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by Samy Abdelmoezz

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New Mathematical Properties of the Kumaraswamy Lindley distribution

Samy Abdelmoezz [1*], and Salah M. Mohamed [2]

[1,2]Department of Applied Statistics and Econometrics, Faculty of Graduate Studies for Statistical Research (FSSR), Cairo University, Egypt.

*Corresponding author: samy_ez2010@yahoo.com

Abstract. The Kumaraswamy Lindley distribution is a generalized distribution that has many applications in various fields, including physics, engineering, and chemistry. so in This paper introduces a new mathematical properties for Kumaraswamy Lindley distribution (KLD) such as , probability weighted moments , moments of residual life, mean of residual life, reversed residual life, cumulative hazard rate function, mean deviation.

Keywords: probability weighted moments, moments of residual life, mean of residual life, reversed residual life, cumulative hazard rate, reversed hazard rate, mean divation

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1. Introduction

There are many researchers dealt with this type of similar distributions to the proposed Kumrsawamy Lindely distribution (KLD). Gauss and Castro (2009) have proposed a new family of generalized distributions. Elbatal et al. (2013)) presented a new generalized Lindley distribution. Çakmakyapan and Özel Kadılar (2014) proposed a new customer lifetime duration distribution for the Kumaraswamy Lindley distribution. Oluyede et al. (2015.) devoted a generalized class of Kumaraswamy Lindley distribution with applications to lifetime data. Riad et al. (2015) analyzed a log-beta log-logistic regression model. Cordeiro et al. (2017) presented the Kumaraswamy normal linear regression model with applications. Cakmakyapan, et al. (2017) presented the Kumaraswamy Marshall-Olkin log-logistic distribution with application. Vigas1 et al. (2017) presented the Poisson-Weibull regression model. Nofal et al. (2017) presented the transmuted Geometric-Weibull distribution and its regression model. Rocha et al. 2017) presented a Negative Binomial Kumaraswamy-G cure rate regression model. Handique et al. (2017) presented Marshall-Olkin-Kumaraswamy-G family of distributions Eissa (2017) presented exponentiated Kumaraswamy-Weibull distribution with application to real data. Elgarhy (2017) proposed Kumaraswamy Sushila distribution. Altun1 et al. (2018) presented a new generalization of generalized half-Normal distribution. Abed et al. (2018) proposed a new mixture statistical distribution Exponential – Kumaraswamy. Fachini-Gomes, et al. (2018) presented the Bivariate Kumaraswamy Weibull regression model. Arshad, et al. (2019) presented the gamma kumarsawmy-G familey distribution, theory, inference and applications. Mdlongwa et al. (2019) presented Kumaraswamy log-logistic Weibull distribution, model theory and application to lifetime and survival data. Pumi1 et al. (2020) presented Kumaraswamy regression model with Aranda-Ordaz link function. Safari et al. (2020) presented Robust reliability estimation for Lindley distribution, a probability integral transform statistical approach. Hafez et al. (2020) presented a study on Lindley Distribution accelerated life tests, application and numerical simulation. AlgarniID(2021 devoted a new generalized Lindley distribution properties, estimation and applications.

1.1 Definition Lindley Distribution. The Lindley distribution was introduced in 1958, but it was used as an alternative to the exponential distribution, where the Lindley distribution was used to study many characteristics such as data modeling and other characteristics.. In this section, the definition and properties of Lindley distribution are provided. Equation (1) presents the pdf of the Lindley distribution with parameter θ :

$$g\,(x) = \frac{\theta^2}{\theta + 1}\,(1+x)e^{-\theta x} \quad x > 0\,, \theta > 0 \qquad \textbf{(1)}$$

The corresponding cdf function is:

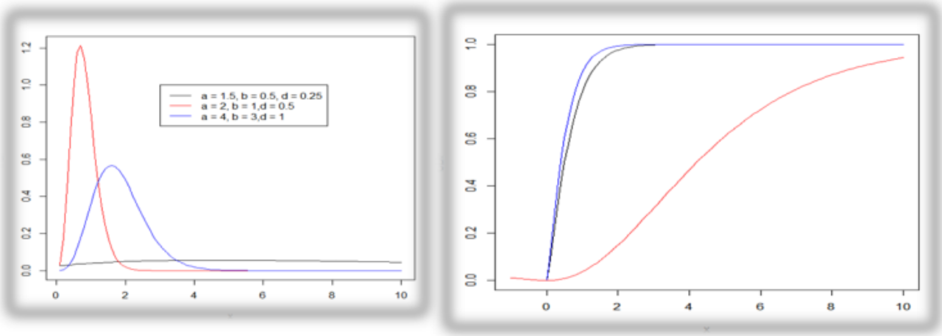
$$G\,(x) = 1 - e^{-\theta x}\left(1 + \frac{x\,\theta}{\theta + 1}\right) \;\; x > 0\,, \theta > 0 \qquad (2)$$

