approach. As shown in Figure 6, learners determined $Q_1$and Q_3 by finding the middle values of the data to the left and right of the median, respectively, using the same method they applied to find the median.

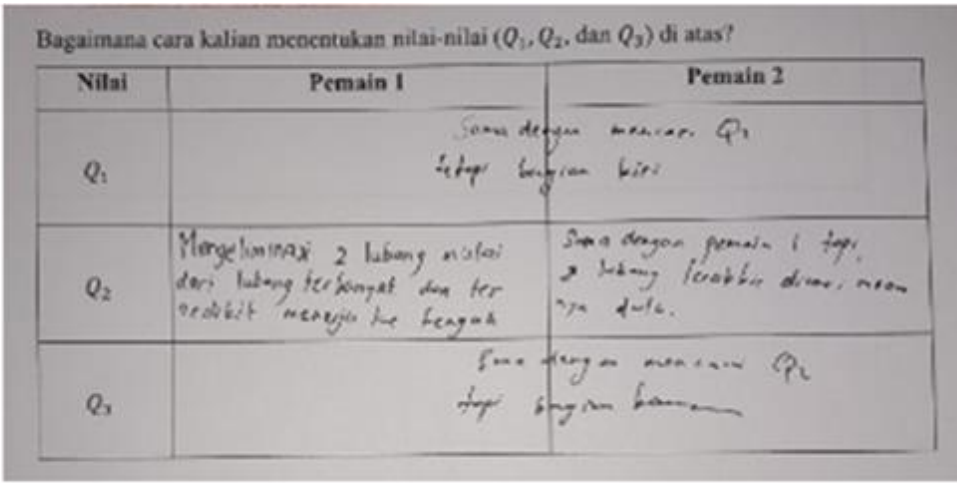

Jika banyak data dapat dituliskan dengan n data ke-n adalah X_n, maka rumus untuk mencari nilai median adalah

Data Ganjil	Data Genap
$Me = X_4$	$Me = \frac{X_{10} + X_{11}}{2}$

Jika banyak data dapat dituliskan dengan n data ke-n adalah X_n, maka rumus untuk mencari nilai median adalah

Data Ganjil	Data Genap
$Me = X_4$	$Me = \frac{X_n + X_n}{2}$

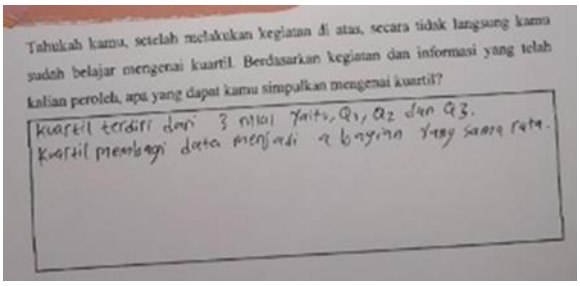
Figure 5: Various Learner Responses in the Discovery of the Mathematical Formula for the Median Concept


Bagaimana cara kalian menentukan nilai-nilai (Q_1, Q_2, dan Q_3) di atas?

Nilai	Pemain 1	Pemain 2
Q_1	Sama dengan mencari Q_1 tetapi bagian kiri	
Q_2	Mengeliminasi 2 lubang nilai dari lubang terbanyak dan ter sedikit menuju ke tengah	Sama dengan pemain 1 tapi 2 lubang terakhir dimasukkan dulu.
Q_3	Sama dengan mencari Q_2 tapi bagian kanan	

Figure.6: Learners' Answers on How to Find the Values of Q_1, Q_2, and Q_3

Interestingly, none of the three groups used the median formula previously derived, which was contrary to the expectations in the HLT. The HLT anticipated that learners would apply the median formula from the earlier activity. During the formal activity, learners successfully connected their experimental results with the concept of quartiles. All three groups correctly identified the quartile values as Q_1, Q_2, and Q_3,


Tahukah kamu, setelah melakukan kegiatan di atas, secara tidak langsung kamu sudah belajar mengenai kuartil. Berdasarkan kegiatan dan informasi yang telah kalian peroleh, apa yang dapat kamu simpulkan mengenai kuartil?

Kuartil terdiri dari 3 nilai yaitu, Q1, Q2 dan Q3.
Kuartil membagi data menjadi 4 bagian yang sama rata.

