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Modeling and Classification Multicollinear Variables using Multinomial Ridge Logistic Regression Aprroach

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Abstract

Multinomial logistic regression is a method used to find relationships between nominal or multinomial response variables (Y) with one or more predictor variables. Logistics regression is a classic method that is often used to solve classification problems. Assumptions on logistics regression are models containing multicollinearity. Ridge logistic estimator (RLE) is methods to solve multicollinearity cases in Logistic Regression. Wu & Asar proposed a new ridge value that can also reduce bias in parameter estimation. Therefore, this research will discuss about multinomial ridge logistic and selection the best of ridge constant values. The performance test of the ridge value will be applied to the Iris Dataset in R software. The best criteria for improvement ridge constant value by looking at the smallest standard error. The calculation results show that the Wu-Asar approach is the best ridge constant and Wald individual test shows significant results. Based on the result, show that the Wu-Asar ridge constant value on multinomial ridge logistic regression are very good performance in estimated smaller standar error. The classification for dataset shows high results with 98% global accuracy.

Keywords: multinomial; ridge logistic regression; Wu-Asar; standard error; classification.

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1. INTRODUCTION

The multiclass classification assumes that each sample is assigned to one and only one label. Classification is the problem of identifying which of a set of categories (sub-populations) an observation variable belongs to [1]. Logistics regression is a statistical method that is often used to solve classification problems. Logistics regression method is a method that can be used to find the relationship between dichotomous (scaled with two categories) or polychotomous (having a nominal or ordinal scale with more than two categories) response variables with one or more predictor variables [2]. Logistic Regression based on the type of data scale can be divided into three, binary logistic regression, multinomial, and ordinal logistic regression. Nominal scale is a measurement that is categorized into more than two categories. Parameter estimation in Logistic Regression uses maximum likelihood estimation (MLE) and iterative reweighted least squares (IRLS) approaches. The MLE approach is used because the distribution of data on the response variable is known.

Assumptions on logistics regression, apart from the characteristics of the response variables, are models containing multicollinearity. The case of multicollinearity is a condition when the predictor variable is not independent, or there is a high correlation between the predictor variables [3]. If multicollinearity is not resolved, it can cause the standard error in parameter estimate to be wide, which would the results of the estimation irrelevant/biased. Even though the coefficient of determination (R^2)

value is high, multicollinearity may cause the parameter significance test to be insignificant. In the presence of multicollinearity, parameter estimations using MLE are known to have high variance. The problem can be solved by the ridge regression approach. The purpose of the ridge regression method is to add a positive constant called the ridge parameter to the parameter estimation process by modifying the OLS method.

Ridge regression method is used to solve multicollinearity problems by providing a biased estimator with a smaller variance than the least squares method. Determining the ridge value can use a Ridge Trace, by selecting a ridge parameter with a small value based on the plot between the regression coefficients and the constant ridge. Khalaf & Iguernane [4] stated that the selection of ridge value can be done iteratively based on the HKB estimator (Hoerl, Kennard and Baldwin). This approach can be done by calculating the estimated regression parameters obtained through the least squares method. The selection of the bias constant (ridge) with this iteration is a method that is solved analytically.

Schaefer, Roi & Wolfe [5] apply ridge parameters to resolve multicollinear problems in logistic regression. Ridge logistic estimator (RLE) is an approach that can solve multicollinearity cases. The SRW (Schaefer, Roi, and Wolfe) constant formula from RLE is used to calculate the ridge value. The theory is an analytical calculation based on least squares and eigen value. Even though the RLE approach can resolve multicollinearity, this parameter estimator produces a greater bias. Therefore, Wu & Asar [6] proposed a new method, namely the almost unbiased ridge logistic estimator (AURLE). The AURLE method not only resolved multicollinearity, but also reduced bias in parameter estimation. The ridge value is chosen by calculating the bias constant value using a new approach, the Wu-Asar estimator.

