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Rigorous Mathematical Thinking Aprroach for Support Relational Understanding: Designing a Learning Trajectory on Linear Equation

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ABSTRACT

The equation of a straight line is a mathematics topic that presents significant learning challenges. These difficulties often arise because the topic is frequently taught through memorization of numerous formulas, without fostering a deep understanding or adequately developing students' reasoning and problem-solving skills. This study aims to develop a learning trajectory for the equation of a straight line using the Rigorous Mathematical Thinking (RMT) approach to support students' relational understanding. The research method employed was design research, focusing on the development of a learning trajectory through three stages: preliminary design, experimental implementation, and retrospective analysis, conducted over two cycles. The participants were 8th-grade students from a Junior High School in Palembang, with 32 students from class 8.2 participating in the pilot experiment and 36 students from class 8.1 in the teaching experiment, selected through purposive sampling. Data were collected through observations, written tests, and interviews, then analyzed qualitatively and presented narratively. The outcome of this study is a learning trajectory consisting of two key activities: solving problems related to the equation of a line passing through two points, and solving problems related to the equation of a line perpendicular to another line. The developed learning trajectory effectively supports students' relational understanding by guiding them to utilize their existing knowledge, determine the most effective and efficient strategies, provide detailed solutions, and have confidence in the accuracy of their answers in problem-solving.

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1. INTRODUCTION

The equation of a straight line is a key mathematical topic taught at the junior high school level. Mastery of linear functions is initially grounded in understanding the various representations of functions. Students must be familiar with three primary representations of a function: algebraic expressions, tabular data, and graphical depictions. These concepts are essential for determining the equation of a line, as highlighted by Kadarisma(2020). The study of the equation of a straight line must be thorough to prevent conceptual errors during learning, as noted by Nafisa and Rochmad (2021). This involves presenting the concept in a coherent manner, beginning with determining the gradient and extending to the equation of the line through either one point and the gradient or through two points. However, this topic is often presented with numerous formulas that students are required to memorize, rather than understanding deeply, which can lead to a lack of reasoning and problem-solving skills (Ibrahim, 2020). To develop students' competencies in understanding the equation of a straight line, it is crucial that they possess the ability to grasp the problems presented to them.

Understanding is a fundamental aspect of achieving learning objectives in mathematics (Saparida et al., 2022; Senita, 2021). In mathematical contexts, understanding is divided into relational understanding and instrumental understanding (Skemp, 1978). Relational understanding refers to students' ability to comprehend the appropriate steps or procedures for solving a problem and the logical reasons behind selecting these procedures (Skemp, 2006). On the other hand, instrumental understanding refers to a person's ability to apply learned concepts without necessarily understanding the underlying reasons for their application (Skemp, 2006). This research focuses on relational understanding.

Relational understanding plays a crucial role in comprehending mathematical concepts, as it enables students to grasp the underlying reasons for each procedure rather than merely applying the concepts without

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deeper insight. Relational understanding in mathematics offers several key benefits: it simplifies the process of solving complex problems, aids in the retention and comprehension of mathematical concepts, facilitates the achievement of learning objectives, and fosters the creation of original ideas (Skemp, 2006). Therefore, it is essential for students to develop relational understanding, particularly when learning about the equation of a straight line.

However, observations by Rahmah & Rahardi (2021) indicate that current learning activities often fail to foster relational understanding, as they predominantly focus on delivering formulas and practice problems without adequately connecting these to the underlying concepts. Ganing et al. (2020) further highlight that students' relational understanding remains weak, as evidenced by their struggles to identify the reasoning behind the procedures used in problem-solving.

To address this gap, an instructional approach that helps students connect mathematical concepts and understand the rationale behind problem-solving procedures is needed. One effective method is the implementation of the RMT approach. RMT is a teacher- and student-centered approach that guides students in defining problems and outlining appropriate solutions (Jannah & Salwah, 2020). The application of RMT in the classroom can promote positive interactions and enhance students' focus during the learning process (Hidayat et al., 2021). Research by Nur'Asyiah et al. (2023) supports the effectiveness of the RMT approach in mathematics education, showing that it leads to high levels of student engagement and positive attitudes towards learning, with 93% of students exhibiting a favorable response to this method. Previous research on learning design using the RMT approach has primarily focused on topics like geometry, particularly in supporting qualitative thinking within the RMT framework (Pratiwi et al., 2022). However, Weni et al.'s study only addressed one of the three levels of thinking in the RMT approach. The study highlighted the effectiveness of the RMT approach in enhancing students' problem-solving skills, suggesting that further research should explore the use of RMT to improve other student abilities, such as conceptual understanding and critical thinking skills (Aulia et al., 2019).