Figure.7: Group 1 Answers Related to Understanding the Concept of Quartiles

As shown in **Figure 7**, Group 1 noted that the quartile value divides the data into four equal parts, but omitted the crucial step of sorting the data from smallest to largest. At the conclusion of the activity, learners were instructed to write down the steps for determining quartile values, beginning with Q_2, followed by Q_1 and

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Q_3, which matched the HLT conjecture. The next activity involved understanding the concept of the interquartile range, building upon the previous quartile activity. Learners discovered that the interquartile range is the difference between Q_3 and Q_1. All three groups correctly understood this concept, in alignment with the HLT conjecture. While the HLT activities effectively helped learners grasp the concepts of quartile values and the interquartile range, some improvements are needed, particularly for the interquartile range activity. During the teaching experiment cycle, the revised HLT (HLT 1) was implemented in a lesson attended by thirty learners. The findings regarding learners' understanding of the concepts of mean, mode, median, range, quartiles, and interquartile range within the HLT 1 design are detailed below.

The experimental stage focused on discovering the concept of mean, most learners carried out the activities as expected in HLT 1. However, some learners collected all the Congklak seeds and distributed them one by one into each hole until the seeds were evenly distributed, as depicted in **Figure 8.**

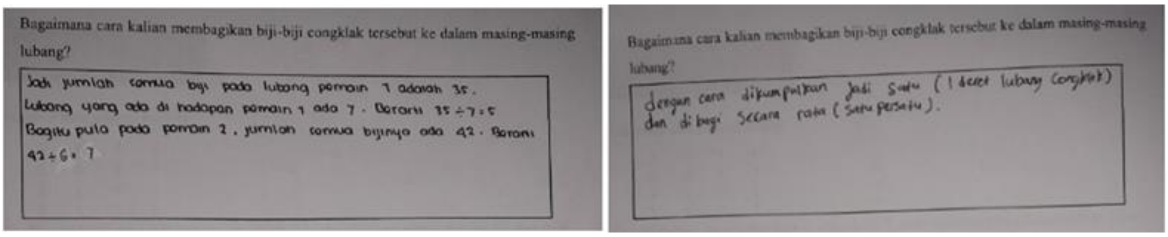

Figure 8. Variations in Learners' Answers to the Mean Concept Discovery Process

In the formal activities, most learners correctly derived the mathematical formula for the mean through the activities outlined in HLT 1. However, some learners struggled with this task, as shown in **Figure 9.**

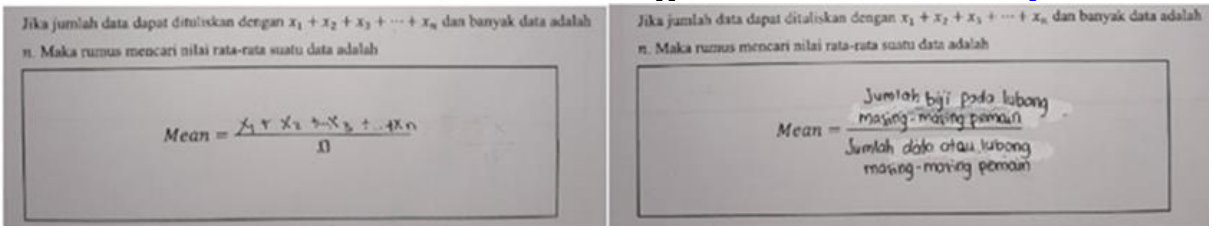

Figure 9. Variation in Learners' Answers to the General Formula for the Mean

The teaching experiment demonstrated that HLT 1 effectively supported learners in understanding the concept of the mean. However, there is room for improvement. It is recommended to add a new conjecture, where learners distribute Congklak seeds one by one until all seeds are used up. Additionally, the mean formula should be presented both as a mathematical equation and in sentence form, as follows:

$$mean = \frac{sum\ of\ all\ data}{number\ of\ data}$$

During the learning experiment focused on discovering the concept of mode, some learners recorded all holes and the number of seeds, while others only recorded the holes with the same number of seeds, as depicted in **Figure 10.**

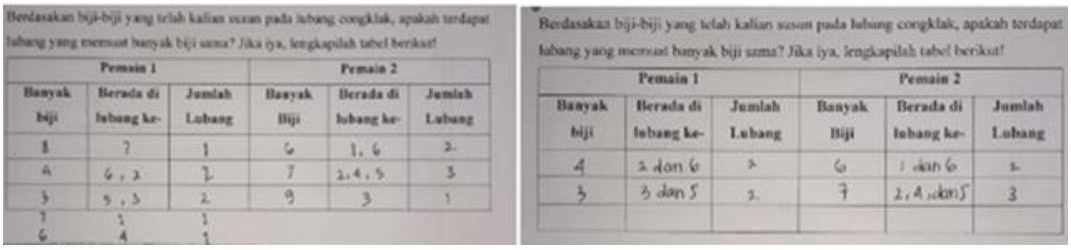

Figure.10: Learners' Answers on the Data Collection of Congklak Seeds and Holes

Based on their data, learners identified the value (the number of Congklak seeds) that most frequently appeared in the Congklak holes. Most learners responded in accordance with the conjecture in HLT 1, but some

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wrote down the number of holes containing the same number of Congklak seeds. This response did not align with the given question, possibly due to a misunderstanding of the instructions on the worksheet.

