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BOLTZMANN-GIBBS ENTROPY TO MEASURE FLUCTUATION OF STOCK INDEX

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ABSTRACT

The disorder of a physical system can be measured based on the entropy of the system. The greater the entropy value of a system, the more disorder the system increases. In Physics, the disorder of system is measured by Boltzmann-Gibbs Entropy. Boltzmann-Gibbs entropy relates the disorder of a system to the probability distribution of the system's states. The Boltzmann-Gibbs entropy can be extended to more general systems to become the Shanon entropy widely used in information theory. In this research, Boltzmann-Gibbs entropy is used to measure stock market disorder or the fluctuation of stock price. The fluctuation in the stock market are more focused on disorder movements, therefore Boltzman Gibbs entropy is developed into time-dependent entropy using a sliding window technique. The samples in this study were stock indices in Germany (GDAXI), China (HIS), Japan (Nikkei), the United States (Dow-Jones), and Indonesia (IDX). The calculation results show an increase in Boltzman-Gibbs entropy when the economic crisis begins. Entropy decreases when the crisis begins to subside towards a more stable.

Keywords: Boltzman-Gibbs Entropy, stock, fluctuation

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INTRODUCTION

Physics has now been applied to fields outside of physics, including in economics, especially in financial markets. One of the interesting things in economics for physicists is the abundance of data in financial markets. This data is very interesting to process so that regular patterns appear that can explain various phenomena in the financial markets [1-9]. Data in financial markets can be assumed to be like data in a system of many particles so that data regularity patterns can be approximated by statistical mechanics and stochastic processes. Variables in financial markets such as security prices, index prices, and exchange rates fluctuate greatly so they are often assumed to follow a stochastic process, namely a diffusion process [10-14].

The disorder of a physical system in physics can be expressed by the entropy of the system. Changes in irregularities in a system can occur in various cases, for example in systems that experience disturbances and systems that experience changes in form or phase. The presence of disturbances that cause an increase in the kinetic energy of the particles in a system will cause a transition from order to disorder. In an isothermal process, for example, an object's phase changes from solid to liquid, the addition of heat increases the entropy without increasing the temperature, while in the system there is an irregular transition from a more regular form (solid phase) to a more irregular form (liquid phase). In the case of expansion or increase in volume, the increase in entropy caused by the addition of heat is related to the transition of disorder from a state that is less free to move (small volume) to a state that is more free to move (larger volume). Entropy thus expresses a measure of the disorder of a system. The greater the entropy of a system, the greater the disorder of the system [15].

The entropy used in physics, namely Boltzmann-Gibbs Entropy, has a form similar to Shanon's entropy which is widely used in information theory. Shanon's entropy is derived based on well-established probability theory [16]. Shanon's entropy has been widely used in various fields such as weather and the environment [17, 18] and has also been used in physics related to probability density distributions [19,20-21]. Boltzmann-Gibbs entropy has also been widely used in various fields including transportation [22].

Shannon's entropy has been used to measure the irregularity of currency exchange rates. There is a connection between the increase in Shanon's entropy and a crisis situation. When the economic crisis approaches, there is an increase in Shanon's entropy, which is calculated based on the currency exchange rate, when the crisis begins to end, there is a decrease in Shanon's entropy value [23]. In the situation leading up to the crisis, there was an increase in disorder or fluctuation in financial markets, which was characterized by increasing randomness in currency exchange rates.

Both currency exchange rates and stock prices/stock indices are one of variables in the financial market that fluctuate. Shanon entropy based on currency exchange rates is related to the level of regularity of currency exchange rates, thus disorder in the stock market can be measured using Shanon entropy. Measuring disorder in the stock market can show the economic situation of a country more objectively considering that stock prices are more related to the economic situation of a country itself. Meanwhile, the currency exchange rate not only depends on the country's finances but also depends on the economy of the currency exchange pairing country. In other words, entropy based on prices or stock indices shows the economic condition of a country more precisely than entropy based on money exchange rates. As previously mentioned, Shanon Entropy has a similar form to Boltzmann- Gibbs entropy, so it is possible for Boltzmann-Gibbs Entropy to be used to measure irregularities in the stock market. Boltzman-Gibbs entropy makes it possible to divide groups into smaller ones so that more refined monetary probability values can be obtained.

In this research, the possibility of using Boltzman-Gibbs entropy to measure disorder in the stock market was examined, using stock index samples in four countries, namely China, Germany, America, and Indonesia, especially irregularities during the economic crisis. Index irregularities have increased during the world economic crisis. In the period from 1995-2020, there were several economic crises in the world, including the Asian crisis (1997-1998) and the global crisis (2007-2009). The crisis affected commodity prices, currency exchange rates, GDP and also stock prices in various countries [23,24,25]. The Asian crisis not only affected countries in Asia but also countries outside Asia such as Latin America, Russia, North America, and Europe [24]. Meanwhile, the global crisis started in the United States and then spread throughout the world.

The research method begins with an explanation of the relationship between Shanon entropy and Boltzman-Gibbs entropy and the sliding window method, then explains how to carry out the calculations. In the results and discussion section, the entropy movement will be shown for 25 years, that is from 1995 to 2020.