In this context, the researcher applied the RMT approach to support relational understanding of the equation of a straight line. RMT can aid students in developing a deeper understanding of this mathematical concept through a more formal and structured learning process. By employing RMT, students can gain a thorough and detailed understanding of the equation of a straight line, which is critical for mastering the subject. Nuraisyah et al. (2023) have emphasized that RMT fosters effective interactions between students and the learning material. The application of RMT involves three phases and six steps that guide students towards improving their relational understanding. The first phase, Cognitive Development, involves students adapting models or ways of thinking to complete cognitive tasks related to straight line equations. Through teacher-provided stimuli, students use psychological tools such as symbols, diagrams, or graphs to carry out these tasks.

The second phase, Content as Process Development, involves students building fundamental concepts related to the slope of a line (gradient) and the equation of a straight line based on known points or perpendicular lines. Under teacher guidance, students discover patterns or formulas to solve problems and adapt specific mathematical psychological tools to consider alternative solutions. The third phase, Cognitive Conceptual Construction Practice, students are encouraged to solve problems independently or in groups, with the teacher serving as a facilitator. Despite the effectiveness of RMT in various educational contexts, there has been limited research specifically designing lessons using RMT to support relational understanding. This gap in the literature prompted the researcher to conduct a study aimed at developing a learning trajectory for the equation of a straight line using the RMT approach, with the goal of enhancing students' relational understanding.

2. MATERIAL AND METHOD

Research Design

This study employs a design research methodology, focusing on the development and validation of learning trajectories. The primary goal is to create and refine theories regarding the learning process and the design of effective learning trajectories. The research flow chart is illustrated in **Figure 1.**

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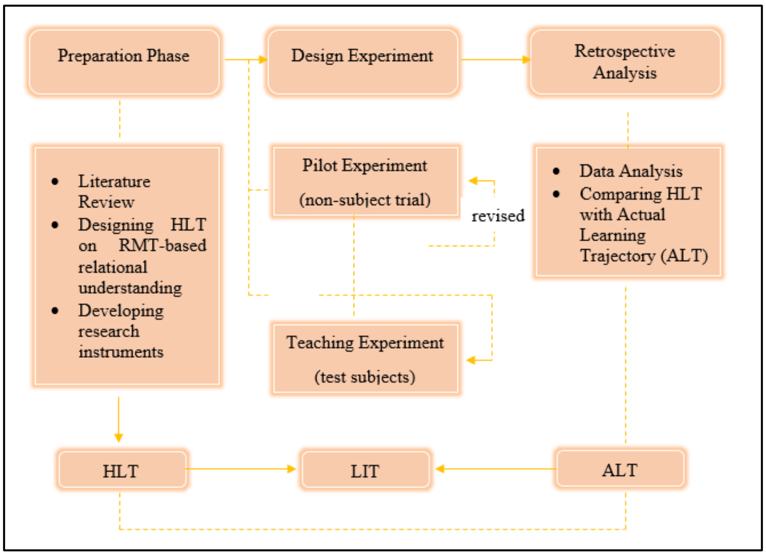

Figure 1. Research Flow Chart

The validation studies are conducted in several stages: the preparation phase, the design experiment phase (which includes both pilot and teaching experiments), and the retrospective analysis (Gravemeijer & Eerde, 2009). During the preparation phase, the researcher conducted a comprehensive literature review, designed the Hypothetical Learning Trajectory (HLT), and developed the research instruments. The pilot experiment phase involved the initial testing and refinement of the HLT, evaluating its effectiveness, and assessing the feasibility of the instruments. Following this, the teaching experiment phase tested the refined HLT in the actual classroom setting with the selected research participants.

Participants and Setting

The participants in this study were 8th-grade students from a junior high school in Palembang, selected through purposive sampling based on the recommendations of the mathematics teacher and the availability of the students. Two classes were chosen for the study: 32 students participated in the pilot experiment, and 36 students took part in the teaching experiment. Additionally, a mathematics teacher was involved to provide insights into the curriculum, teaching materials, and classroom conditions, and another researcher acted as an observer.

