



Preservice Mathematics Teachers' Perceptions of School-Related Content Knowledge Tasks

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ABSTRACT

School-related Content Knowledge (SRCK) encompasses profession-specific content knowledge essential for teaching secondary mathematics and bridging the gap between university and school mathematics. Preparing preservice teachers with this knowledge is vital for effective teaching. This study explores the perceptions and performance of preservice mathematics teachers regarding SRCK tasks. Using a qualitative approach and case study design, the research involved twenty third-semester preservice mathematics teachers selected through convenience sampling at a state university in Yogyakarta. Instruments included tests, questionnaires, and interviews to assess SRCK. Data analysis involved content analysis, inductive category formation, thematic categorization, and data triangulation. The findings reveal that preservice mathematics teachers consider SRCK tasks relevant and beneficial for their professional development. However, differences emerge in their evaluation of task relevance and realism, highlighting the need to incorporate more university-level mathematical concepts into SRCK tasks to bridge the gap between theory and practice. Despite recognizing the importance of university mathematics, there is a tendency to rely predominantly on school-level mathematics knowledge. This suggests that teacher preparation programs must address the challenge of mastering more complex university-level concepts to ensure that future educators are adequately prepared to teach advanced mathematical ideas. Varying levels of success in solving SRCK tasks underscore the necessity for a comprehensive integration of university-level mathematical concepts. Such integration aligns with efforts to enhance the relevance of university mathematics education for aspiring teachers. It is recommended that university mathematics be blended with school mathematics and future professional roles to improve conceptual understanding and teaching readiness.

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1. INTRODUCTION

Teacher education programs in mathematics require aspiring educators to develop a deep and flexible understanding of mathematical content tailored for secondary school instruction. This is essential for preparing teachers to convey mathematical concepts effectively, bridging the gap between school-level and university-level mathematics and ensuring their readiness for future educator roles (Darling-Hammond, 2020; Ball & Forzani, 2009). Therefore, teacher candidates must take university mathematics courses, including number theory, abstract algebra, and real analysis (Pramasdyahsari et al., 2019; Wasserman, 2016, 2018). Research indicates that preservice teachers often struggle to relate university mathematics to teaching in secondary school (Agustyaningrum et al., 2018; Malambo, 2020, 2021; Morali & Filiz, 2023). Specific challenges include applying abstract concepts in a classroom setting, connecting advanced theories to essential school-level topics, and translating symbolic language into understandable lessons for students. Some believe that advanced mathematical content knowledge is irrelevant to classroom practice (Even, 2022; Zazkis & Leikin, 2010) and not necessary for teaching advanced mathematics, which exceeds what school teachers typically teach (Allmendinger, 2016; Wasserman, 2016). This perspective can result in shallow instruction, limiting students' ability to grasp complex concepts and prepare for higher education. As a result, some preservice teachers lack

proficiency in certain content areas (Cofer, 2015; Wasserman, 2016). The disparity between university mathematics courses and their practical application in school teaching is evident.

A key difference lies in the content focus: school mathematics emphasizes real-life applications and understanding reality, often introduced through real-world experiences and contexts using inductive methods and prototypes (Bromme & Steinbring, 1994; Wu, 2011). University mathematics follows an axiomatic-deductive structure, prioritizing formal proofs, abstract concepts, and symbolic language (Tall, 1992; Wu, 2011). This gap underscores the need for teacher education programs to integrate university-level content with school-level teaching practices better, ensuring that future educators can bridge theoretical knowledge and classroom application (Blömeke et al., 2016; Tatto et al., 2012). As the Association of Mathematics Teacher Educators (AMTE, 2017) emphasized, preservice teachers need more than robust content knowledge. They require comprehensive training to address specific challenges in applying this knowledge in the classroom. These challenges include bridging the gap between abstract university-level mathematics and practical school-level applications, managing diverse student needs, and using pedagogical strategies to convey complex concepts (Ball et al., 2008; Tatto et al., 2012). Addressing these challenges in teacher education programs is crucial for preparing effective educators.

One potential solution is the development of professional development programs designed to help teachers connect university mathematics with school curricula. These programs could focus on practical applications of advanced theories in everyday teaching, demonstrating how concepts like group theory or linear algebra can be simplified and made relevant for school topics such as identity and inverse functions. When teaching university mathematics, lecturers should recognize the connections between school mathematics and university mathematics, understanding that this awareness will influence their teaching as secondary school educators and positively impact student learning (Murray et al., 2017). Preservice mathematics teachers require a particular type of mathematical Content Knowledge (CK) to establish meaningful connections between university mathematics and school mathematics. Outlining the specific knowledge essential for secondary school mathematics teachers is crucial. Within the domain of university mathematics and the learning undertaken by mathematicians, a category of Content Knowledge (CK) tailored for secondary school teachers is conceptualized and referred to as SRCK (Woehlecke et al., 2017). SRCK represents a distinctive form of mathematics CK designed for teaching school mathematics.

SRCK addresses essential aspects of teacher knowledge, including the connection between school and university mathematics. By highlighting these connections, SRCK establishes a strong foundation for teachers to apply university-level mathematics in teaching school mathematics, facilitating a deeper understanding of relevant content. The SRCK framework, conceptualized by Dreher et al. (2018), includes three components: curricular knowledge, top-down direction, and bottom-up direction. Figure 1 illustrates the SRCK conceptualization, encompassing these three elements and their interrelations with content knowledge in school and university mathematics.