1.2 Kumaraswamy Lindley Distribution (KLD). Suppose G(x, θ) be the cdf of the Lindley distribution given by (2). The cdf of Kw-Lindley distribution can and (pdf) of Kw-Lindley are:

$$F\,(x) = 1 - \left[1 - \left(1 - e^{-\theta x}\left(1 + \frac{\theta x}{\theta + 1}\right)\right)^a\right]^b \qquad (3)$$

$$f\,(x) = ab\left(\frac{\theta^2}{\theta + 1}\,(1+x)e^{-\theta x}\right)\left(1 - e^{-\theta x}\left(1 + \frac{\theta x}{\theta + 1}\right)\right)^{a-1}$$

$$\left[1-\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^{a}\right]^{b-1} \quad (4)$$



1.3

Figure 1. pdf (lenght) and cdf (right) plot of (KWL) Kumaraswamy Lindley distribution

2. Mathematical Properties

2.1 probability weighted moments

The probabilities weighted moments (PWM) are used to derive parameters of generalized probability distributions and this method is used to compare with the parameters obtained by using the probabilities weighted moments are defined as follows

$$\rho_{s,r} = E(x^s F(x)^r) = \int_0^{\infty} x^s F(x)^r f(x)dx \quad (5)$$

From the definition of the cumulative distribution function of the Kumaraswamy Lindley, we find that

$$F(x) = 1 - \left[1-\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^{a}\right]^{b}$$

$$(1-z)^d = \sum_{i=0}^{\infty}(-1)^i \binom{d}{i} z^i, \ |z|<1$$

$$\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^{a} = \sum_{n=0}^{\infty}(-1)^n \binom{a}{n} e^{-n\theta x}\left(1+\frac{\theta x}{\theta+1}\right)^{n}$$

Using the binomial theorem on the following

$$\left(1+\frac{\theta x}{\theta+1}\right)^{n} = \sum_{l=0}^{\infty}\binom{n}{l}\left(\frac{\theta}{\theta+1}\right)^{l} x^l$$

Substitute the value of the term into equation number (3) we get the following

$$\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^{a} = \sum_{n=0}^{\infty}(-1)^n \binom{a}{n} \sum_{l=0}^{\infty}\binom{n}{l}\left(\frac{\theta}{\theta+1}\right)^{l} x^l e^{-n\theta x} \quad (6)$$

Simplifying the equation number , we get

$$\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^{a} = W_{nl} x^l e^{-n\theta x}$$

Where

$$W_{nl} = \sum_{n=0}^{\infty} (-1)^n \binom{a}{n} \sum_{l=0}^{\infty} \binom{n}{l} \left(\frac{\theta}{\theta + 1}\right)^l$$

Substituting into the cumulative distribution function for a value

$$\left(1 - e^{-\theta x}\left(1 + \frac{\theta x}{\theta + 1}\right)\right)^a$$

We get

$$F(x) = 1 - \left[1 - W_{nl} x^l e^{-n\theta x}\right]^b \quad (7)$$

Applying the binomial theorem to the following expression $\left[1 - W_{nl} x^l e^{-n\theta x}\right]^b$

$$\left[1 - W_{nl} x^l e^{-n\theta x}\right]^b = \sum_{\delta=0}^{\infty} (-1)^\delta \binom{b}{\delta} \left(W_{nl} x^l e^{-n\theta x}\right)^\delta$$

Simplifying the previous expansion, we get

$$\left[1 - W_{nl} x^l e^{-n\theta x}\right]^b = \sum_{\delta=0}^{\infty} (-1)^\delta \binom{b}{\delta} (W_{nl})^\delta \quad e^{-\delta n\theta x} x^{\delta l}$$

$$\left[1 - W_{nl} x^l e^{-n\theta x}\right]^b = W_{nl\delta} e^{-\delta n\theta x} x^{\delta l} \quad (8)$$

Where

$$W_{nl\delta} = \sum_{\delta=0}^{\infty} (-1)^\delta \binom{b}{\delta} (W_{nl})^\delta$$

Thus, the final formulation of the cumulative distribution function of the Kumaraswamy Lindley distribution becomes as follows

$$F(x) = 1 - W_{nl\delta} e^{-\delta n\theta x} x^{\delta l} \quad (9)$$

And now we show the components of the general formulation the probabilities weighted moments (PWM)
The first component is

$$F(x)^r = \left(1 - W_{nl\delta} e^{-\delta n\theta x} x^{\delta l}\right)^r \quad (10)$$

Applying the binomial theorem to the following expansion

$$F(x)^r = \sum_{\pi=0}^{\infty} (-1)^\pi \binom{r}{\pi} \left(e^{-\pi\delta n\theta x} x^{\pi\delta l}\right)(W_{nl\delta})^\pi$$

So the final form of the function $F(x)^r$ is

$$F(x)^r = \left(e^{-\pi\delta n\theta x} x^{\pi\delta l}\right) W_{nl\delta\pi} \quad (11)$$

$$Where \; W_{nl\delta\pi} = \sum_{\pi=0}^{\infty} (-1)^\pi \binom{r}{\pi} (W_{nl\delta})^\pi$$