Data Collection Methods

Data for this study were collected through observations, written tests, and interviews. Observations of the learning process were recorded using observation sheets and video recordings, which captured the students' attitudes and skills, with evaluations based on predetermined criteria. The written tests were administered using learner worksheets (LKPD), which were analyzed to assess students' conjectures, solution strategies, and the achievement of learning objectives. The analysis of LKPDs was supplemented with narrative texts that explained the students' responses. Interviews were conducted using semi-structured guidelines, which were adapted based on the development of students' answers during the study.

Data Analysis Procedures

Data from the interviews were analyzed and presented in narrative text form. The analysis of observation and interview data provided a clearer understanding of the results from the LKPDs completed by students. All data were analyzed using a descriptive method, aiming to describe students' thinking during the learning process and the emergence of relational understanding. The indicators of relational understanding used in this study are outlined in Table 1.

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Table 1. Indicators of Relational Understanding

Indicators of Relational Understanding	Descriptor
Ability to perform detailed procedures	- Students can write down what they know from the problem given. - Students can articulate the problem clearly.
Obtaining the correct result	- Students can provide the correct answer. - Students can draw conclusions based on their results.
Ability to use appropriate formulas	- Students can apply suitable methods to solve problems.
Ability to link various concepts	-Students can solve problems by utilizing their existing understanding.

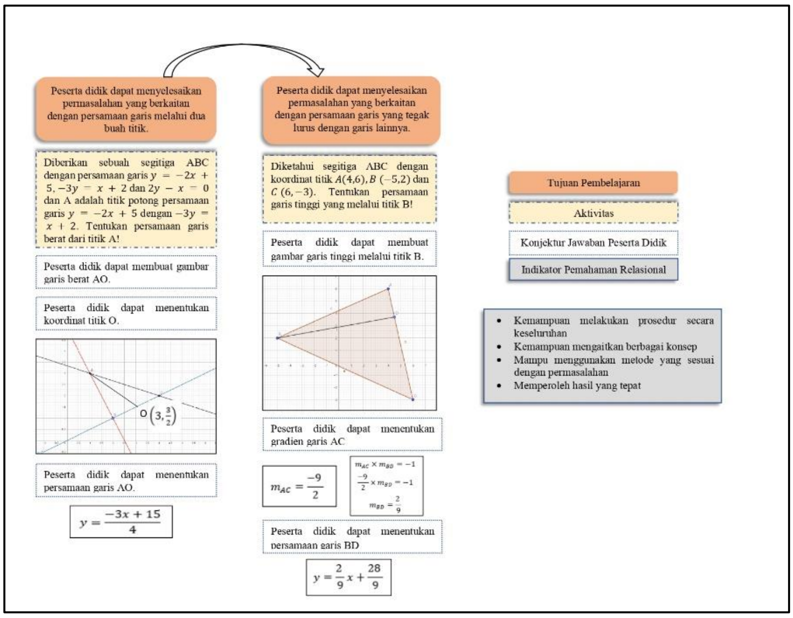
This detailed analysis aims to shed light on how students develop relational understanding through the designed learning trajectory, offering insights into both their cognitive processes and the instructional strategies that support their learning.

3. FINDINGS

Preparation Phase

During the preparation phase, an extensive literature review was conducted to identify common challenges students face when learning about the equation of a straight line. This phase also explored the effectiveness of the RMT approach in addressing these challenges. Based on this review, HLT were developed, encompassing lesson plans, learning materials, and assessment tools. These materials underwent validation by two experts in mathematics education, who assessed their clarity, relevance, and potential effectiveness. The HLT design was structured to include specific learning objectives, detailed activities, and conjectured learner thinking patterns. The framework of the HLT for the line equation is depicted in **Figure 2.**

Figure 2. HLT Line Equation



Pilot Experiment

The pilot experiment was implemented in class 8.2 at a junior high school in Palembang and consisted of three sessions. In the first session, students engaged with problems related to determining the equation of a line passing through two points. They were instructed to draw a line through a given point and identify the coordinates of the intersection. The second session focused on problems involving the equation of a line perpendicular to another line, where students worked on identifying the gradient and applying the appropriate formula for the equation of the line. The third session was dedicated to consolidating the learning outcomes and addressing any remaining difficulties.