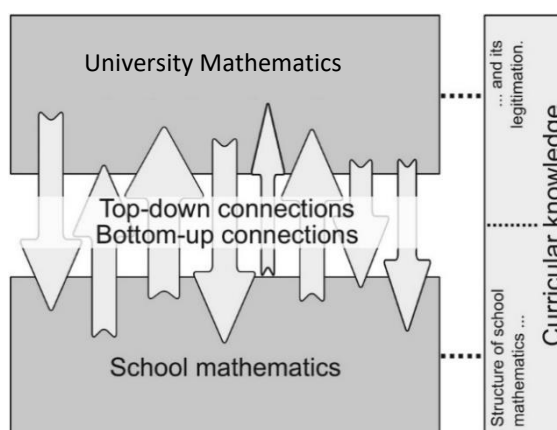


Figure 1. SRCK conceptualization (Dreher et al., 2018)

Curricular knowledge involves understanding the organization of school mathematics and its foundational legitimacy from the perspective of university mathematics (Weber et al., 2023). This knowledge helps bridge the

gap between university and school mathematics, enabling teachers to align advanced mathematical theories with basic concepts taught in schools. It creates a cohesive learning progression, ensuring students build a solid foundational understanding supporting their future studies. Additionally, it empowers teachers to introduce advanced topics in a simplified manner, fostering a deeper appreciation and comprehension of mathematics among students. For example, teachers use curricular knowledge to determine which ideas can explain mathematical concepts (such as infinity) in a given class. They consider previously covered concepts and anticipate those that may follow, ensuring a logical and progressive learning experience.

Top-down knowledge involves recognizing relationships originating from university mathematics. It helps preservice teachers understand and explain complex ideas, connecting them to students' existing knowledge. This approach enhances teaching by improving comprehension and engagement. For example, understanding top-down relationships is crucial when introducing and adapting specific mathematical concepts for instructional purposes. Research has shown how algebraic limit theorems for sequences in real analysis can inform a teacher's response to using rounded numbers in basic equations (Wasserman & Weber, 2017).

Bottom-up relationships become relevant when examining elements of school mathematics in connection to university mathematics. For instance, a student questioned why $a^0 = 1$ for all nonzero real values of a . *he student reasoned* that a^0 means a multiplied by itself zero times, which they thought should result in zero. The formal $\epsilon - \delta$ definition of continuity to explain why $a^0 = 1$ makes sense, as the function $f(x) = a^x$ is continuous for all real (Wasserman, 2024). This example started with a school-level context and further explored related university-level content, showing a reverse connection from school to university concepts.

This study will explore preservice mathematics teachers' perceptions and performance in completing SRCK tasks. Specifically, it aims to investigate their views on SRCK tasks, their value of applying mathematical concepts, their attitudes toward university mathematics, and their approaches to solving SRCK tasks. It will examine how preservice teachers perceive the significance of SRCK tasks concerning their future careers and assess the extent to which they value applying mathematical concepts from university and school in task-solving. Additionally, the study seeks to uncover preservice teachers' attitudes toward concepts typically encountered in university mathematics. Furthermore, it will analyze how these teachers approach SRCK tasks, drawing upon their understanding of mathematical principles from school and university education.

Perceptions and performance in SRCK tasks can significantly impact teaching effectiveness, as a strong appreciation for SRCK tasks and the application of mathematical concepts can enhance instructional strategies and student engagement. The broader implications for mathematics education include informing teacher training programs, improving curriculum design, and elevating the quality of mathematics education by ensuring teachers are well-prepared and confident in their subject matter. This exploration provides insights into the preparation and readiness of preservice mathematics teachers for their future roles in education. This study contributes to the field of mathematics education by highlighting key competencies and areas for improvement, thereby informing teacher training programs and educational policies.

2. MATERIAL AND METHOD

Research Design

The study uses a qualitative research approach with a case study design. Qualitative research is chosen for its ability to generate feedback data related to group perceptions, beliefs, and experiences (McDuffie & Scruggs, 2008). The case study design is employed due to its capacity to provide detailed and in-depth analyses, which contribute to educational development (Nieveen & Folmer, 2013). This research method is selected to explore the perspectives and abilities of aspiring mathematics teachers in addressing SRCK tasks.

Participant

Twenty third-semester students from the Mathematics Education program at a state university in Yogyakarta participated in this study. These participants were identified as preservice mathematics teachers. The inclusion criteria were students who had completed core university-level mathematics courses such as number theory, calculus, and discrete mathematics. Students who had yet to complete these core courses or had limited experience studying mathematics were excluded to strengthen the generalization of the research findings.

Participants were selected using a convenience sampling technique, which involves choosing participants easily accessible to the researcher. This method was chosen for its practicality and efficiency in reaching many participants within a limited timeframe. However, this technique can introduce bias and lack broad applicability

because the selected participants may not represent the entire population. To address this, efforts were made to include participants from diverse backgrounds and demographics to enhance the sample's representativeness. All response sheets were collected and meticulously examined. Interviews were conducted with some participants based on their answers and ability to communicate their responses. This selection allowed for an in-depth exploration of critical themes and provided a comprehensive understanding of the research topic. The interviews were crucial for data triangulation and validation by cross-verifying the information gathered through other methods.