Nisa & Hastuti (2023) [7] simulated multicollinearity variables in multiple regression using ridge regression and adjusted ridge regression methods. The result shows that the adjusted ridge regression method and the ridge regression method produce a smaller MSE value when the sample size used is larger. Sari (2018) [8] doing a research study about parameter estimation in logistic regression with multicollinearity features. The method of parameter estimation was using maximum likelihood estimation and ridge logistic estimator. The results show that the ridge logistic estimator is better than the maximum likelihood estimator for parameter estimation. Putra [9] conducted research on HDI modeling in East Java using ridge logistic regression method. The response variable is binary and the selection of ridge parameters uses the principal component approach. Classification accuracy obtained by ridge logistic regression was 97.37%. In this paper, we examine the performance of Wu-Asar constant and its application to ridge logistic regression for modeling and classification.

2. METHODS

2.1. Multinomial Logistic Regression

The multinomial distribution known that a variable criterias has more than two categories with nominal scale. There is a random variable known Y with q categories, the the probability relationship can be written as $P(Y=r)=P(y_r=1)=\pi_r$ and $r=1,2,\dots,J$. Multinomial distribution function can be written as follows (1).

$$P(y_1,y_2,\dots y_J)=\pi_1^{y_1}\pi_2^{y_2}\dots\pi_J^{y_J}(1-\pi_1-\pi_2-\dots-\pi_J)^{1-y_1-y_2-,\dots-y_J} \quad (1)$$

with $E(Y_r)=\pi_r$ and $Var(Y_r)=\pi_r(1-\pi_r)$ and then $Cov(Y_r,Y_t)=-\pi_r\pi_t$.

Multinomial logistic regression is a method used to find relationships between nominal or multinomial response variables (Y) with one or more predictor variables [2]. The response variable (Y) consists of more than 2 categories are usually code by 0, 1, or 2. The general equation form of logistic regression model with p predictors is expressed in equation below [10].

$$\pi(\boldsymbol{x})=\frac{exp(\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_px_p)}{1+exp(\beta_1x_1+\beta_2x_2+\beta_3x_3+\dots+\beta_px_p)} \quad (2)$$

Equation (2) can be transformed using logit transformation.

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$$g_1(\boldsymbol{x}) = ln\left(\frac{\pi_1(\boldsymbol{x})}{\pi_0(\boldsymbol{x})}\right) = \beta_{10} + \beta_{11}x_1 + \ldots + \beta_{1p}x_p = \mathbf{x}^{\mathrm{T}}\boldsymbol{\beta}_1$$

$$g_2(\boldsymbol{x}) = ln\left(\frac{\pi_2(\boldsymbol{x})}{\pi_0(\boldsymbol{x})}\right) = \beta_{20} + \beta_{21}x_1 + \ldots + \beta_{2p}x_p = \mathbf{x}^{\mathrm{T}}\boldsymbol{\beta}_2 \quad (3)$$

Based on (3), the logistic regression model is obtained as follows (4).

$$\pi_0(\boldsymbol{x}) = \frac{1}{1 + exp\, g_1(\boldsymbol{x}) + exp\, g_2(\boldsymbol{x})}$$
$$\pi_1(\boldsymbol{x}) = \frac{exp\, g_1(\boldsymbol{x})}{1 + exp\, g_1(\boldsymbol{x}) + exp\, g_2(\boldsymbol{x})}$$
$$\pi_2(\boldsymbol{x}) = \frac{exp\, g_2(\boldsymbol{x})}{1 + exp\, g_1(\boldsymbol{x}) + exp\, g_2(\boldsymbol{x})} \quad (4)$$

Parameters estimation for multinomial logistic regression was performed using the maximum likelihood estimation (MLE) method. Based on (1), we can form a likelihood function for the categorical response J = 3.

$$L(\boldsymbol{\beta}) = \prod_{i=1}^{n} \pi_{1i}{}^{y_{1i}}\pi_{2i}{}^{y_{2i}}(1 - \pi_{1i} - \pi_{2i})^{1-y_{1i}-y_{2i}} \quad (5)$$

Equation (5) will be differentially respect to β to determine the first and second derivatives. The calculation of the estimation process will produce a non-closed form, therefore it will be approached by numerical iterations with iterative reweighted least squares (IRLS).