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Throughout the cycle 1 activities, observations revealed that students initially encountered difficulties with algebraic manipulations and grasping the geometric implications of the equations. Feedback from the pilot experiment highlighted the need for more precise instructions and additional direct explanations from the teacher on certain concepts (John,2014). Based on these insights, revisions were made to the HLT and associated learning materials to enhance clarity and instructional effectiveness. The implementation process during the pilot experiment stage is illustrated in **Figure 3.**

Figure 3. Pilot Experiment Stage

The findings from the pilot experiment were crucial in reviewing and refining the HLT, particularly regarding the conjectures made in the LKPD activities. The researcher used these findings to revise the HLT before advancing to the teaching experiment stage. Observation during the pilot experiment was that students did not redraw the line as required in problem No. 1; instead, they directly drew a line through point A on the provided diagram. This issue prompted a reflection and subsequent adjustment for the teaching experiment stage, where students were explicitly guided to redraw the line in the designated space (Dristian,2023). Additionally, the pilot experiment revealed that no students applied the formula for the equation of a line through two points. Consequently, adjustments were made for the teaching experiment stage to prevent students from being overly reliant on this specific formula. Instead, they were encouraged to find the equation of the line using the more flexible formula formula $y - y_1 = x - x_1$.

Teaching Experiment

The refined HLT was implemented in the teaching experiment with 36 eighth-grade students to evaluate its effectiveness in enhancing students' relational understanding of the equation of a straight line.

Activity 1: Solving Problems Related to the Equation of a Line Through Two Points

The first activity involved solving problems related to determining the equation of a line through two points. The teacher distributed LKPD Activity 1 to 36 students, who were divided into nine groups. The lesson commenced with a discussion of the learning objectives and an exploration of the students' prior knowledge regarding the equation of a straight line. Students were then asked to understand and solve the problems presented in the LKPD. The problem required determining the equation of a straight line that serves as the median of a triangle, which leveraged their prior understanding of medians in triangles. Details of the first meeting's LKPD activities are depicted in **Figure 4.**

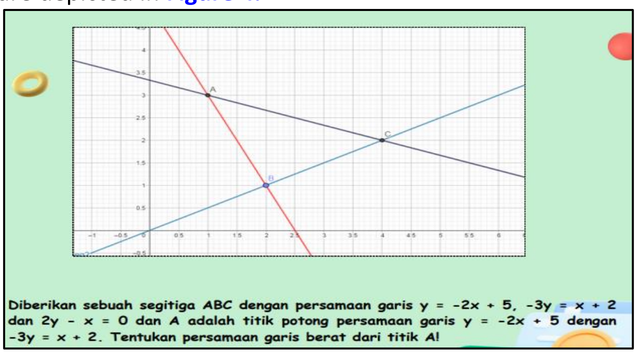

Figure 4. LKPD Activities 1

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Students were instructed to solve the problems by following the steps outlined in the LKPD. The first question asked students to draw a median from point A to the opposite side BC, with the median intersecting BC at point O. They were then required to determine the coordinates of point O. This problem encouraged students to connect their previous knowledge, such as the concept of medians in triangles and how to determine the coordinates of points on a graph. **Figure 5** shows the results of the students' responses.

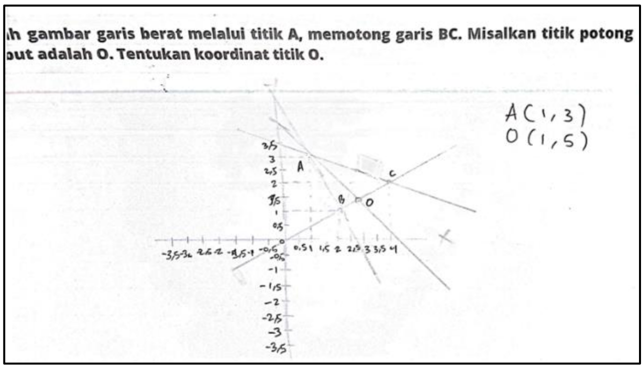

Figure 5. Results of Student's Answer Number 1

From the students' answers, it was evident that they could successfully draw the median through point O, intersecting line BC. The students correctly determined the coordinates of point O as (3;1.5) by observing the diagram provided and using the corresponding x and y coordinaes. This demonstrates

the students were able to apply their prior knowledge to solve the problem, specifically in determining the equation of the median AO. Figure 6 shows the students' responses to the second question.