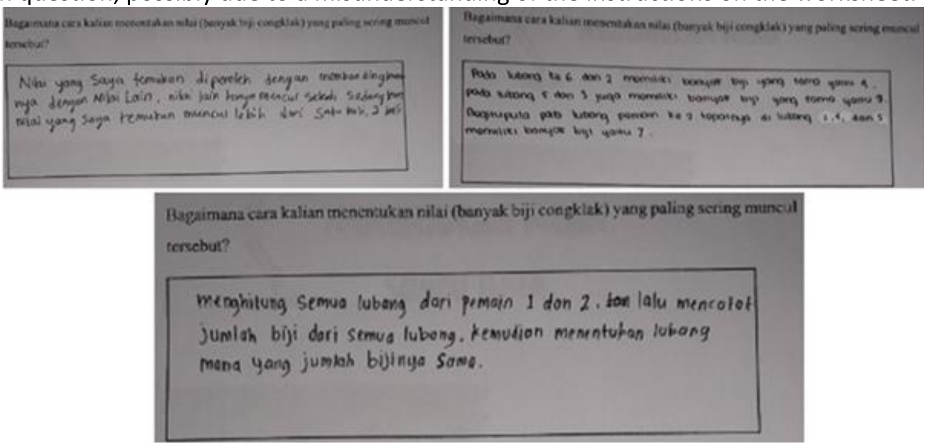

Figure 11: Variation in Learners' Answers on How to Find the Mode Value

In the formal activity, learners connected their experimental work with the concept of mode to determine how to find the mode. As shown in **Figure 11**, some learners correctly identified the mode as the value with the highest frequency, while others struggled, merely noting the steps involved in recording the holes with the same number of Congklak seeds. However, after further investigation through interviews, all learners were able to accurately define the mode. The results of this teaching experiment suggest that the activities in HLT 1 effectively supported learners in discovering the definition of mode. In the experiment to understand the median concept, learners employed several different methods, as illustrated in **Figure 12.**

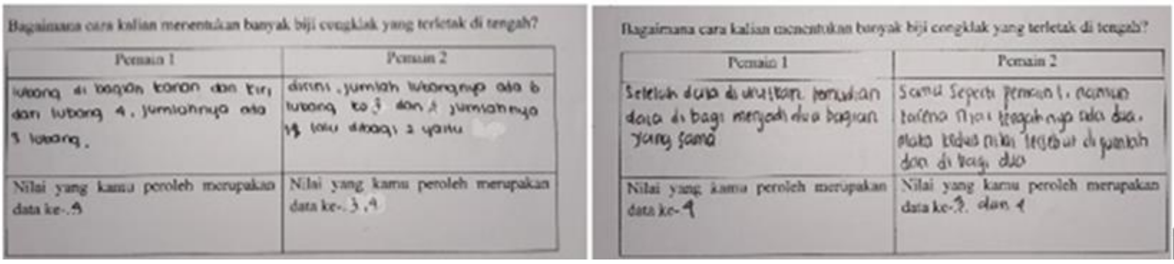

Figure 12: Learners' Answers in Determining the Median Value of a Data Set

In this activity, learners demonstrated two variations in their understanding of the median. One approach involved sorting the data first and then calculating the median by counting from both ends toward the center. Learners also noted that if the dataset contained an odd number of values, the middle value could be directly identified, whereas for an even number of values, the median was calculated as the mean of the two middle data points. However, the second method did not emphasize the importance of first sorting the data from smallest to largest. Despite this, both methods generally aligned with the conjecture in HLT 1. In the formal activity, learners connected their experimental findings to the concept of the median, generalizing their results and using teacher instructions to symbolize the number of data points as nnn and the n^{th} -data as x_n. . All groups wrote the formula as expected in HLT 1.

The next activity involved finding the range of a data set. Learners identified the largest value as the number of seeds in the last hole and the smallest value as the number of seeds in the first hole, concluding that the range is the difference between these values. In the formal activities, learners wrote the mathematical formula for the range, using $x_{min}$for the smallest value and x_{mak} for the largest value. All learners successfully derived the range formula, although some expressed it in sentence form. The teaching experiment concluded that the activities in HLT 1 were effective in helping learners understand the concepts of the median and range. However, improvements are needed in the median activity to ensure that learners remember to sort the data from smallest to largest.