Instrument

The study utilized tests, questionnaires, and interviews as instruments. Ensuring the reliability and validity of these instruments involved rigorous validation processes conducted by experts. The test evaluated preservice mathematics teachers' ability to solve three SRCK-related problems. **Table 1** outlines the SRCK components and the associated questions used in the study.

Table 1. SRCK Task (Weber et al., 2023)

Component	Problem	Description						
Bottom-up knowledge (SRCK Problem 1)	<p>Observe how Alex, Jordan, and Kelly find the inverse function of $f(x) = \frac{2}{3}x + 1$</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <th>Alex's Work</th> <th>Jordan's Work</th> <th>Kelly's Work</th> </tr> <tr> <td> $y = \frac{2}{3}x + 1$ $x = \frac{3}{2}y - 1$ $x - 1 = \frac{3}{2}y$ $\frac{x-1}{\frac{3}{2}} = y$ $\frac{2}{3}(x-1) = y$ </td> <td> $f \circ f^{-1}(y) = y$ $f(y) = \frac{2}{3}y + 1$ So: $f(f^{-1}(y)) = \frac{2}{3}f^{-1}(y) + 1 = y$ $\frac{2}{3}f^{-1}(y) = y - 1$ $\frac{2}{3} \cdot \frac{3}{2}f^{-1}(y) = \frac{3}{2}(y-1)$ $f^{-1}(y) = \frac{3}{2}(y-1)$ </td> <td> $y = \frac{2}{3}x + 1$ $y - 1 = \frac{2x}{3}$ $3(y-1) = 2x$ $\frac{3(y-1)}{2} = x$ But: $f(x) = y \Rightarrow f^{-1}(y) = x$ $x = f^{-1}(y)$ So $f^{-1}(y) = \frac{3}{2}(y-1)$ </td> </tr> </table> <p>Compare and contrast the main mathematical ideas used by Alex, Jordan, and Kelly to find the inverse function of $f(x) = \frac{2}{3}x + 1$. Be sure to identify which properties of inverse functions each student used, if any.</p>	Alex's Work	Jordan's Work	Kelly's Work	$y = \frac{2}{3}x + 1$ $x = \frac{3}{2}y - 1$ $x - 1 = \frac{3}{2}y$ $\frac{x-1}{\frac{3}{2}} = y$ $\frac{2}{3}(x-1) = y$	$f \circ f^{-1}(y) = y$ $f(y) = \frac{2}{3}y + 1$ So: $f(f^{-1}(y)) = \frac{2}{3}f^{-1}(y) + 1 = y$ $\frac{2}{3}f^{-1}(y) = y - 1$ $\frac{2}{3} \cdot \frac{3}{2}f^{-1}(y) = \frac{3}{2}(y-1)$ $f^{-1}(y) = \frac{3}{2}(y-1)$	$y = \frac{2}{3}x + 1$ $y - 1 = \frac{2x}{3}$ $3(y-1) = 2x$ $\frac{3(y-1)}{2} = x$ But: $f(x) = y \Rightarrow f^{-1}(y) = x$ $x = f^{-1}(y)$ So $f^{-1}(y) = \frac{3}{2}(y-1)$	<p>These tasks illustrate the preservice teachers' understanding of inverse functions, bridging the gap between procedural knowledge and theoretical understanding and enhancing their preparedness to explain and apply mathematical principles in a classroom setting.</p>
Alex's Work	Jordan's Work	Kelly's Work						
$y = \frac{2}{3}x + 1$ $x = \frac{3}{2}y - 1$ $x - 1 = \frac{3}{2}y$ $\frac{x-1}{\frac{3}{2}} = y$ $\frac{2}{3}(x-1) = y$	$f \circ f^{-1}(y) = y$ $f(y) = \frac{2}{3}y + 1$ So: $f(f^{-1}(y)) = \frac{2}{3}f^{-1}(y) + 1 = y$ $\frac{2}{3}f^{-1}(y) = y - 1$ $\frac{2}{3} \cdot \frac{3}{2}f^{-1}(y) = \frac{3}{2}(y-1)$ $f^{-1}(y) = \frac{3}{2}(y-1)$	$y = \frac{2}{3}x + 1$ $y - 1 = \frac{2x}{3}$ $3(y-1) = 2x$ $\frac{3(y-1)}{2} = x$ But: $f(x) = y \Rightarrow f^{-1}(y) = x$ $x = f^{-1}(y)$ So $f^{-1}(y) = \frac{3}{2}(y-1)$						
Curricular knowledge (SRCK Problem 2)	<p>There is a problem as follows.</p> <div style="border: 1px solid black; padding: 5px;"> <p>A student will measure the height of a tree that is $4\sqrt{3}$ meters away from them. An angle of elevation of 30° is formed between the student's eyes and the top of the tree. If the height of the student measured up to their eyes is 1.6 meters, what is the height of the tree?</p> </div> <ol style="list-style-type: none"> How would you solve this problem if you were a high school student? How would you solve this problem if you were a middle school student? If the problem cannot be solved for middle school or high school students, what needs to be added to the problem to make it solvable for middle school or high school students? What concept would you use? (Note: you are not allowed to add the distance from the child to the tree). 	<p>Tasks involving the measurement of a tree's height using trigonometric concepts, foundational in both school and university math.</p>						
Top-down knowledge (SRCK Problem 3)	<p>In a lecture, there is a theorem as follows. For every natural number n, it is calculated:</p> $1 + 2 + \dots + n = \frac{n \cdot (n + 1)}{2}$ <p>and prove it by induction. Basically, $1 + 2 + 3 + \dots + n$ is a series of natural numbers. If this problem is given in school, express this theorem to middle school students in a concrete way to make it more understandable. You can use numbers or diagrams to make this theorem make sense.</p>	<p>Tasks demonstrating how to convey advanced mathematical concepts, such as the summation of natural numbers, to middle school students.</p>						