The second component $f(x) = \frac{ab\,\theta^2}{\theta + 1} w_j \, q \, x^{m+j} \, (1 + x) \, e^{-\theta x(k+\gamma)}$

Substituting in the general form for probability weighted moments, we get

$$\rho_{s,r} = \int_0^\infty x^s \left(e^{-\pi\delta n\theta x} x^{\pi\delta l}\right) W_{nl\delta\pi} \frac{ab\,\theta^2}{\theta + 1} w_j \, q \, x^{m+j} \, (1 + x) \, e^{-\theta x(k+\gamma)} dx \quad (12)$$

Simplifying the equation, we get

$$\rho_{s,r} = \frac{ab\,\theta^2}{\theta + 1} w_j \, q W_{nl\delta\pi} \int_0^\infty e^{-\pi\delta n\theta x} \left(x^{\pi\delta l}\right) x^{s+m+j} \, (1 + x) \, e^{-\theta x(k+\gamma)} dx$$

$$\rho_{s,r} = \frac{ab\,\theta^2}{\theta + 1} w_j\, qW_{nl\delta\pi} \int_0^\infty x^{\pi\delta l+s+m+j} \ (1+x)\, e^{-\theta x(k+\gamma)-\pi\delta n\theta x} dx$$

$$\rho_{s,r} = \frac{ab\,\theta^2}{\theta + 1} w_j\, qW_{nl\delta\pi} \int_0^\infty (x^{\pi\delta l+s+m+j+1} + x^{\pi\delta l+s+m+j})\, e^{-\theta x(k+\gamma)-\pi\delta n\theta x} dx$$

$$\rho_{s,r} = \frac{ab\,\theta^2}{\theta + 1} w_j\, qW_{nl\delta\pi} \int_0^\infty (x^{\pi\delta l+s+m+j+1} + x^{\pi\delta l+s+m+j})\, e^{-(\theta(k+\gamma)+\pi\delta n\theta)x} dx \qquad (13)$$

$$\rho_{s,r} = \tau \int_0^\infty (x^{\pi\delta l+s+m+j+1})\, e^{-(\theta(k+\gamma)+\pi\delta n\theta)x}\, dx \quad + \tau \int_0^\infty e^{-(\theta(k+\gamma)+\pi\delta n\theta)x}\, x^{\pi\delta l+s+m+j} dx$$

assuming that $K_1 = \tau \int_0^\infty (x^{\pi\delta l+s+m+j+1})\, e^{-(\theta(k+\gamma)+\pi\delta n\theta)x}\, dx$ and $K_2 = \tau \int_0^\infty e^{-(\theta(k+\gamma)+\pi\delta n\theta)x}\, x^{\pi\delta l+s+m+j} dx$

It becomes probabilities Weighted moments (PWM) as following $\rho_{s,r} = K_1 + K_2$ And we calculate the value of each K_1 and K_2

$$K_1 = \tau \int_0^\infty (x^{\pi\delta l+s+m+j+1})\, e^{-(\theta(k+\gamma)+\pi\delta n\theta)x}\, dx$$

assuming that $y = (\theta(k+\gamma) + \pi\delta n\theta)x$ Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma) + \pi\delta n\theta)dx$, $x = \frac{y}{(\theta(k+\gamma)+\pi\delta n\theta)}$, $dx = \frac{dy}{(\theta(k+\gamma)+\pi\delta n\theta)}$ By substituting the value of each of. x, dx.. and simplifying, we get the following.

$$K_1 = \tau \int_0^\infty \left(\left(\frac{y}{(\theta(k+\gamma) + \pi\delta n\theta)}\right)^{\pi\delta l+s+m+j+1} \right) e^{-y}\, \frac{dy}{(\theta(k+\gamma) + \pi\delta n\theta)}$$

$$K_1 = \frac{\tau}{(\theta(k+\gamma) + \pi\delta n\theta)^{\pi\delta l+s+m+j+2}} \int_0^\infty y^{\pi\delta l+s+m+j+1}\ e^{-y}\, dy$$

From the gamma integral we get $K_1 = \frac{\tau}{(\theta(k+\gamma)+\pi\delta n\theta)^{\pi\delta l+s+m+j+2}} \boldsymbol{\Gamma}\, (\pi\delta l + s + m + j + 2)$

And by calculating the value of (K_2)... we get

$$K_2 = \tau \int_0^\infty e^{-(\theta(k+\gamma)+\pi\delta n\theta)x}\, x^{\pi\delta l+s+m+j} dx$$

Substituting in $(..K_2)$.. for the hypothesis and differentiation, we get the following

$$K_2 = \tau \int_0^\infty e^{-(y)} \left(\frac{y}{(\theta(k+\gamma) + \pi\delta n\theta)}\right)^{\pi\delta l+s+m+j} \frac{dy}{(\theta(k+\gamma) + \pi\delta n\theta)}$$