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In formal activities linking experimental results to the concept of quartiles, learners employed various methods to determine quartile values. These methods included ensuring the number of holes to the right and left of the quartile value were equal, linking quartile values with the concept of the median, identifying Q_2 as the median of the entire data set, Q_1 as the median of the data to the left of Q_2, and Q_3 as the median of the data to the right of Q_2. Learners also found quartile values by associating them with the median and deriving formulas for , Q_1, Q_3, and Q_3. Although few learners initially wrote down the requirement that data must be sorted from smallest to largest, interviews confirmed that all learners understood this necessity. This is illustrated in **Figure 13** below.

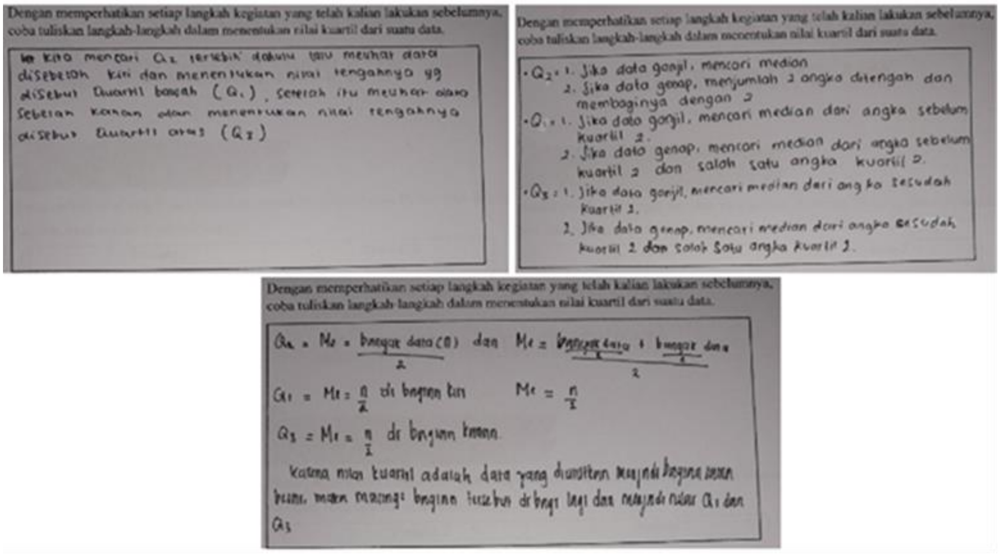

Figure 13. Learners' Answers in Determining Steps to Find Quartile Values

Determining the concept of the interquartile range builds on an understanding of quartiles and involves a related experiment. Learners used the conjecture in HLT 1 to determine Q_1, Q_2, and Q_3. With the given instructions, most learners concluded that the interquartile range is the difference between Q_3 and Q_1, while others described it as the difference between the middle value of the data to the right of Q_2 and the middle value to the left of Q_2. In the subsequent formal activity, all learners correctly wrote the interquartile range formula based on the previously discussed definition, following the conjecture in HLT 1. The teaching experiment concluded that the activities designed in HLT 1 were effective in helping learners understand quartiles and interquartile ranges. However, since many learners did not initially note the requirement for sorting data before determining quartile values, adjustments to HLT 1 are necessary.

The statistics learning trajectory outlines how learners grasp the concepts of mean, mode, median, range, quartile, and interquartile range. This learning trajectory aids and facilitates learners in understanding and defining these concepts, as well as in deriving the mathematical formulas associated with them. The activities developed within the learning trajectory enhance learner engagement and reasoning skills, making the learning process more meaningful. Below is the learning trajectory for the concepts of mean, mode, median, range, quartile, and interquartile range.

The Learning Trajectory of the Mean Concept

The core activity for understanding the concept of the mean involves having learners rearrange Congklak seeds so that each hole contains the same number of seeds. Initially, the seeds in each hole are arranged differently, so learners must determine how to redistribute the seeds evenly across all holes. Learners generally approach this task in two ways. Some divide the total number of seeds by the number of Congklak holes and then fill each hole with the corresponding number of seeds based on their calculations. Others collect all the Congklak seeds first and then distribute them one by one into each hole until all the seeds are used up. Although the methods differ, both effectively guide learners in discovering the concept of the mean. Through these activities, learners not only arrive at the definition of the mean but also derive its mathematical formula. The learners typically express the formula in two forms:

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$$mean = \frac{sum\ of\ all\ data}{number\ of\ data} \text{ and } \bar{x} = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$$

The Learning Trajectory of the Mode Concept

The objective of the learning trajectory for the mode concept is to help learners explore and understand the concept of mode through hands-on activities. Learners begin by arranging Congklak seeds into holes according to a specified number, then present their findings in a table with two columns: one for the number of Congklak seeds and the other for the frequency of each seed count. Learners are asked to identify the value that appears most frequently. They generally understand the instructions well and are able to effectively organize the data in the table, allowing them to easily identify the mode. In doing so, learners come to understand that the mode is the value that appears most frequently in a data set. They also learn that a data set can have more than one mode (McKenney, & Reeves, 2018).