Each problem was designed to assess different aspects of SRCK, evaluating candidates' readiness for future roles in mathematics education. The problems aimed to measure their understanding and application of relevant mathematical concepts, problem-solving skills, and ability to apply theoretical concepts from school and university mathematics.

A questionnaire was developed using a 4-point Likert scale to evaluate student performance, following the approach outlined by Allen and Seaman (2007). The questionnaire included 11 items adapted from Hermans (2021) to gauge the perceived relevance of tasks. Separate questionnaires were also created to assess the perspectives of preservice mathematics teachers on university and school mathematics. These questionnaires aimed to capture participants' views, opinions, and insights on the nature and relevance of mathematics at both

levels. The interview questions were based on the participants' test results and questionnaire responses. These semi-structured interviews aimed to gather additional information and clarify understanding, ensuring the research results were comprehensive and precise. The interview process encouraged informal, comfortable, and stress-free communication. Interview questions focused on key themes, such as participants' comprehension of concepts and the relevance of university-level mathematical concepts to their future teaching roles. For example, participants were asked: "Do you comprehend the concepts? Did university-level mathematical concepts assist you in completing tasks? Why?" and "Do you think the mathematics you are studying at the university (such as calculus, number theory, algebraic structures, etc.) plays a role in your future profession as a teacher? Why?" These carefully crafted questions, validated by experts, ensured that the interviews effectively gathered detailed and relevant information, enhancing the overall precision and depth of the research findings.

Data Analysis

The data analysis in this study aimed to explore the viewpoints and capabilities of aspiring mathematics teachers concerning SRCK tasks. The analysis began with initial data processing, which included transcribing interviews, converting questionnaire responses, and coding SRCK task answers. Each instrument was coded at an appropriate level, enabling the identification of emerging findings and patterns. Thematic categorization organized responses into predefined themes based on the SRCK framework. Multiple coders worked on the data, resolving discrepancies through discussion to enhance validity and reliability. Data triangulation further validated the findings by comparing and contrasting tests, questionnaires, and interview results. These key steps—coding, thematic categorization, and data triangulation—ensured a comprehensive understanding of participants' perspectives and minimized potential biases. Despite limitations such as potential subjectivity in coding and limited generalizability, the study provided valuable insights into the SRCK-related competencies of aspiring mathematics teachers, contributing to mathematics education research.

3. FINDINGS

This section explores how preservice teachers perceive the significance of SRCK tasks concerning their future careers. It examines how much they value applying mathematical concepts from university and school in task-solving. Additionally, it investigates their attitudes toward concepts found in university-level mathematics. Lastly, it analyzes how they solved the SRCK tasks, drawing on their understanding of mathematical principles acquired in school and university mathematics.

Student Evaluation of SRCK Task

A questionnaire was administered to understand preservice teachers' perceptions of the significance of SRCK tasks in their future careers. The results are shown in [Table 2](#).

Table 2. Student evaluation of SRCK Task

Indicator	Task 1	Task 2	Task 3
	M (SD)	M (SD)	M (SD)
Importance for Future Profession	3.26 (0.73)	3.47 (0.51)	3.47 (0.51)
Realistic Task	3.21 (0.71)	3.37 (0.68)	3.32 (0.67)
Relevance to Teaching Issues in Schools	3.33 (0.49)	3.33 (0.49)	3.39 (0.50)
Enhancement of Communication Skills	2.95 (0.85)	2.95 (0.78)	3.05 (0.85)
Challenge in Simplifying Mathematical Concepts	3.37 (0.60)	3.26 (0.56)	3.32 (0.58)

Strongly agree = 4; agree = 3; disagree = 2; strongly disagree = 1.

An essential criterion for evaluating relevance involves aligning with the preservice profession, focusing on explicit references to the teaching context. The indicators "Importance for Future Profession" and "Relevance to Teaching Issues in Schools" averaged 3.26 to 3.47. This suggests that foundational mathematical concepts should be taught and applied explicitly in university mathematics to enhance their relevance to future careers. The indicator "Enhancement of Communication Skills" received a low rating, which may be due to some preservice mathematics teachers misunderstanding the meaning of communication, encompassing both oral and written forms. The online teaching format also limited direct communication opportunities, making it challenging for preservice teachers to express their mathematical ideas and concepts clearly. The average score for the "Realistic

Task" indicator was also low compared to the others, ranging from 3.21 to 3.37. This suggests that these tasks may not be well connected to real-world teaching situations.