2.2. Multicollinearity

Multicollinearity is a condition where there is a high correlation between predictor variables or the predictor variables are not independent. One of the criteria that may be used to detect multicollinearity is the variance inflation factor (VIF). The VIF value is formulated as follows [3].

$$VIF = \frac{1}{1 - R_k^2} \quad (6)$$

R_k^2 is the coefficient of determination between predictor variables in the regression model, where $k = 1,2, \ldots, p$. For VIF values greater than 10, the predictors are multicollinear. The estimations of the parameters fall short of the actual value.

2.3. Ridge Regression

The ridge regression is the method used to resolve multicollinearity cases. The $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ matrix is almost singular, and the regression parameter estimation results are unstable due to high correlation between several predictor variables. Ridge regression method is designed to address these problems [11]. The parameters are estimated using the least squares method by adding the ridge parameter (θ) to the diagonal elements of the $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ matrix. The ridge parameter is a small positive number between 0 and 1. If the value is zero, the ridge regression estimation is equivalent to the least-squares linear regression [11]. Parameter estimate for regression ridge is shown in (7).

$$\widehat{\boldsymbol{\beta}}^* = (\mathbf{X}^{\mathrm{T}}\mathbf{X} + \theta\mathbf{I})^{-1}\mathbf{X}^{\mathrm{T}}\boldsymbol{y} \quad (7)$$

With $\hat{\beta}^*$ is an estimator for ridge regression. The ridge regression method will increase the eigenvalue, so it can reduce MSE.

2.4. Ridge Logistic Regression

Schaefer, Roi & Wolfe [5] introduced the ridge logistic estimator (RLE), by adding the ridge parameter to the logistic regression estimation covariance variance matrix. The general form of parameter estimation with RLE is as follows:

$$\hat{\boldsymbol{\beta}}_{RLE} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X} + k\mathbf{I}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{z} \quad (8)$$

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where $\boldsymbol{k}$ is a constant called the ridge parameter. The RLE method is used to deal with multicollinearity problems by providing a biased estimator but has a smaller variance. Wu & Asar [6] introduced a new method almost unbiased ridge logistic estimator (AURLE) to minimize bias variance. Ridge value is determined by calculating the Wu-Asar estimator constant (k_{WA}).

$$k_{WA} = \frac{p}{\sum_{l=1}^{p}\left[\frac{a_l^2}{(1+(1+\lambda_l a_l^2)^{1/2})}\right]} \tag{9}$$

Where $\boldsymbol{\alpha} = \boldsymbol{\gamma}\widehat{\boldsymbol{\beta}}$ and α_i is an element of $\boldsymbol{\alpha}$. It is known that $\boldsymbol{\gamma}$ and λ are eigen vectors and eigen values of the covariance matrix.

2.5. Parameter Significance Test

Parameter significance tests consist of overral fit tests/joint test and individual/partial tests. Overral fits test was carried out using the likelihood ratio test or also called statistical G test, with the following hypothesis is:

$$H_0 : \beta_{j1} = \beta_{j2} = \cdots = \beta_{jp} = 0 , j = 1,2,\ldots,J-1$$

$$H_1 : at\ least\ on\ of\ \beta_{jl} \neq 0 \ \ ; l = 1,2,\ldots,p$$

Statistics test

$$\text{G} = -2\left(lnL(\widehat{\omega}) - lnL(\widehat{\Omega})\right) \tag{10}$$

with $L(\omega)$: the value of the likelihood function for a model where all parameters are equal to zero. Meanwhile, $L(\Omega)$: the value of the likelihood function for the complete model. The G test follows a chi-square distribution with degrees of freedom (v). H_0 is rejected if $G \geq \chi^2{}_{(\alpha,v)}$, which means the overral fits test has significant effect [2].