The Learning Trajectory of the Median Concept

The objective of this learning trajectory is to help learners discover the concept of the median. The activity begins with learners arranging Congklak seeds in the holes in ascending order, from the fewest to the most seeds. Once the seeds are ordered, learners then identify the number of Congklak seeds in the middle hole. Learners typically use two strategies to determine this middle value: some count the Congklak holes from both the right and left ends toward the center, while others divide the total number of holes into two equal parts to locate the middle hole.

Learners also encounter differences in finding the middle value depending on whether the number of holes is odd or even. If the number of Congklak holes is odd, learners can easily identify the median by counting the seeds in the middle hole. However, if the number of holes is even, learners find two middle holes and must add the seeds in these holes and then divide the total by two to calculate the median. While this sequence of activities effectively guides learners in deriving the mathematical formula for the median, it does not fully ensure that they grasp the definition of the median. Some learners still define the median as simply the middle value without noting the crucial requirement that the data must first be sorted from smallest to largest.

The Learning Trajectory of the Range Concept

The activity designed to teach the concept of range is straightforward. Learners are asked to determine the difference between the number of Congklak seeds in the last hole and those in the first hole. Given that the Congklak seeds are arranged from the smallest to the largest number, learners must find the difference between the largest and smallest quantities of seeds. Through this activity, learners are able to grasp the definition of range and derive the mathematical formula for the range in two ways: $Range = x_{mak} - x_{min}$

The Learning Trajectory of Quartile and Interquartile Range Concepts

The learning objective for this trajectory is to understand quartiles. Learners begin by arranging the Congklak seeds in ascending order from the smallest to the largest quantity. They then identify the median, or Q_2, as the value in the center. Afterward, learners determine the median of the data to the left of Q_2 (which is Q_1) and the median of the data to the right of Q_2 (which is Q_3). Learners typically use two methods to calculate the quartile values: some count the holes from both ends toward the center to find the middle hole, while others divide the total number of holes into two equal parts to identify the center hole. Through these activities, learners discover the steps for determining Q_1, Q_2, and Q_3. They come to understand that Q_1 is the lower quartile, Q_2 is the median or middle quartile, and Q_3 is the upper quartile. The activities related to the interquartile range build upon the quartile concept. Learners calculate the difference between the middle value of the data to the right of Q_2 and the middle value to the left of Q_2. This exercise helps learners realize that the interquartile range is the difference between the values of Q_3 and Q_1. Consequently, learners derive the formula for the interquartile range =Q_3 - Q_1.

4. DISCUSSION

The statistics curriculum for eighth-grade junior high school students includes the concepts of mean, median, mode, range, quartiles, and interquartile range. Current teaching methods often fail to integrate real-world contexts familiar to learners, leading to weaknesses in solving contextual problems. These shortcomings

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are largely due to a lack of mathematical reasoning skills, such as making conjectures, performing mathematical manipulations, providing evidence for solutions, and drawing conclusions or generalizations.
Realistic Mathematics Education (RME) utilizes contexts and environments familiar to learners, facilitating a more effective learning process (Garfield & Ben Zvi,2008). This approach employs a learning trajectory that aligns well with the use of the traditional game of Congklak in teaching statistics. The method proves effective in achieving the expected competencies and enhancing learners' mathematical reasoning skills. The following discussion outlines the learning trajectory for understanding and discovering the concepts of mean, mode, median, range, quartiles, and interquartile range (Gravemeijer,2004)

Based on the improvements made in earlier stages, a learning trajectory was developed to help learners determine the concept of mean. Learners were instructed to rearrange the Congklak seeds so that each hole contained the same number of seeds. Given that the original arrangement of seeds varied, learners explored different methods to redistribute the seeds evenly. Some learners employed mathematical calculations, while others did not. However, they all recognized that to achieve an even distribution, the total number of seeds needed to be divided by the number of holes. This observation aligns with the conclusions drawn by Macross and Russell, who assert that averages reflect fair portions and balance, allowing learners to grasp the concept of averages informally through fair division (Fowler, 2023). Through this activity, learners were able to understand the concept of the mean and derive its mathematical formula. The learning trajectory for discovering the mean is illustrated in the concept map of the thinking flow in **Figure 14.**