Simplifying the previous equation, we get

$$K_2 = \frac{\tau}{(\theta(k+\gamma) + \pi\delta n\theta)^{\pi\delta l+s+m+j+1}} \int_0^\infty e^{-(y)}\, (y)^{\pi\delta l+s+m+j} dy$$

From the gamma integral we get $K_2 = \frac{\tau}{(\theta(k+\gamma)+\pi\delta n\theta)^{\pi\delta l+s+m+j+2}} \boldsymbol{\Gamma}\, (\pi\delta l + s + m + j + 1)$

The value of probabilities Weighted moments (PWM) as following $\rho_{s,r} = K_1 + K_2$

$$\rho_{s,r} = \frac{\tau \boldsymbol{\Gamma}\, (\pi\delta l+s+m+j+2)}{(\theta(k+\gamma)+\pi\delta n\theta)^{\pi\delta l+s+m+j+2}} + \frac{\tau\, \boldsymbol{\Gamma}\, (\pi\delta l+s+m+j+1)}{(\theta(k+\gamma)+\pi\delta n\theta)^{\pi\delta l+s+m+j+2}} \qquad (14)$$

The nth moments of residual life denoted by $E[(x-t)^n | \, x > t]$ where $n = 1,2,3, \ldots$..

Is defined by $m(t)_n = \frac{1}{R(T)} \int_t^\infty (x-t)^n\, f(x)dx$ *(15)*

Substitute in the general form for the residuals life for the probability density function of the Kumaraswamy Lindley distribution

$$m(t)_n = \frac{1}{R(T)}\int_t^{\infty} (x-t)^n \frac{ab\,\theta^2}{\theta\ +\ 1}\ w_j\, q\, x^{m+j}\,(1+x)\, e^{-\theta x(k+\gamma)}dx$$

From the binomial theorem we get

$$(x-t)^n = (-t)^n(1-\frac{x}{t})^n = (-t)^n \sum_{h=0}^{\infty}(-1)^h \binom{n}{h} (\frac{x}{t})^h$$

Using the binomial expansion of the expression ((x − t)^n) and the substitution of the general form for residual life

$$m(t)_n = \frac{1}{R(T)}\int_t^{\infty}(-t)^n \sum_{h=0}^{\infty}(-1)^h\binom{n}{h}(\frac{x}{t})^h\ \frac{ab\,\theta^2}{\theta\ +\ 1}\ w_j\, q\, x^{m+j}\,(1+x)\, e^{-\theta x(k+\gamma)}dx \qquad (16)$$

Simplifying the least equation we get

$$m(t)_n = \frac{\frac{ab\,\theta^2}{\theta\ +\ 1}\ w_j\, q(-t)^n \sum_{h=0}^{\infty}(-1)^h\binom{n}{h}}{R(T)}\int_t^{\infty}(\frac{x}{t})^h\ x^{m+j}\,(1+x)\, e^{-\theta x(k+\gamma)}dx$$

Let $k_3 = \frac{\frac{ab\,\theta^2}{\theta\ +\ 1} w_j\, q(-t)^n \sum_{h=0}^{\infty}(-1)^h\binom{n}{h}}{t^h}$ the equation number ( )becomes

$$m(t)_n = \frac{k_3}{R(T)}\int_t^{\infty}\ x^{m+j+h}\,(1+x)\, e^{-\theta x(k+\gamma)}dx$$

$$m(t)_n = \frac{k_3}{R(T)}[\int_t^{\infty}\ x^{m+j+h}\, e^{-\theta x(k+\gamma)}dx + \int_t^{\infty} x^{m+j+h+1}\ e^{-\theta x(k+\gamma)}dx] \qquad (17)$$

Let $m(t)_n = \frac{k_3}{R(T)}[k_4 + k_5]$ where$k_4 = \int_t^{\infty}\ x^{m+j+h}\ e^{-\theta x(k+\gamma)}dx$ and $k_5 = \int_t^{\infty}\ x^{m+j+h+1}\, e^{-\theta x(k+\gamma)}dx$

Now calculating the value of k_4 and using the general form of the gamma function $\boldsymbol{\Gamma}(a,b) = \int_b^{\infty} y^{a-1}e^{-y}\,dy$ we get

$$k_4 = \int_t^{\infty}\ x^{m+j+h}\ e^{-\theta x(k+\gamma)}dx$$

Assuming that $y = (\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $y = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. x, dx and simplifying, we get the following

$$k_4 = \int_t^{\infty}\ \left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+h}\ e^{-y}\frac{dy}{(\theta(k+\gamma))}$$

$$k_4 = \frac{1}{(\theta(k+\gamma))^{m+j+h+1}}\int_{t\theta(k+\gamma)}^{\infty}\ [y]^{m+j+h}\ e^{-y}\,dy$$

Applying the general form of the gamma function $-1 = m+j+h, b = t\theta(k+\gamma)$, the value of

$$k_4 = \frac{\boldsymbol{\Gamma}\big(m+j+h+1, t\theta(k+\gamma)\big)}{(\theta(k+\gamma))^{m+j+h+1}}$$