Individual testing was carried out to determine whether the predictor variables individually affect the model independently. This individual test is performed using the Wald test with the following hypotheses:

$$H_0 : \beta_{jl} = 0$$
$$H_1 : \beta_{jl} \neq 0 \ \ ; l = 1,2,\ldots,p \ \ j = 1,2,\ldots,J-1$$

p is number of amount predictor variable. Statistics Wald test as follows:

$$W_{jl} = \frac{\widehat{\beta_{jl}}}{SE(\widehat{\beta_{jl}})} \tag{11}$$

with $\hat{\beta}_{jl}$ is a parameter estimator and $SE(\hat{\beta}_{jl}) = \sqrt{\widehat{Var}(\hat{\beta}_{jl})}$. H_0 rejected if $|W_{jl}| \geq Z_{\frac{\alpha}{2}}$ or can uses $W_{jl}^2 \geq \chi^2{}_{(\alpha,1)}$ that follows a chi-square distribution with degrees of freedom 1.

2.6. Classification Evaluation

Evaluation of classification accuracy aims to determine the percentage of error classification or the accuracy percentage of the classification result performed by the classification method [12]. Classification measurements are performed using a confusion matrix approach.

$$Accuracy = \frac{n_{11} + n_{22} + \ldots + n_{kl}}{N} \times 100\%$$
$$Sensitivity = \frac{TP}{TP + FN}$$
$$Specificity = \frac{TN}{FP + TN} \tag{12}$$

n_{kl} : variables classified in categories-*k,l*. TP: true positive, TN: true negative, FP: false positive and FN: false negative [1].

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2.6. Data

Iris data is a dataset in Software R package that is commonly used for classification. This dataset consists of 3 types of Iris species, and each species consists of 50 samples. The response variable is Species with three category (0: Setosa, 1: Versicolor, 2: Virginica) and four features that are measured as predictor. The structure of the observation variables is explained in Table 1.

Table 1. Research variables

Variable	
Y	Species (0: Setosa, 1: Versicolor, 2: Virginica)
X_1	Sepal length
X_2	Sepal width
X_3	Petal length
X_4	Petal width

2.6. Research procedure

The research procedure consists of several stages to ensure that the results of this search run smoothly**.** The steps in this research are as follows:

1) Parameter estimation for multinomial logistic regression uses MLE (5).
2) Find the first and second derivatives of the likelihood function. Numerical iteration based on IRLS and Taylor series expansion.
3) Obtain the estimated parameter equation of multinomial logistic regression.
4) Obtain the estimated parameter form of the multinomial ridge logistic regression parameters with the RLE according to (8).
5) Multicollinearity check on Iris data using VIF.
6) Modeling the Iris data with multinomial ridge logistic regression.
7) Selecting the ridge parameter value: calculating the eigen vector and eigen values from $\mathbf{X}^{\mathbf{T}}\mathbf{W}_{\mathbf{j}}\mathbf{X}$**.** Determine the best ridge value from the ridge trace (k_{rt}), Schaefer, Roi & Wolfe formula (k_{SRW}) and new approach constants Wu-Asar (k_{WA}).
8) Parameter significance test, overall and individual test for Iris data.
9) Test the accuracy of classification with the confusion matrix (12).

3. RESULTS AND DISCUSSION

3.1. Parameter Estimation of Multinomial Logistic Regression

Parameter Estimation for multinomial logistic regression uses the MLE approach and IRLS. The following distribution for response is multinomial distribution. First step, construct the likelihood function of parameter $\boldsymbol{\beta}$ with category J=3.