Figure 6. Results of Student's Answer Number 2

For the second question, students were tasked with determining the equation of the line passing through two points, A and O. They correctly calculated the gradient of the line and then used it to derive the equation of the line. The students used the formula for the equation of a line when one point and the gradient are known, which they had learned during their previous lessons. All groups used the same formula, $y - y_1 = x -$, to arrive at the equation of the line, though there were differences in how the answers were presented. As shown in Figure 6, one group presented their result as a fraction, while another presented it in decimal form; however, both answers were correct and met the indicators of relational understanding. These indicators included the ability to perform detailed procedures, obtain the correct result, use the appropriate formula for the problem, and link various concepts(Nopriana, 2017).

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Activity 2: Solving Problems Related to the Equation of a Line Perpendicular to a Line

The second activity focused on determining the equation of a line perpendicular to another line. The lesson began with a recap of the previous learning and a discussion of the new learning objectives. The activity, presented in LKPD 2, is shown in **Figure 7.**

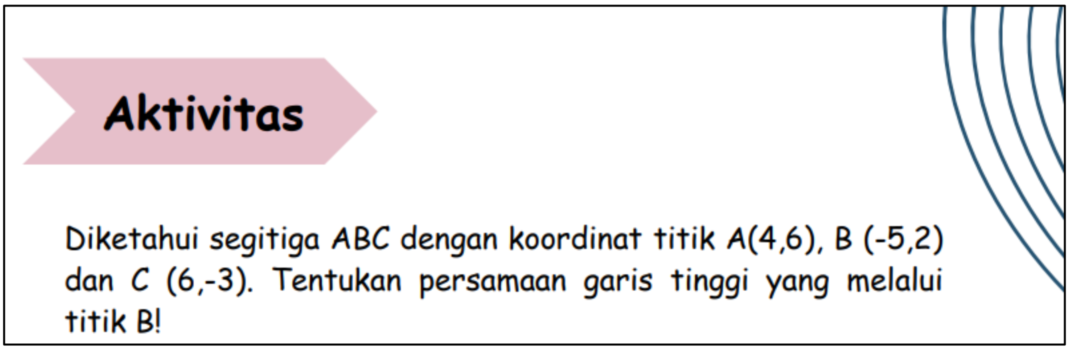

Figure7. LKPD Activities 2

Students were asked to determine the equation of a line perpendicular to a given line. In this activity, problems were designed to further support students' relational understanding by encouraging them to apply their existing knowledge, such as the concept of the altitude in a triangle. Students were also asked to illustrate the triangle based on the provided coordinate points, which is an application of the RMT approach to support relational understanding. **Figure 8** illustrates the students' responses to the first question in this activity.

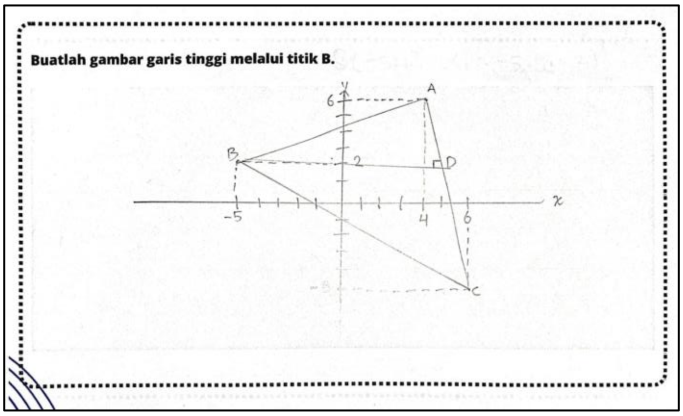

Figure 8. Results of Student's Answer in LKPD 2

Through these activities, students were able to demonstrate a deeper understanding of how to determine the equation of a line perpendicular to another line, reinforcing their grasp of the geometric and algebraic concepts involved. The teaching experiment showed that the revised HLT effectively supported students in developing a relational understanding of the equation of a straight line.