Evaluation of the Usefulness of SRCK Tasks

Preservice mathematics teachers also assessed the usefulness of SRCK tasks through a questionnaire. **Table 3** shows their evaluations of using university and school mathematical concepts in completing SRCK tasks.

Table 3. Preservice teacher evaluation of the usefulness of SRCK tasks

Indicator	Task 1 M (SD)	Task 2 M (SD)	Task 3 M (SD)
University-Level Mathematical Skills Required	3.37 (0.60)	3.32 (0.67)	3.37 (0.68)
High School Knowledge Required	3.63 (0.60)	3.68 (0.58)	3.58 (0.61)
Alignment with School-Level Mathematics	3.50 (0.51)	3.58 (0.51)	3.53 (0.61)
Support for Understanding School-Level Concepts	3.37 (0.60)	3.42 (0.61)	3.42 (0.61)
Bridging University and School-Level Mathematics	3.42 (0.69)	3.42 (0.69)	3.32 (0.67)

Strongly agree = 4; agree = 3; disagree = 2; strongly disagree = 1

Preservice mathematics teachers evaluated the usefulness of SRCK tasks by considering their application of mathematical concepts. They highlighted the foundational importance of concepts like inverse functions, trigonometry, Pythagoras' theorem, and mathematical induction in completing the given tasks. The evaluation revealed a significant reliance on high school-level knowledge rather than university-level skills. This is reflected in the average scores for the "High School Knowledge Required" indicator, which ranged from 3.58 to 3.63. In contrast, the scores for applying university-level mathematical knowledge ranged from 3.32 to 3.37.

One of the preservice teachers made the following statement during an interview:

"Yes, I understand the mathematical concepts used to complete the task. Concepts such as inverse functions, inverse operations, rules of inverse functions, composition, substitution, and equation solving, as well as topics like trigonometry, Pythagoras' theorem, arithmetic series, and mathematical induction, serve as the foundation for preservice mathematics teachers when tackling tasks involving function calculations, finding the inverse of functions, and understanding basic mathematical operations."

This statement showed that preservice mathematics teachers tend to rely more on the mathematical knowledge acquired at the school level to complete these tasks. The evaluation concerns the value of applying mathematical concepts from both university and school in task-solving. Preservice mathematics teachers needed to consistently assess which basic concepts they used to complete the tasks. This suggests a gap in transitioning from high school to university-level mathematical thinking, highlighting the need for curricular adjustments to better bridge this gap and enhance the preparedness of preservice teachers for advanced mathematical tasks.

Attitudes Towards University and School Mathematics

Preservice mathematics teachers were given a questionnaire to assess their views on university and school mathematics. The assessment results are presented in **Table 4**.

Table 4. Evaluation of preservice teacher attitudes towards university and school mathematics

Statement	Mean	SD
Correlation between University and School Mathematics	3.23	0.72
Improvement of Correlation through Mathematics Education	2.60	1.03
Encouragement to Think Ahead	3.42	0.65
Relevance of University Mathematics to Teaching Activities	3.37	0.68

Strongly agree = 4; agree = 3; disagree = 2; strongly disagree = 1

Preservice teachers generally perceive a weak correlation between university and school mathematics, as reflected in an average score of 3.23 with a standard deviation of 0.72. Views on improving this correlation

through mathematics education show more variability, with an average score of 2.60 and a standard deviation of 1.03. Preservice teachers agree that mathematics courses encourage them to think ahead of their students, indicated by an average score of 3.42 and a standard deviation of 0.65. They also consider the relevance of university mathematics to teaching activities significant, with an average score of 3.37 and a standard deviation of 0.68, showing a relatively stable consensus among respondents.

Most preservice mathematics teachers (15 out of 19) believe that concepts taught in school mathematics are relevant to university mathematics. However, these basic concepts are only sometimes explicitly taught and applied. Some of their opinions on the relevance of university mathematics for their future teaching profession include:

"It is very important because to teach, one must understand the concepts and be able to convey them."

"It is important because the course delves deeper into the subjects taught in school, making it relevant to future professions as a teacher."

Some preservice teachers, however, argue that university-level mathematics is less relevant, expressing views such as:

"Teachers should have a deeper understanding of the subject matter they teach."

"The student's level of understanding is below all the material mentioned."

"It needs to be clarified how much it helps."

In conclusion, the survey results in Table 4 reveal mixed perspectives among preservice mathematics teachers regarding the relevance of university-level mathematics education to their future roles as educators.

Results of SRCK Tasks

The preservice mathematics teachers' solutions to the SRCK tasks were analyzed using qualitative content analysis, as outlined by Kuckartz (2012). The coding process for the solutions encompassed three categories: predominantly incorrect, partially correct, and predominantly correct. "Predominantly incorrect" indicates that the solutions have more inaccurate or incomplete responses than accurate ones. "Partially correct" means that the solutions include more accurate answers than inaccurate ones or have only minor errors. "Predominantly correct" denotes that the solutions are primarily precise and comprehensive. The outcomes of this analysis are presented in Table 5.