$$L(\boldsymbol{\beta}) = \prod_{i=1}^{n} \pi_{1i}{}^{y_{1i}}\pi_{2i}{}^{y_{2i}}(1-\pi_{1i}-\pi_{2i})^{1-y_{1i}-y_{2i}}$$

$$\ln L(\boldsymbol{\beta}) = \sum_{i=1}^{n}[y_{1i}\ln\pi_{1i} + y_{2i}\ln\pi_{2i} + (1-y_{1i}-y_{2i})\ln(1-\pi_{1i}-\pi_{2i})]$$

$$\ln L(\boldsymbol{\beta}) = \sum_{i=1}^{n}[y_{1i}g_1(\boldsymbol{x}_i) + y_{2i}g_2(\boldsymbol{x}_i) - \ln(\exp g_1(\boldsymbol{x}_i) + \exp g_2(\boldsymbol{x}_i))] \quad (13)$$

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With $\boldsymbol{\beta} = [\boldsymbol{\beta}_1{}^T \quad \boldsymbol{\beta}_2{}^T]^{\boldsymbol{T}}$ and $\mathbf{x} = [1 \quad x_{1i} \quad x_{2i} \quad \cdots \quad x_{pi}]^T$. Second step, find the first and second derivatives of equation (13) to $\boldsymbol{\beta}$. The first derivative as follows.

$$\frac{\partial \ln L\,(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_j} = \sum_{i=1}^{n} \mathbf{x}_i(y_{ij} - \pi_{ij}) = \mathbf{X}^{\mathrm{T}}(\mathbf{Y} - \boldsymbol{\pi}) \tag{14}$$

Then the second derivative is performed to estimate the variance covariance.

$$\frac{\partial^2 \ln L\,(\boldsymbol{\beta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'} = \mathbf{X}^{\mathrm{T}}\mathbf{V}\mathbf{X} \tag{15}$$

Third step, get the estimate form $\widehat{\boldsymbol{\beta}}$ multinomial logistic regression based on numerical iteration. Taylor series expansion is carried out.

$$\left.\frac{\partial \ln L\,(\boldsymbol{\beta}_j)}{\partial \boldsymbol{\beta}_j}\right|_{\hat{\beta}=\hat{\beta}_0} = \left.\frac{\partial^2 \ln L\,(\boldsymbol{\beta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'}\right|_{\hat{\beta}=\hat{\beta}_0} \widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}_0 \tag{16}$$

Substitution equation (14) and (15) to (16)

$$\mathbf{X}^{\mathrm{T}}(\boldsymbol{Y} - \boldsymbol{\pi}) = \mathbf{X}^{\mathrm{T}}\mathbf{V}\mathbf{X}(\widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}_0)$$
$$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^{\mathrm{T}}\mathbf{V}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{V}\mathbf{z} \tag{17}$$

The form parameter estimation for multinomial logistic regression can be written as.

$$\widehat{\boldsymbol{\beta}}_j = (\mathbf{X}^{\mathrm{T}}\mathbf{V}_{\mathrm{j}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{V}_{\mathrm{j}}\mathbf{z}_{\mathrm{j}} \tag{18}$$

j=1,2, $\mathbf{z}_{\mathrm{j}}$ is a vector with length n × 1.

$$z_j = Logit[\widehat{\pi_{ij}}] + \frac{y_{ij} - \hat{\pi}_{ij}}{\hat{\pi}_{ij}(1 - \hat{\pi}_{ij})}$$

3.2. Parameter Estimation of Multinomial Ridge Logistic Regression

After obtaining the estimation form of the multinomial logistic regression, an estimation model for multinomial ridge logistic regression will be formed. The first step is to obtain estimation parameters using the ridge logistic estimator (RLE). The general form of the RLE for $\widehat{\boldsymbol{\beta}}$ estimation by [5] is equation (8).

$$\hat{\boldsymbol{\beta}}_{RLE} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X} + k\mathbf{I}\right)^{-1} \mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{z} \tag{19}$$

As we know, k is a constant called ridge parameter. The second step is to obtain the parameter estimation equation of the multinomial ridge logistic regression. From (18) can be defined $\boldsymbol{W_j} = \boldsymbol{V_j} = \pi_{ij}(1 - \pi_{ij})$. The form of parameter estimation for multinomial ridge regression is as follows (20).