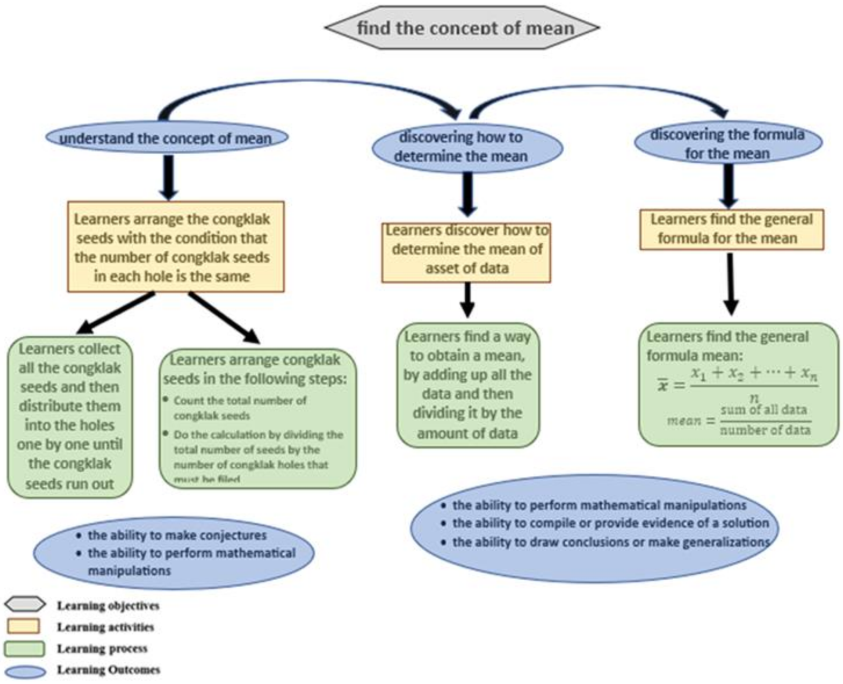

Figure 14. The Learning Trajectory of the Mean Concept

In the learning trajectory for determining the mode, learners began by placing Congklak seeds into the holes according to a specified number. Next, they identified which holes contained the same number of seeds and recorded their findings in a table. This exercise helped learners to understand that there can be more than one mode and that the mode is the value that appears most frequently in a dataset. This understanding is consistent with the three possible outcomes identified in Fakhmi's (2021) research on learning trajectories in statistics: there may be no mode if all data points have the same frequency, there may be a single mode, or there may be multiple modes. **Figure 15** depicts the concept map of the learning trajectory for determining the mode.

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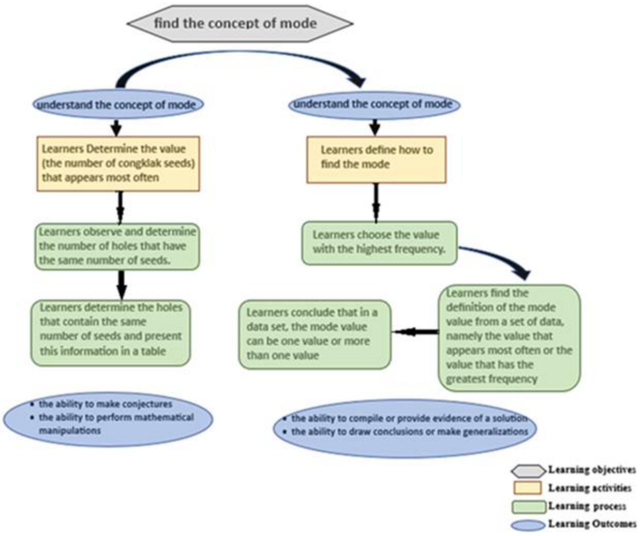

Figure 15. The Learning Trajectory of the Mode Concept

As part of the learning trajectory to determine the median concept, learners are instructed to place Congklak seeds in holes, starting from the fewest to the most seeds. Once the seeds are arranged, the number of Congklak seeds in the middle hole is identified. Learners typically use two methods to find this median value: either by splitting the total number of holes into two equal parts or by counting the Congklak holes from the left and proceeding toward the center hole. Learners also noted the difference in determining the middle value when dealing with an even or odd number of holes. If the number of holes is odd, learners simply count the seeds in the middle hole to find the median value(Lithner,2008). However, if there is an even number of holes, learners find the two center holes, add the number of seeds in both, and then divide the result by two to calculate the median. This exercise helps learners to calculate the median, understand the different approaches required for even and odd data sets, and derive the mathematical formula for the median. It is important to note that some learners concluded that the median is merely the value in the middle, without recognizing the crucial step of ordering the data from smallest to largest. This oversight can lead to errors when solving problems. According to Collins (2004), learners often make the mistake of not sorting the data before calculating the median, resulting in inaccurate conclusions. **Figure 16** displays the learning trajectory for understanding the median concept.