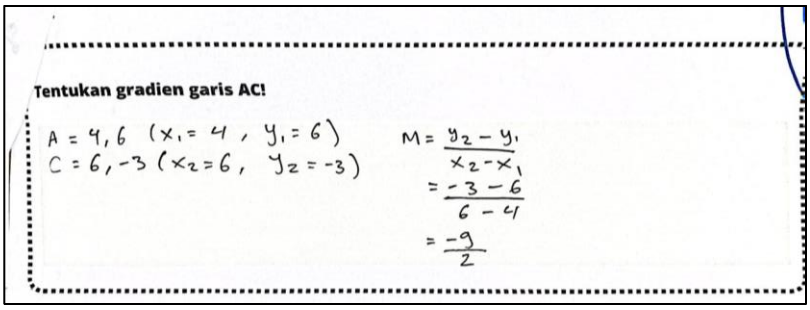

Figure 9. Results of Student's Answer Number 2

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In the second question, depicted in Figure 9, learners were required to determine the gradient of line AC. The students accurately calculated the gradient of line AC using points A and C, arriving at the correct value of $\frac{-9}{2}$.. This step is crucial for relational understanding, as it involves linking the gradient concept to the subsequent calculation of the line's equation. The gradient of line AC is then used to find the equation of line BD, as shown in the next question.

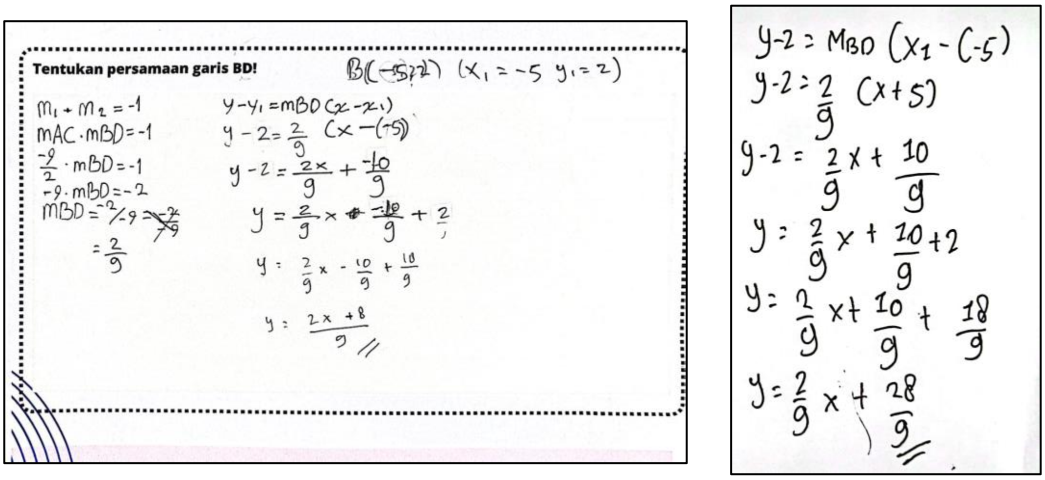

Figure 10. Results of Student's Answer Number 3

In the third question, illustrated in **Figure 10**, learners were asked to determine the equation of line BD, which is the altitude given in the problem. Since line BD is perpendicular to line AC, the gradient relationship $mAC \times mBD = 1$is used, leading to the calculation $mBD = \frac{2}{9}$. Subsequently, students used this gradient to find the equation of line BD, resulting $y = \frac{2x+8}{9}$ Similar to the first activity, while learners generally used the correct formulas, there were differences in the precision of their calculations. For instance, one learner made an operational error in the expression $\frac{2}{9}(x - (-5)$where the correct form should be $\frac{2}{9}x + \frac{10}{9}$. Despite this, students overall demonstrated all the indicators of relational understanding: performing detailed procedures, obtaining correct results, using appropriate formulas, and linking various concepts effectively.