$$\hat{\boldsymbol{\beta}}_{MLR} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{W}_j\mathbf{X} + k\mathbf{I}\right)^{-1} \mathbf{X}^{\mathrm{T}}\mathbf{W}_j\mathbf{z}_j \tag{20}$$

Varians $\widehat{\boldsymbol{\beta}}$

$$var(\widehat{\boldsymbol{\beta}_j}) = (\mathbf{X}^{\mathbf{T}}diag(\mathbf{W}_{\mathrm{j}})\mathbf{X} + k\mathbf{I})^{-1}\mathbf{W}_{\mathrm{j}}(\mathbf{X}^{\mathbf{T}}diag(\mathbf{W}_{\mathrm{j}})\mathbf{X} + k\mathbf{I})^{-1}$$

Wu-Asar Constant

The third step is to determine the value of the ridge constant in the multinomial ridge logistic regression model. In this paper, ridge values were determined using a new method, the Wu-Asar estimator (k_{WA}). The (k_{WA}) constant value is given by the following equation (9). Then the ridge value will be compared with other methods, ridge trace (k_{rt}) and SRW constant (k_{SRW}).

Ridge Trace

The ridge trace is a plot between the ridge $\hat{\beta}$ against k values, with $k \in \{0,1\}$. Through the ridge trace, the goal is to choose k with a small value, where at this k it is considered that the regression coefficient is starting to stabilize [4].

SRW Constant

Another analytic method for selecting the ridge constant was introduced by [6], calculating the k_{SRW} value using the SRW formula.

$$k_{SRW} = \frac{1}{a^2{}_{max}}$$

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3.3. Modeling with Multinomial Logistic Regression

We use the response to have three categories, and Setosa is used as a comparison between the Versicolor and Virginica categories. The results of the estimation and modeling of Iris data with multinomial regression are explained as follows.

Table 2. Result of logistic regression

Respon		$\hat{\beta}$	std. error	Wald
Versicolor	X_1	-5.9063	61.1412	0.0093
	X_2	-6.2887	81.0242	0.006
	X_3	11.941	70.2095	0.0289
	X_4	0.3984	130.524	0
Virginica	X_1	-8.3739	61.1632	0.0187
	X_2	-12.924	81.1539	0.0254
	X_3	21.3062	70.4283	0.0915
	X_4	18.5437	130.664	0.0201

Table 2 shows the results of parameter estimation and Wald test statistics. It is known that the value of the standard error $\hat{\beta}$ is quite large, this causes the variance to widen, and the test statistic becomes insignificant.

3.4 Multicollinearity Checking

The multicollinearity test is performed by examining the VIF values. Table 3 shows that at VIF > 5 there is evidence of multicollinearity in the variable sepal length (X_1). Meanwhile, VIF values greater than 10 indicate high multicollinearity in the petal length and petal width variables.

Table 3. VIF value

X_1	X_2	X_3	X_4
7.073	2.1	31.262	16.09

3.5 Modeling with Multinomial Ridge Logistic Regression

The initial step of modeling with ridge method is to estimate the parameters based on equation (20). The selection of the ridge value will be done with ridge trace (k_{rt}), SRW formula (k_{SRW}) and new constants Wu-Asar (k_{WA}). Ridge value calculation is done in R software and produces the following output Table 4.

Table 4. Selection ridge value

Respon	Predictor	(k_{rt}) Std. error	(k_{SRW}) Std. error	(k_{WA}) Std. error
Versicolor	X_1	1.22	1.11	0.59
	X_2	1.51	1.37	0.73
	X_3	1.19	1.09	0.58
	X_4	2.06	1.87	0.97
Virginica	X_1	1.35	1.23	0.66
	X_2	1.74	1.59	0.87
	X_3	1.47	1.34	0.73
	X_4	2.16	1.96	1.01

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Ridge values for each constant are $k_{rt} = 0.056$, $k_{SRW} = 0.0698$ and $k_{WA} = 0.305$. The lowest standard error of $\widehat{\boldsymbol{\beta}}$ value is the best ridge constant value. From Table 4, we can see that the new approach k_{WA} has the best ridge constant with a ridge value of 0.305.