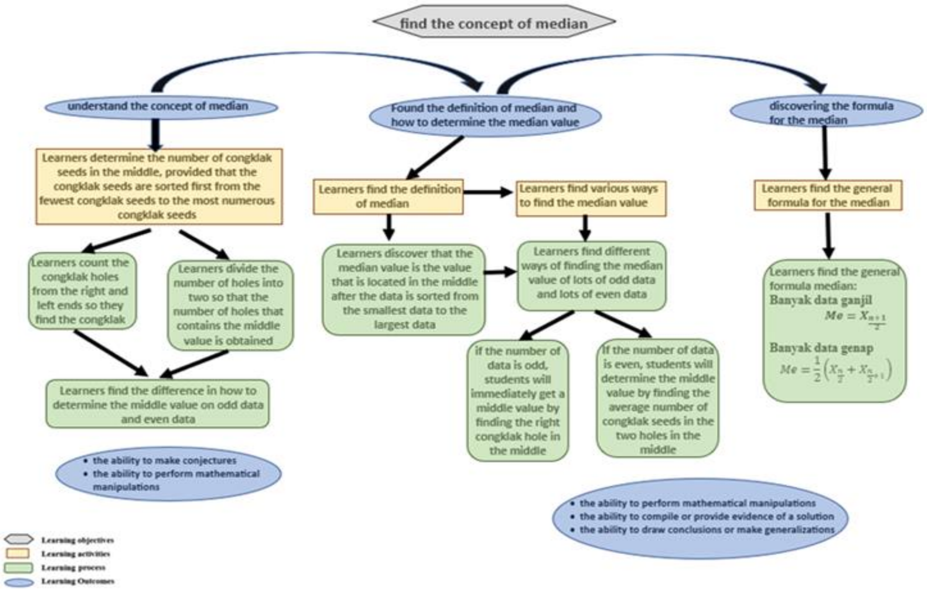

Figure 16. Learning Trajectory of the Median Concept

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The learning activities in the trajectory for understanding the concept of range are relatively straightforward. Learners are asked to calculate the difference between the number of Congklak seeds in the first and last holes. Based on the arrangement of the seeds, starting from the smallest quantity, learners must determine the difference between the largest and smallest numbers of Congklak seeds. Through this exercise, learners can grasp the definition of range and derive its mathematical formula(Marande & Diana, 2022). **Figure.17** shows the learning trajectory for the concept of range.

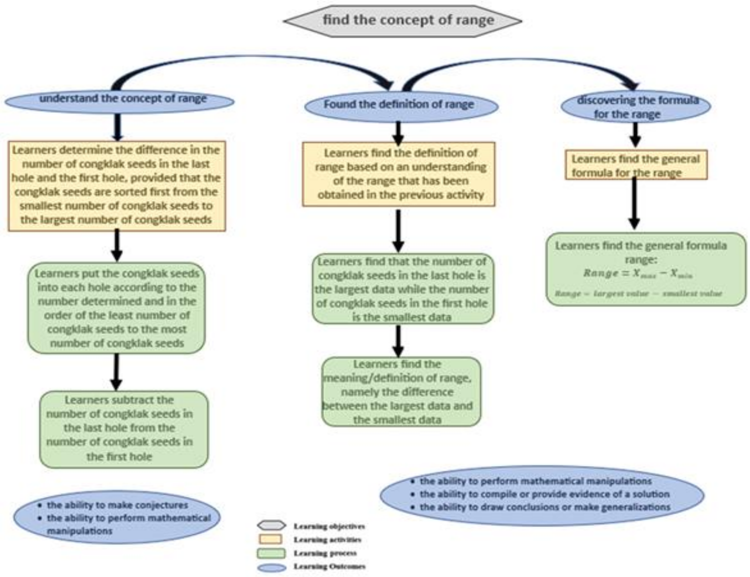

Figure 17. Learning Trajectory of the Range Concept

In the learning trajectory for discovering the concept of quartiles, learners begin by arranging the Congklak seeds in the holes from the least to the most. They then determine the value located in the middle, known as Q_2. Following this, learners identify the middle values of the data to the left and right of Q_2, which correspond to Q_1 and Q₃, respectively. The strategy employed by learners to find $Q_{1,}Q_2$, and Q_3 is similar to the method used for finding the median, as both involve identifying the middle value within a set of data. Learners may find it challenging to draw conclusions about quartiles on their own, so teacher guidance is crucial in helping them grasp the concept (**Figure 18**).