To compute $k_5 = \int_t^{\infty} x^{m+j+h}\, e^{-\theta x(k+\gamma)} dx$

assuming that $y = (\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. $x, dx. In\ k_5$. and simplifying, we get the following

$$k_5 = \int_{\theta(k+\gamma)t}^{\infty} \left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+h+1} e^{-y} \frac{dy}{(\theta(k+\gamma))}$$

$$k_5 = \frac{1}{[\theta(k+\gamma)]^{m+j+h+2}} \int_{(\theta(k+\gamma))t}^{\infty} [y]^{m+j+h+1}\, e^{-y}\, dy$$

Applying the general form of the gamma function $a - 1 = m + j + h + 1, b = t\theta(k+\gamma)$

The value of k_5 equal

$$k_5 = \frac{\Gamma(m+j+h+2, t\theta(k+\gamma))}{[\theta(k+\gamma)]^{m+j+h+2}}$$

The Moments of Residual life equal the following

$$m(t)_n = \frac{k_3}{R(T)}\left[\frac{\Gamma(m+j+h+1, t\theta(k+\gamma))}{(\theta(k+\gamma))^{m+j+h+1}} + \frac{\Gamma(m+j+h+2, t\theta(k+\gamma))}{[\theta(k+\gamma)]^{m+j+h+2}}\right] \quad (18)$$

2.2 Mean of Residual Life. The mean residual of (KWL) distribution is defined by

$$m(t) = \frac{1}{R(T)} \int_t^{\infty} x\, f(x) dx - t \quad (19)$$

Substituting the probability density function for the(KLD) kumaraswamy Lindley distribution in equation (19), we get

$$m(t) = \frac{1}{R(T)} \int_t^{\infty} x \frac{ab\,\theta^2}{\theta + 1} w_j\, q\, x^{m+j}\, (1+x)\, e^{-\theta x(k+\gamma)}\, dx - t$$

Simplifying the equation, we get the following

$$m(t) = \frac{\frac{ab\,\theta^2}{\theta + 1} w_j\, q}{R(T)} \int_t^{\infty} x^{m+j+1}\, (1+x)\, e^{-\theta x(k+\gamma)}\, dx - t$$

$$m(t) = \frac{\frac{ab\,\theta^2}{\theta + 1} w_j\, q}{R(T)} \left[\left(\int_t^{\infty} x^{m+j+1}\, e^{-\theta x(k+\gamma)} dx + \int_t^{\infty} x^{m+j+2} e^{-\theta x(k+\gamma)}\right) dx\right] - t \quad (20)$$

Let $k_6 = \int_t^{\infty} x^{m+j+1}\, e^{-\theta x(k+\gamma)} dx$, $k_7 = \int_t^{\infty} x^{m+j+2} e^{-\theta x(k+\gamma)})\ dx$ and $\delta = \frac{\frac{ab\,\theta^2}{\theta + 1} w_j\, q}{R(T)}$

The mean of residuals life becomes as follows

$$m(t) = \delta[\,k_6 + k_7] - t$$

Now we calculate the value of k_6, which is

$$k_6 = \int_t^{\infty} x^{m+j+1}\, e^{-\theta x(k+\gamma)} dx$$

assuming that $y = (\theta(k+\gamma))$x Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. $x, dx. In\ k_6$. and simplifying, we get the following

$$k_6 = \int_t^{\infty} \left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+1} e^{-y} \frac{dy}{(\theta(k+\gamma))}$$

$$k_6 = \frac{1}{[(\theta(k+\gamma))]^{m+j+2}} \int_{(\theta(k+\gamma))t}^{\infty} [y]^{m+j+1} e^{-y}\, dy$$

Applying the general form of the gamma function $a-1 = m+j+1, b = (\theta(k+\gamma))t$

$$k_6 = \frac{\Gamma(m+j+2, (\theta(k+\gamma))t)}{[(\theta(k+\gamma))]^{m+j+2}}$$

Now calculate the value of k_7

$$k_7 = \int_t^{\infty} x^{m+j+2} e^{-\theta x(k+\gamma))}\; dx$$

assuming that $y = (\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. x, dx. *In* k_7. and simplifying, we get the following

$$k_7 = \frac{1}{(\theta(k+\gamma))^{m+j+3}} \int_{(\theta(k+\gamma))t}^{\infty} y^{m+j+2} e^{-y})\; dy$$

Applying the general form of the gamma function $a-1 = m+j+2, b = (\theta(k+\gamma))t$

$$k_7 = \frac{\Gamma(m+j+3, (\theta(k+\gamma))t)}{(\theta(k+\gamma))^{m+j+3}}$$

The mean of residuals life becomes as follows

$$m(t) = \delta\left[\frac{\Gamma(m+j+2, (\theta(k+\gamma))t)}{[(\theta(k+\gamma))]^{m+j+2}} + \frac{\Gamma(m+j+3, (\theta(k+\gamma))t)}{(\theta(k+\gamma))^{m+j+3}}\right] - t \qquad (21)$$