Table 5. The results of SRCK task

Rating	Bottom Up	Curricular knowledge			Top Down
		a	b	c	
Predominantly correct	0%	42%	21%	11%	11%
Partly correct	47%	11%	0%	5%	11%
Predominantly wrong	53%	47%	79%	84%	79%

In SRCK Problem 1, the bottom-up task, many preservice teachers provided partially correct solutions (47%), successfully identifying and comparing inverse properties and demonstrating competence in recognizing mathematical concepts. In SRCK Task 3, the top-down task, significant weaknesses were evident. Most responses (79%) were predominantly incorrect, indicating a substantial lack of comprehension in addressing problems that integrate school- and university-level mathematics. Only a small portion (11%) of responses were predominantly correct, highlighting the need to improve understanding and application of top-down knowledge.

4. Discussion

Perception of SRCK Tasks and Future Occupational Paths

The results indicate that preservice mathematics teachers generally perceive SRCK tasks as relevant and beneficial. However, differences in assessing task relevance and realism highlight the need to integrate university-level mathematics with school-level applications better. This finding aligns with previous research, which also found that SRCK questions were ineffective in improving communication skills among preservice

mathematics teachers (Hermanns, 2021; Hermanns & Ermler, 2021). The indicator for realistic tasks also received low scores.

SRCK tasks, referred to as School-Related Mathematics Problems (SRMPs) by Weber and Lindmeier (2022), do not always meet the criterion of being realistic. Preservice mathematics teachers often find problems unrealistic if they cannot recall similar content from their experiences, indicating a low personal association. This obstacle to perceived relevance was significantly highlighted during interviews and has yet to be extensively covered in studies on physics issues (Massolt & Borowski, 2018, 2020). Incorporating real-world scenarios like classroom management and curriculum design is crucial to address this. Emphasizing practical applications of theoretical concepts and integrating technology can bridge the gap between theory and practice. Using case studies and simulations to mimic real classroom challenges and promoting reflective practice can enhance the preparedness and effectiveness of future educators.

Value of Applying Mathematical Concepts from University and School in Task-Solving

The evaluation focuses on the significance of applying mathematical concepts from both university and school in task-solving. Preservice teachers tend to depend more on mathematical knowledge acquired at the school level to complete tasks. Statements about using university-level mathematical knowledge scored lower, indicating a preference for high school-level mathematical skills in addressing tasks. As a result, the solutions may not incorporate or require the application of higher-level mathematical concepts taught at the university level, thus not fully supporting the understanding of school-level mathematics.

Research suggests that preservice mathematics teachers often lack proficiency in certain university-level mathematical topics (Agustyaningrum et al., 2019; Malambo, 2020; Malambo, 2021; Morali & Filiz, 2023). Some preservice teachers mentioned in interviews that, despite having a strong foundation from school, they encountered significant disparities in the complexity and depth of mathematical content at the university level, realizing the need for a deeper understanding and application of more advanced concepts.

Ensuring preservice teachers can transfer university-level mathematical knowledge to school-level problems is crucial for preparing confident educators. Many rely on high school-level knowledge, leading to less effective solutions due to a lack of proficiency in advanced topics. Teacher education programs should integrate advanced concepts into practical tasks, support reflective practice, and enhance communication skills through interactive methods and blended learning. Incorporating realistic school contexts into university courses with case studies and simulations and focusing on transferable skills through problem-solving workshops will be beneficial. Regular assessments with detailed feedback will help address misconceptions, ensuring a stronger connection between theoretical knowledge and teaching practice. Enhancing curricular design and targeted support will equip preservice teachers to bridge the gap between university and school-level mathematics, improving their proficiency and confidence in teaching.

Attitudes Towards University-Level Mathematics Concepts

Preservice mathematics teachers have a mixed perspective regarding the role of university-level mathematics concepts. While most preservice mathematics teachers acknowledge the relevance of concepts learned in school mathematics to university mathematics, views differ on the explicit teaching and application of these basic concepts in the university setting. Some preservice mathematics teachers emphasize the importance of university mathematics in sharpening problem-solving skills and preparing them for their future profession as teachers. They recognize the depth of university-level courses and their relevance to their preservice careers, stressing the importance of understanding mathematical concepts comprehensively for effective teaching. In contrast, there are dissenting opinions suggesting that some aspects of university-level mathematics may be perceived as irrelevant or too advanced for the role of a teacher (Cofer, 2015). Concerns are raised about the unclear impact of certain topics and the appropriateness of specific mathematical areas for teachers (Allmendinger et al., 2023; Wasserman, 2016). This diversity in perspectives underscores the need to refine and align university-level mathematics education with the practical needs of future educators. Clear communication and explicit integration of relevant concepts from school mathematics into university courses may address some of the concerns of preservice mathematics teachers, ensuring a more cohesive and beneficial transition from school to university mathematics education.

These attitudes impact their performance and confidence in using university-level concepts. When preservice teachers lack confidence in their university-level knowledge, they tend to rely on simpler, school-level

methods that they find more familiar and less intimidating. This reliance can lead to inadequate problem-solving strategies, as seen in the high percentage of incorrect responses. Reluctance to fully engage with more complex, university-level mathematics may stem from a lack of confidence or insufficient practice with these concepts. This affects their overall performance and diminishes their ability to apply advanced mathematical theories in practical scenarios. Enhancing confidence through targeted training and support in applying university-level mathematics in varied contexts can bridge this gap, leading to more accurate and effective problem-solving skills.