Here are the results of a hypothesis test on the significance parameter of the multinomial ridge logistic regression with the Wu-Asar ridge constant. The result of overall fit test is a p-value 0.000 < $\alpha(0.05)$ indicates that there is at least one predictor that has a simultaneous/significant influence on the model. Individual tests were performed by the Wald test described in Table 5.

Table 5. Parameter estimation

Respon Category	variable	$\hat{\beta}$	Std. error	Wald	p-value
Versicolor	Intercept	0.685	1.167	0.345	0.557
	Sepal Length	-0.163	0.591	0.077	0.782
	Sepal Width	-1.749	0.731	5.73	0.017
	Petal Length	2.064	0.581	12.60	0.0004
	Petal Width	-0.22	0.973	0.052	0.82
Virginica	Intercept	-1.66	1.187	1.9644	0.161
	Sepal Length	-1.87	0.66	8.0845	0.0045
	Sepal Width	-2.85	0.866	10.8578	0.001
	Petal Length	4.226	0.732	33.31	0
	Petal Width	3.18	1.009	9.941	0.0016

Individual test using the Wald test and p-value. In Table 5, it is known that all predictors had an effect on the Virginica category. Meanwhile, in the response of the Versicolor category, only sepal length had no significant effect. When Setosa is compared, it is known that the larger the petal length and width, the higher the probability of being included in Virginica.

From the results obtained were the standard error value of the parameter with ridge method was smaller than that of the model before using the ridge constant. Rahmawati & Suratman [13] tested the performance of the ridge regression and the lasso approach on data with multicollinearity. The result of mean squared error (MSE) shows that the performance of ridge regression is better than lasso regression. Ridge regression will provide a better performance on data with a lot of predictors and relatively equal coefficients. Other research [14] is modelling poverty in South Sulawesi, where three of the seven predictor indicates multicollinearity. The results obtained were the MSE value of the parameter estimator smaller than the weighted logistic regression model before using the ridge value. This shows that the ridge method is more effective if there are multicollinearity problems. Fitri et al. [15] found the same thing, in their analysis of poverty levels in West Sumatra Province where the predictor variables contained multicollinearity, the ridge regression model was the best model.

3.6. Classification Accuracy

The classification results of the multinomial ridge logistic regression method based on the confusion matrix are shown in Table 6.

Table 6. Confusion matrix performance

Observed	Predicted		
	Setosa	Versicolor	Virginica
Setosa	50	0	0
Versicolor	0	48	2
Virginica	0	1	49

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Table 6 shows that the classification accuracy is 98%, which means that up to 147 species variable observations are correctly classified. Sensitivity values for Setosa (100%), Versicolor (98%), and Virginica (96%) were obtained. The specificity values for each category are Setosa (100%), Versicolor (98%), and Virginica (99%).

4. CONCLUSION

The estimation results for multinomial ridge logistic regression were obtained using maximum likelihood estimation (MLE) and iterative reweighted least squares (IRLS) approaches. The experiment was conducted on the Iris dataset with response variables consisting of 3 categories. The application to the Iris dataset was analyzed using multinomial logistic regression (MLR) and multinomial ridge logistic regression. (MRLR). Performance tests using R software obtained the best ridge constant, the Wu-Asar estimator (k_{WA}) with a value of 0.305. The results of modeling and statistical tests of Iris data based on MRLR are better than MLR because the standard error of the estimated parameter becomes smaller. This causes the parameter test statistic that was previously insignificant to become significantly influential. The confusion matrix test shows an overall classification accuracy score of 98% for the Setosa (100%), Versicolor (96%), and Virginica (98%) classifications.

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