Definition A distribution is said to be DMRL if the mean residual life function m(t) is decreasing in t , $\acute{m}(t) < 0 \;\forall\, t > 0$ and (IMRL) if $\acute{m}(t) > 0 \;\forall\, t > 0$ Differentiate the equation m(t) with respect to x

$$\acute{m}(t) = \acute{\delta}\left[\frac{\Gamma(m+j+2, (\theta(k+\gamma))t)}{[(\theta(k+\gamma))]^{m+j+2}} + \frac{\Gamma(m+j+3, (\theta(k+\gamma))t)}{(\theta(k+\gamma))^{m+j+3}}\right] + \delta\left[\frac{\acute{\Gamma}(m+j+2, (\theta(k+\gamma))t)(\theta(k+\gamma))}{[(\theta(k+\gamma))]^{m+j+2}} + \frac{\acute{\Gamma}(m+j+3, (\theta(k+\gamma))t)(\theta(k+\gamma))}{(\theta(k+\gamma))^{m+j+3}}\right] \qquad (22)$$

Note that $\acute{m}(t) > 0 \;\; t > 0$ thus mean residual life is increasing (IMRL)

2.3 Reversed Residual Life. The nth moments of residual life denoted by $E[(t-x)^n \mid x \le t]$ *where* $n = 1,2,3, \ldots$ *Is defined by* $M(t)_n = \frac{1}{F(T)} \int_0^t (t-x)^n f(x)dx (23)$. Substitute in the general form for the residuals life for the probability density function of the Kumaraswamy Lindley distribution the reversed residual life becomes

$$M(t)_n = \frac{1}{F(T)}\int_0^{t\infty}(t-x)^n \frac{ab\,\theta^2}{\theta\,+\,1}\,w_j\,q\,x^{m+j}\,(1+x)\,e^{-\theta x(k+\gamma)}dx$$

Using the binomial expansion of the expression

$$(t-x)^n = (t)^n(1-\frac{x}{t})^n = (t)^n\sum_{\omega=0}^{\infty}(-1)^{\omega}\binom{n}{\omega}(\frac{x}{t})^{\omega}$$

$$M(t)_n = \frac{1}{F(T)}\int_0^t (t)^n\sum_{\omega=0}^{\infty}(-1)^{\omega}\binom{n}{\omega}(\frac{x}{t})^h \frac{ab\,\theta^2}{\theta\,+\,1}\,w_j\,q\,x^{m+j}\,(1+x)\,e^{-\theta x(k+\gamma)}dx$$

Simplifying the least equation we get

$$M(t)_n = \frac{\frac{ab\,\theta^2}{\theta\,+\,1}\,w_j\,q(t)^n\sum_{\omega=0}^{\infty}\frac{(-1)^{\omega}\binom{n}{\omega}}{t^{\omega}}}{F(T)}\int_0^t x^{\omega}\;x^{m+j}\,(1+x)\,e^{-\theta x(k+\gamma)}dx$$

Suppose that $\varphi = \frac{\frac{ab\,\theta^2}{\theta\,+\,1}\,w_j\,q(t)^n\sum_{\omega=0}^{\infty}\frac{(-1)^{\omega}\binom{n}{\omega}}{t^{\omega}}}{F(T)}$ the last equation becomes

$$M(t)_n = \varphi\int_0^t x^{m+j+\omega}\,(1+x)\,e^{-\theta x(k+\gamma)}dx$$

$$M(t)_n = \varphi\left[\int_0^t x^{m+j+\omega}\,e^{-\theta x(k+\gamma)}dx + x^{m+j+\omega+1}e^{-\theta x(k+\gamma)}dx\right]$$

Let $k_8 = \int_0^t x^{m+j+w}\,e^{-\theta x(k+\gamma)}dx$, $k_9 = \int_0^t x^{m+j+\omega+1}e^{-\theta x(k+\gamma)})\;dx$

The reversed of residuals life becomes as follows

$$M(t) = \varphi[\,k_8 + \,k_9] \qquad (24)$$

Now calculate the value of k_8

$$k_8 = \int_0^t x^{m+j+w}\,e^{-\theta x(k+\gamma)}dx$$

assuming that $y = (\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. $x, dx. In\;\;k_8$. and simplifying, we get the following

$$k_8 = \int_0^t \left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+w} e^{-y}\frac{dy}{(\theta(k+\gamma))}$$

$$k_8 = \frac{1}{(\theta(k+\gamma))^{m+j+w+1}}\int_0^{(\theta(k+\gamma))t}[y]^{m+j+w}\,e^{-y}dy$$

From the general definition of the gamma function $a-1 = \;m+j+w$ and $b = \theta(k+\gamma))b$

$$k_8 = \frac{\boldsymbol{\Gamma}\,(m+j+w+1, \theta(k+\gamma))b)}{(\theta(k+\gamma))^{m+j+w+1}}$$