Approaches to Solving SRCK Tasks

The analysis of how preservice teachers approached solving SRCK tasks reveals varied levels of understanding and application of mathematical principles acquired in school and university settings. The data indicates that while preservice teachers perform better when dealing with familiar, school-level concepts (Curricular Knowledge), they struggle significantly with more abstract, university-level concepts (Top-Down). For instance, 42% of responses were correct in Curricular Knowledge, whereas 79% were incorrect in Top-Down approaches. This shows a reliance on familiar methods and confidence in school-level knowledge but a significant difficulty in applying abstract, university-level concepts. Additionally, the Bottom-Up approach, which involves building solutions from fundamental principles, showed moderate performance, with 47% of responses being partly correct. These findings highlight the need to integrate university-level concepts with practical problem-solving strategies better. Teacher education programs should focus on enhancing the application of university-level mathematical principles in real-world, school-level contexts through targeted practice, real-world problem-solving exercises, and continuous feedback to build confidence and competence.

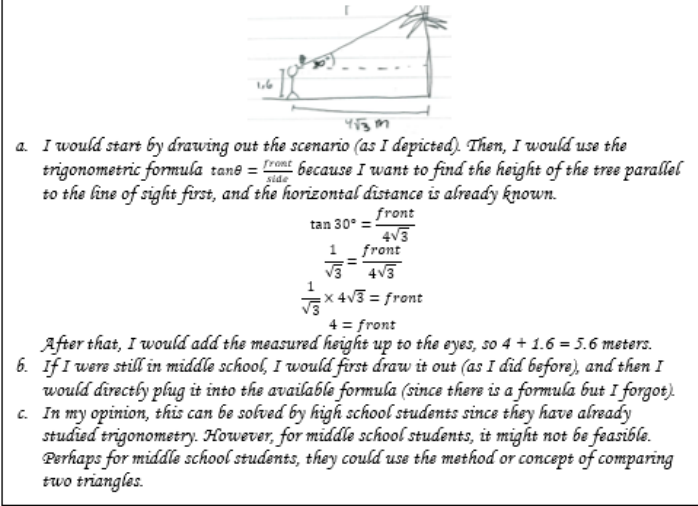
Since preservice teachers' future profession is secondary teaching, they must analyze student answers, as seen in SRCK's Problem 1. Solving SRCK's Problem 1 requires Bottom-Up knowledge. **Figure 2** shows an example of the answers of preservice mathematics teachers.

Alex	Jordan	Kelly
In Alex's work, he immediately applied the concept of inversion at the beginning by swapping x and y , so there was an immediate solution without considering other examples.	Jordan employs one composition with equations by replacing $f(x) = y$ with the composition $f(f^{-1}(y)) = y$. This is then transformed into another composition: $x = f^{-1}(y)$. From this formula, $x = f^{-1}(y)$ is obtained. Subsequently, the value of x is substituted back into $f(x)$.	In Kelly's approach, she first transforms it into $x = \frac{2(y-1)}{2}$ because the inverse of y is x . Therefore, the final result is the inverse of y . Based on the results presented in Table 5, most of the tasks were completed with partial correctness. Task number 1 tested bottom-up knowledge. The prospective mathematics teacher's answers to task 1 are shown in Figure 2.

Figure 2. Example answer of SRCK Problem 1 by preservice teachers

Figure 2 displays preservice teacher responses that have been assessed as partially correct. The analysis of responses assessed as partially correct, which constituted 47% of the total, indicates that while many students possess a foundational understanding of the tasks, they need help with complete and accurate application. Common misconceptions include the misapplication of inverse functions, as students might understand the concept but need to implement it correctly, thinking that inversion is merely a straightforward swap. Additionally, the complexity of function composition poses challenges; students often grasp the initial steps but need clarification during subsequent compositions or inversions. Incorrect initial transformations further contribute to errors, as seen in Kelly's approach, where students might understand the need for transformation but apply incorrect formulas. These misconceptions highlight the significance of SRCK Problem 1 in revealing deeper comprehension levels and pinpointing specific areas for improvement. The high percentage of partially correct responses underscores the importance of reinforcing foundational concepts and ensuring students can accurately execute mathematical processes. This suggests the need for enhanced instructional strategies and targeted practice.

SRCK Problem 2 requires Curricular Knowledge. It involves understanding the organization of school mathematics and its foundational validity as viewed from the standpoint of university-level mathematics (Weber et al., 2023). **Figure 3** shows an example of one preservice teacher's answer to SRCK Problem 2.



a. I would start by drawing out the scenario (as I depicted). Then, I would use the trigonometric formula $\tan \theta = \frac{\text{front}}{\text{side}}$ because I want to find the height of the tree parallel to the line of sight first, and the horizontal distance is already known.

$$\tan 30^\circ = \frac{\text{front}}{4\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{\text{front}}{4\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \times 4\sqrt{3} = \text{front}$$

$$4 = \text{front}$$

After that, I would add the measured height up to the eyes, so $4 + 1.6 = 5.6$ meters.

b. If I were still in middle school, I would first draw it out (as I did before), and then I would directly plug it into the available formula (since there is a formula but I forgot).

c. In my opinion, this can be solved by high school students since they have already studied trigonometry. However, for middle school students, it might not be feasible. Perhaps for middle school students, they could use the method or concept of comparing two triangles.