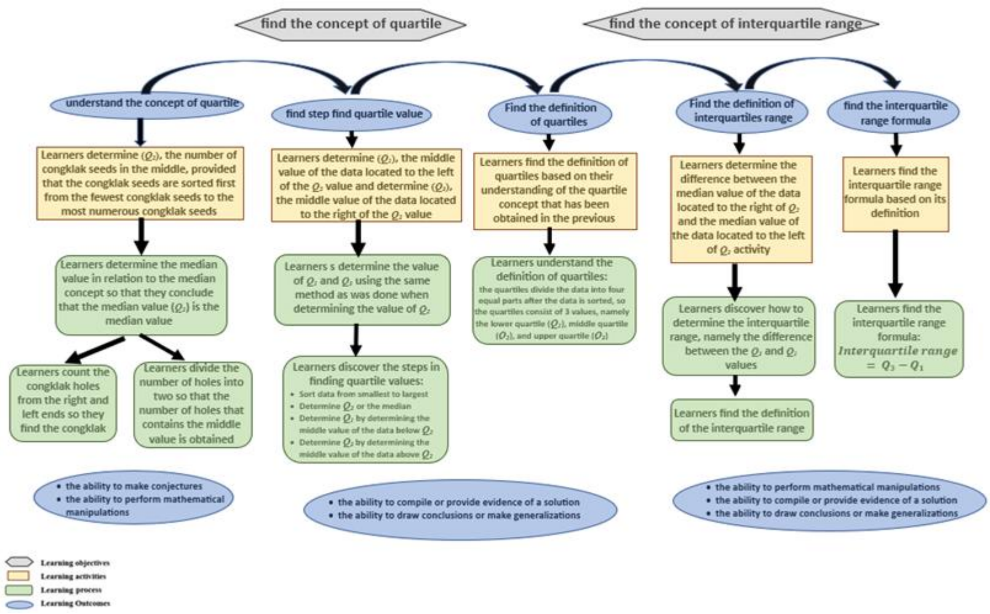

Figure 18. Learning Trajectory Concept of Quartiles and Interquartile Range

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In the learning trajectory for understanding the concept of the interquartile range, learners continue from where they left off with the quartile values. They are asked to determine the difference between the median value to the right of Q_2 and the median value to the left of Q_2. Through these activities, learners come to understand that this difference corresponds to the values of Q_3 and Q_1. By exploring this difference, they discover the meaning of the interquartile range and its general formula. The learning trajectory for the concepts of quartiles and interquartile range developed in this study is presented in Figure.18.

The application of a realistic mathematics learning approach, using a learning trajectory that incorporates the traditional game of Congklak for teaching statistics at a junior high school in Wonogiri City, has proven effective in enhancing learners' mathematical reasoning skills. This effectiveness is evidenced by the t-test results conducted on the experimental and control classes, as outlined below.

Table 1. Independent Samples Test

Test	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference
Levene's Test for Equality of Variances	0.705	0.404						
t-test for Equality of Means			2.007	60	0.049	8.910	4.440	Lower: 0.030 Upper: 17.791
Equal variances not assumed			2.007	59.541	0.048	8.910	4.418	Lower: 0.073 Upper: 17.748

Based on **Table 1**, the Levene's test shows a significance value (Sig.) of 0.404, which is greater than 0.05. This indicates that the data between the experimental and control classes have equal variances. Therefore, to determine whether there is a significant difference in the average mathematical reasoning skills between learners in the experimental and control classes, the data in the first row is used. Since the Sig. value is 0.049, which is less than 0.05, the null hypothesis (H_0) is rejected. This indicates that there is a significant difference in the average mathematical reasoning skills between learners in the experimental and control classes. The output table also shows a mean difference of 8.910, suggesting that there is a noticeable difference in mathematical reasoning skills between learners taught using the learning trajectory within the context of the traditional Congklak game and those taught through direct instruction. Given that the marginal mean of the experimental class—where the Realistic Mathematics Education (RME) approach with a learning trajectory assisted by the traditional game of Congklak was applied—is 71.57, compared to 62.66 for the control class using direct learning, it can be concluded that the application of a realistic mathematics approach with a learning trajectory incorporating the traditional game of Congklak effectively enhances learners' mathematical reasoning skills (Nickerson, & Whitacre,2010)..

5. CONCLUSION

The application of Realistic Mathematics Education (RME) using a learning trajectory based on the traditional Congklak game for teaching statistics in a junior high school in Wonogiri City necessitates active learner engagement in both informal and formal activities. Informal activities involve experiments with the Congklak game to explore and understand key statistical concepts, including mean, mode, median, range, quartiles, and interquartile range. Formal activities require learners to connect their experimental findings with mathematical concepts, using these results to define or derive mathematical formulas. This approach effectively enhances learners' mathematical reasoning skills, encompassing the ability to make conjectures, perform mathematical manipulations, provide evidence for solutions, and draw conclusions or make generalizations.

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