Now calculate the value of k_9

$$k_9 = \int_0^t x^{m+j+\omega+1}e^{-\theta x(k+\gamma)})\;dx$$

assuming that $y=(\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $dy=(\theta(k+\gamma))dx$, $x=\frac{y}{(\theta(k+\gamma))}$, $dx=\frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. $x, dx. In\ k_8$. and simplifying, we get the following

$$k_9=\int_0^{(\theta(k+\gamma))t}\left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+\omega+1} e^{-y})\ \frac{dy}{(\theta(k+\gamma))}$$

$$k_9=\frac{1}{(\theta(k+\gamma))^{m+j+\omega+2}}\int_0^{(\theta(k+\gamma))t}[y]^{m+j+\omega+1}(e^{-y})\,dy$$

From the general definition of the gamma function $a-1=\ m+j+w+1$ and $b=\theta(k+\gamma))b$

$$k_9=\frac{\boldsymbol{\Gamma}\,(m+j+w+2,\theta(k+\gamma))b)}{(\theta(k+\gamma))^{m+j+\omega+2}}$$

The reversed of residuals life becomes as follows

$$M(t)=\varphi\left[\frac{\boldsymbol{\Gamma}\,(m+j+w+1,\theta(k+\gamma))b)}{(\theta(k+\gamma))^{m+j+w+1}}+\frac{\boldsymbol{\Gamma}\,(m+j+w+2,\theta(k+\gamma))b)}{(\theta(k+\gamma))^{m+j+\omega+2}}\right] \qquad (25)$$

The cumulative hazard function (CHF) of the kumaraswamy Lindley distribution denoted $C\,HF_{KWL}\,(x,\theta)$ *is*

Defined as $CHF_{KWL}\,(x,\theta)=\int_0^x h(x,\theta)dx$ *(26)*

Substituting for hazard rate function.. in Equation No. (26) we get

$$CHF_{KWL}\,(x,\theta)=\int_0^x\frac{f(x)}{1-F(x)}dx$$

Substituting the value of the probability density function for the Kumarasumi-Lindley distribution as well as the cumulative distribution function for it, we get

$$CHF_{KWL}\,(x,\theta)=\int_0^x\frac{\frac{ab\,\theta^2}{\theta+1}w_j\,q\,x^{m+j}\,(1+x)\,e^{-\theta x(k+\gamma)}}{\left[1-\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^a\right]^b}dx$$

Applying the rule of integration which states that if the numerator is the differentiation of the denominator, then the integral is Lin (the denominator)

$$CHF_{KWL}\,(x,\theta)=-lin\left[1-\left(1-e^{-\theta x}\,\left(1+\ \frac{\theta x}{\theta+1}\right)\right)^a\right]^b \qquad (27)$$

$$Reverse\ hazard\ rate\ function{=}\frac{f_{KWL}}{F_{KWL}}=\frac{\frac{ab\,\theta^2}{\theta+1}w_j\,q\,x^{m+j}\,(1+x)\,e^{-\theta x(k+\gamma)}}{1-\left[1-\left(1-e^{-\theta x}\left(1+\frac{\theta x}{\theta+1}\right)\right)^a\right]^b} \qquad (28)$$

Dispersion in a population can be measured by measuring the sum of deviations from the mean and median and if we have X a random variable that is distributed Kamwaswamy Lindley we can derive the mean deviation about the mean $\mu=E(x)$ and the mean deviation about the median $M=F^{-1}(\frac{1}{2})$

$$\delta_1=\int_0^{\infty}|X-\mu|\ f_{kwl}(x)dx \qquad (29)$$

$$\delta_1=\int_0^{\mu}|X-\mu|\ f_{kwl}(x)dx+\int_{\mu}^{\infty}(x-\mu)f_{kwl}\,(x)dx$$

$$\delta_1 = \mu \int_0^{\mu} f_{kwl}(x)dx - \int_0^{\mu} x\ f_{kwl}(x)dx + \int_{\mu}^{\infty} x f_{kwl}(x)dx - \int_{\mu}^{\infty} \mu f_{kwl}(x)dx$$

$$\delta_1 = \mu F(\mu) - \int_0^{\infty} x\ f_{kwl}(x)dx + \int_{\mu}^{\infty} x f_{kwl}(x)dx - \mu \int_{\mu}^{\infty} f_{kwl}(x)dx \quad (30)$$

Substitute in equation No. (30) for the value of the integral $\mu \int_{\mu}^{\infty} f_{kwl}(x)dx = \mu(1 - F(\mu))$

$$\delta_1 = \mu F(\mu) - \int_0^{\infty} x\ f_{kwl}(x)dx + \int_{\mu}^{\infty} x f_{kwl}(x)dx - \mu(1 - F(\mu))$$

$$\delta_1 = 2\mu F(\mu) - 2\mu + \int_{\mu}^{\infty} x f_{kwl}(x)dx$$

Now we calculate the value of the following integral $\pi = \int_{\mu}^{\infty} x f_{kwl}(x)dx \quad (31)$

Substituting the probability density function for the kumaraswamy Lindley distribution into equation No. (31) we get

$$\pi = \int_{\mu}^{\infty} x \frac{ab\,\theta^2}{\theta + 1} w_j\, q\, x^{m+j}\,(1+x)\, e^{-\theta x(k+\gamma)} dx(x)dx$$