Figure 3. Example answer of SRCK Problem 2 by preservice teachers

The performance ratings for SRCK Problem 2 (**Table 5**) reveal essential insights into applying mathematical concepts from university and school education. These ratings underscore the importance of a balanced curriculum that ensures students learn mathematical concepts and apply them effectively in real-world contexts.

Part (a) in **Figure 3** shows that many preservice mathematics teachers provided accurate answers, relying on their memory of high school mathematics concepts. The subjects addressed during high school serve as the basis and are frequently revisited or expanded upon in university-level studies. However, some preservice teachers needed help with part (b) of the problem. Despite their familiarity with the high school curriculum, a subset of preservice teachers needed help to extend the problem-solving approach to a level appropriate for preservice junior high school teachers. They might approach the problem using concepts of comparison or unity but face challenges in identifying the elements needed to make it manageable for junior high school students. This observation highlights a gap in preservice teachers' ability to bridge the curricular knowledge between high and junior high school mathematics. It suggests that while preservice teachers may possess a solid grasp of advanced mathematical concepts, they may face challenges in tailoring their problem-solving strategies to align with students' understanding and proficiency levels at different educational stages. Addressing this gap is crucial for fostering effective communication and teaching practices that cater to diverse mathematical backgrounds and levels of expertise.

SRCK Problem 3 requires top-down knowledge that involves understanding the components of school mathematics in the context of the broader mathematical foundation at the university level.

Suppose we have consecutive positive integers from 1 to 5. Then, if we perform addition operations between these 5 numbers a total of $4(n - 1)$ times, the sum becomes 15 (the sum of the 5 consecutive positive integers). Now, let's create the formula or simplify it in the form of $\frac{n(n+1)}{2}$ where n is the last term or number in the sequence of positive integers. With this formula, we can directly calculate the sum without adding the consecutive positive integers from 1 to 5.

$$1 + 2 + 3 + 4 + 5 = \frac{5(5 + 1)}{2} = \frac{30}{2} = 15$$

Figure 4. Example answer of SRCK Problem 3 by preservice teachers

Addressing SRCK task 3, Top-Down Problem (**Figure 4**), preservice mathematics teachers demonstrated three distinct methods to explain the addition of natural numbers. These methods revealed varying levels of

mathematical understanding, impacting the overall proficiency of future teachers in both school and university contexts.

Some preservice teachers provided the formula for the sum of natural numbers without explaining its derivation. Such an approach might indicate a superficial understanding, posing challenges in university-level mathematics, where deeper comprehension and application of concepts are essential. Another method involved using an illustration, assuming the sum of initial natural numbers and multiplying the outcome by the total number of terms. Effective for certain problems in school settings, this method needs to have mathematical rigor and may hinder performance in university-level mathematics, where generalizing concepts is crucial.

A third approach focused on recognizing the pattern in the sequence of natural numbers and applying the formula for the sum of an arithmetic sequence. This method aligns well with mathematical concepts, fostering a more generalizable understanding. Emphasizing the underlying pattern prepares preservice teachers for success in more advanced mathematical studies at the university level. The choice of approach not only affects immediate performance in school and influences preparedness for advanced mathematical challenges at the university level. The arithmetic sequence approach demonstrates that prioritizing conceptual understanding contributes to a robust and transferable mathematical skill set. Foundational comprehension ensures a smoother transition from secondary to university-level mathematics, highlighting the importance of a deep understanding of concepts and their practical application.

Enhancing the problem-solving skills of preservice teachers requires educational practices that integrate knowledge from both educational levels. Teacher education programs should incorporate real-world problem-solving exercises that require the application of university-level concepts in school-level contexts. Such an approach bridges the gap between theoretical knowledge and practical application (Smith & Stein, 2018). Additionally, continuous feedback and opportunities for reflective practice help preservice teachers identify and address conceptual gaps (Darling-Hammond et al., 2017). Collaborative learning and peer-teaching strategies are also effective, allowing preservice teachers to learn from each other's strengths and perspectives (Johnson & Johnson, 2018). Incorporating technology, such as interactive simulations and virtual manipulatives, enhances understanding and engagement with complex mathematical concepts (Hughes et al., 2008). Integrating case studies and scenario-based learning provides preservice teachers with practical examples of how to apply mathematical principles in real-world teaching situations (Konsin). By implementing these educational practices, teacher education programs can better prepare preservice teachers to apply mathematical concepts effectively and confidently in their future classrooms.

5. CONCLUSION

Preservice mathematics teachers generally perceive SRCK tasks as relevant to their future profession, leading to positive outcomes. Despite variations in perception across specific items, overall, they find the tasks beneficial for their professional development. However, certain areas, particularly mathematical communication skills, need attention due to challenges in online learning. Differences emerge in how preservice mathematics teachers assess task relevance to teaching context and realism. Evaluations indicate a tendency to rely on school mathematics knowledge, emphasizing the need to incorporate more university-level concepts into SRCK tasks. Preservice mathematics teachers recognize the value of university mathematics in enhancing their teaching skills. Analysis of SRCK task solutions reveals varying degrees of success, highlighting the necessity for a more comprehensive infusion of university-level mathematical concepts. This approach aligns with efforts to enhance the applicability of university mathematics education for aspiring teachers in teacher education programs.

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