4. DISCUSSION

Impact of the RMT Approach on Student Engagement and Motivation

To effectively explore the impact of the RMT approach on student engagement and motivation, it is essential to focus on how this approach influences students' behavior during the learning process. The RMT approach, which involves the systematic development of mathematical concepts through stages like cognitive development and cognitive conceptual construction, aims to deepen students' understanding and enhance their cognitive abilities (Ramadhani, 2023;Firmasari et al., 2022). This structured approach allows students to engage more meaningfully with mathematical concepts, which can lead to increased enthusiasm and participation in the classroom (Abdurrahman et al., 2021). Observations have shown that when students are actively involved in such cognitively demanding tasks, they demonstrate a higher level of curiosity and engagement, which is crucial for fostering a positive learning environment (Doño & Mangila, 2021). Student engagement in mathematics is not solely determined by the instructional method but also by other factors such as the characteristics of the learners, the quality of instruction, and the dynamics of peer interactions (Torrejos, 2024). The RMT approach, by emphasizing problem-solving and the connection of various mathematical concepts, supports students in developing a deeper understanding of the material. This understanding, in turn, contributes to their motivation to engage with the subject matter, as they begin to see the relevance and application of what they are learning. The structured nature of RMT, which contrasts with more traditional methods that often emphasize rote memorization, helps students develop a more meaningful connection with the material, thereby enhancing their intrinsic motivation (Delima, 2017).

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The long-term effects of the RMT approach on students' motivation are also noteworthy. Research indicates that when students engage deeply with mathematical concepts and are encouraged to link new knowledge with their existing understanding, they develop a more sustained interest in mathematics (Pratiwi et al., 2022). The ability to make these connections is a key component of relational understanding, which is critical for success in more advanced mathematical studies. The RMT approach's focus on fostering these connections can lead to lasting improvements in students' motivation, as they continue to build on their understanding and develop a genuine interest in mathematics that extends beyond specific topics like line equations (Lazarides et al., 2018). The RMT approach has a significant impact on students' engagement and motivation by enhancing their conceptual understanding, fostering cognitive development, and supporting problem-solving skills. The structured and meaningful nature of this approach not only helps students in their immediate learning but also contributes to long-term motivation and interest in mathematics. These findings underscore the importance of adopting teaching strategies that go beyond surface-level learning and encourage deeper cognitive engagement, ultimately leading to more effective and lasting educational outcomes.

Challenges and Limitations of Implementing the RMT Approach in Diverse Classroom Settings

When implementing the RMT approach in diverse classroom settings, addressing the varied learning paces of students and ensuring teacher preparedness are critical challenges. Diverse learning paces among students can create difficulties in maintaining engagement, as the rigorous nature of the RMT approach may overwhelm some students while others may grasp concepts more quickly. This disparity can lead to frustration or disengagement, which can hinder the effectiveness of the approach (Hendrayana, 2017). To manage these differences, teachers must employ differentiated instruction strategies that cater to the needs of all students, offering extra support where necessary and providing extension activities for advanced learners (Kinard & Kozulin, 2008).

Teacher preparedness and training also play a crucial role in the successful implementation of RMT. Transitioning from traditional to more inquiry-based and conceptual teaching methods requires a significant shift in teaching practices and mindset. Teachers accustomed to rote memorization may struggle to adapt to the demands of RMT, which emphasizes deep conceptual understanding and critical thinking (Hidayat et al., 2021). Specific challenges include restructuring lesson plans to facilitate meaningful discourse and assessing student understanding beyond surface-level knowledge (Bakar,2019, Tillery et al., 2009). Comprehensive training programs focusing on RMT principles are essential to equip teachers with the necessary skills and confidence to effectively implement this approach in diverse educational contexts(Mefiana & Juandi, 2023). Addressing the challenges of diverse learning paces and ensuring that teachers are well-prepared and trained are vital steps in successfully integrating the RMT approach into classrooms. By acknowledging these challenges and providing adequate support and training, schools can create an environment that fosters deep mathematical thinking, enhances student engagement, and ultimately improves learning outcomes.

5. CONCLUSION

The learning trajectory produced in this study is in the form of a learning process trajectory or learning experience that students have gone through to achieve each learning objective, which consists of two activities. First activity aims to solving problems related to the equation of a line through two points and the second activity aims to solving problems related to the equation of a line perpendicular to a line. Based on the results of the study, it can be concluded that the learning design of equation of a straight line that has been designed and tested is able to support students to understand the material and encourage students to use relational understanding during the learning process. Through the activities given in the LKPD, it can help students in solving problems as a whole, be able to use methods that are appropriate to the problem, be able to recognise new forms of problems using the same method, and obtain the right results.

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