Simplifying the previous integration, we get

$$\pi = \frac{ab\,\theta^2}{\theta + 1} w_j\, q \int_{\mu}^{\infty} x^{m+j+1}\,(1+x)\, e^{-\theta x(k+\gamma)} dx(x)dx$$

$$\pi = \frac{ab\,\theta^2}{\theta + 1} w_j\, q \left[\int_{\mu}^{\infty} x^{m+j+1} e^{-\theta x(k+\gamma)} dx + \int_{\mu}^{\infty} x^{m+j+2}\, e^{-\theta x(k+\gamma)} dx\right] \quad \textbf{(32)}$$

Suppose that $k_{10} = \int_{\mu}^{\infty} x^{m+j+1} e^{-\theta x(k+\gamma)} dx$ and$k_{11} = \int_{\mu}^{\infty} x^{m+j+2}\, e^{-\theta x(k+\gamma)} dx$

Equation no(32) . becomes

$$\pi = \frac{ab\,\theta^2}{\theta + 1} w_j\, q\, [\,k_{10} + k_{11}]$$

Now we calculate the value of k_{10}

$$k_{10} = \int_{\mu}^{\infty} x^{m+j+1} e^{-\theta x(k+\gamma)} dx$$

assuming that $y = (\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. $x, dx. In\ \ k_{10}$. and simplifying, we get the following

$$k_{10} = \int_{\mu}^{\infty} \left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+1} e^{-y} \frac{dy}{(\theta(k+\gamma))}$$

$$k_{10} = \frac{1}{(\theta(k+\gamma))^{m+j+2}} \int_{\mu(\theta(k+\gamma))}^{\infty} [y]^{m+j+1} e^{-y} dy$$

Applying the general form of the gamma function

$$k_{10} = \frac{\boldsymbol{\Gamma}(m+j+1, \mu(\theta(k+\gamma)))}{(\theta(k+\gamma))^{m+j+2}}$$

Now we calculate the value of k_{11}

$$k_{11} = \int_{\mu}^{\infty} x^{m+j+2}\, e^{-\theta x(k+\gamma)} dx$$

assuming that $y = (\theta(k+\gamma))x$ Differentiate the hypothesis, we get the following $dy = (\theta(k+\gamma))dx$, $x = \frac{y}{(\theta(k+\gamma))}$, $dx = \frac{dy}{(\theta(k+\gamma))}$

By substituting the value of each of. $x, dx. In\ k_{11}$. and simplifying, we get the following

$$k_{11} = \int_{\mu(\theta(k+\gamma))}^{\infty} \left[\frac{y}{(\theta(k+\gamma))}\right]^{m+j+2} e^{-y} \frac{dy}{(\theta(k+\gamma))}$$

$$k_{11} = \frac{1}{[\theta(k+\gamma)]^{m+j+3}} \int_{\mu(\theta(k+\gamma))}^{\infty} [y]^{m+j+2}\, e^{-y}\, dy$$

Applying the general form of the gamma function

$$k_{11} = \frac{\boldsymbol{\Gamma}(m+j+3, \mu(\theta(k+\gamma)))}{[\theta(k+\gamma)]^{m+j+3}}$$

$$\pi = \frac{ab\,\theta^2}{\theta + 1} w_j\, q\, [k_{10} + k_{11}]$$

Substituting for the valuek_{11}, k_{10} we get

$$\pi = \frac{ab\,\theta^2}{\theta + 1} w_j\, q \left[\frac{\boldsymbol{\Gamma}(m+j+1, \mu(\theta(k+\gamma)))}{(\theta(k+\gamma))^{m+j+2}} + \frac{\boldsymbol{\Gamma}(m+j+3, \mu(\theta(k+\gamma)))}{[\theta(k+\gamma)]^{m+j+3}}\right]$$

Substituting for the value π we get the mean division

$$\delta_1 = 2\mu F(\mu) - 2\mu + \frac{ab\,\theta^2}{\theta + 1} w_j\, q \left[\frac{\boldsymbol{\Gamma}\left(m+j+1, \mu(\theta(k+\gamma))\right)}{\left(\theta(k+\gamma)\right)^{m+j+2}} + \frac{\boldsymbol{\Gamma}\left(m+j+3, \mu(\theta(k+\gamma))\right)}{[\theta(k+\gamma)]^{m+j+3}}\right] \quad (33)$$

3. Conclusions

In this research paper, we present some of the new mathematical properties of the Kumaraswamy Lindley distribution, which was proposed in 1958, Because of the great importance of the distribution in different fields, including physics and engineering, and these properties of Reversed residual life, mean of residual life, moments of residual life, probability weighted moments, cumulative hazard rate function, have been derived further more, we invite researchers to study more mathematical properties of the distribution because of its many applications, which can contribute to solving many life problems.

New Mathematical Properties of the Kumaraswamy Lindley distribution

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