METHOD

Boltzmann-Gibbs Entropy

Boltzmann-Gibbs entropy has a long history. Starting from the thermodynamic entropy stated by Claucius, namely $S = dQ/T$, Boltzmann related the thermodynamic entropy to the number of microstates in a macrostate of the system which is now known as $S_B = k \ln W(a)$. The notation W is the total number of microstates associated with a macroscopic state (a) which is called the thermodynamic probability of the system by Boltzmann. The quantity W itself expresses the degree of disorder of the system so Boltzman's principle can be expressed as $S = k\ ln\ (disorder)$, in other words, entropy measures the disorder of a system [26].

The equation $S_B = k \ln W(a)$ itself was not formulated by Boltzmann but was formulated by Planck who called the equation as Boltzmann Entropy. Boltzmann calculated the value of $\ln W(a)$ in an ideal gas and obtained the energy distribution of ideal gas particles. Gibbs then expanded Boltzmann Entropy so that it applies to more general conditions. The Boltzmann Gibbs entropy for a classical particle system with discrete microstates is

$$S_{BG} = -k \sum_{i=1}^{I} p_i \ln(p_i) \tag{1}$$

where p_i is the probability of state i, and k is the Boltzmann constant which is a thermodynamic unit for entropy.

At the time of Boltzmann, Gibbs and Maxwell discussed about entropy and the distribution of particles, they recognized that the distribution of particles was necessary because of limited information about the state of systems of many particles. Knowing all the information on all the particles in a system is impossible, one of the best ways to describe a system is to use a particle distribution function. Unfortunately, at that time a solid theory of opportunity had not yet emerged. At current masses the Boltzmann-Gibbs Entropy formulation can be carried out by utilizing probability theory [27,28,29].

SHANNON ENTROPY

The formula of Shanon's entropy is [27]

$$S_H = -\sum_{i=1}^{N} p_i \ln p_i \tag{2}$$

where p_i represents the probability of state i from a set of system states. In a system that does not have complete information, S_H represents a measure of the incompleteness of system information. The high form of the p_i distribution indicates less information deficiency compared to the wide form of the distribution. How much less is expressed by S_H. The S_H value produced by a probability distribution that forms a sharp peak will be smaller than the S_H value produced by a wide probability distribution.

It can be seen that equation (2) agrees with equation (1). In the case of very many particles and taking $k = 1$ equation (1) becomes equation (2) in other words the Boltzmann-Gibbs entropy becomes Shanon entropy with $I = N = infinity$.

Viewed from the perspective of order, a system will be orderly if the number of possible states is smaller. In the extreme, a situation will be orderly if there is only one possible condition so that only one condition can occur. If the possible states vary, then the system can change from one state to another possible state, thus the system can become disorderly. The more possibilities that occur, the wider the distribution of system opportunities will be, while the fewer possibilities that can occur, the distribution of opportunities will be narrower and higher [30].

SLIDING WINDOW METHOD

Entropy is a independent time quantity. To determine the increase in entropy at any time, the entropy can be modified to be time dependent using a sliding window method by dividing the data series into several sub-data series [31]. Each sub data series will produce entropy at time *t*. For example, N data series are used from day 1 to N. The data series can be divided into K subsets or K windows with $M = N/K$ members each. The first subset or window is a set of data series from the first data to the M data or $\{d_1, d_2, .., d_M\}$, while the second subset or window is a set consisting of data series values from 2 to $M + 1$ or $\{d_2, d_3, \ldots, d_{M+1}\}$. The entropy value at $t = 1$ is thus obtained from data 1 to N or from the data of members of the first subset, the entropy value at $t = 2$ is obtained from data 2 to data $M + 1$ or from data of members of the second subset, and so on. until all subsets are used. Look at a subset i which contains data values on days i to $M + i$. The probability distribution can be found in that window by grouping the return values in each window with the width of the return values in each group being 0.1. For example, group $j = 1$ contains returns with values -1.2 to -1.1, group $j = 2$ contains return values with values -1.1 – 1.0 and so on. The probability distribution in window *j* is:

$$p_{ji} = \frac{Nji}{M}$$

where N_{ji} is number of members of group *j* in window *i*.

The entropy value for each window is:

$$S(t_i) = -\sum_{j=1}^{K} p_{ji} \ln p_{ji} \tag{3}$$

When the economy is stable, entropy is low because share prices are stable or there is not much variation in share value. This situation will be very different from the situation during the economic crisis. During an economic crisis, price uncertainty and fluctuations will increase, so that entropy will increase. Therefore, in financial markets, the more important thing to study is the change in entropy as a function of time to indicate a situation leading to a crisis. When the situation becomes stable, fluctuations will decrease so that entropy over a certain period will also decrease. The need for entropy values over time intervals requires entropy as a function of time.

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The data used in this research is time series data for the stock index in 5 countries, namely GDAXI (Germany), HSI (China), Nikkei (Japan), Dow Jones (USA), and IHSG (Indonesia) from 1995 to 2020. Each index contains 6716 data points, so the total data used is 33580 data points. Data obtained from Yahoo Finance with link https://finance.yahoo.com/markets/stocks. The index value for each index is shown in Figure 1.

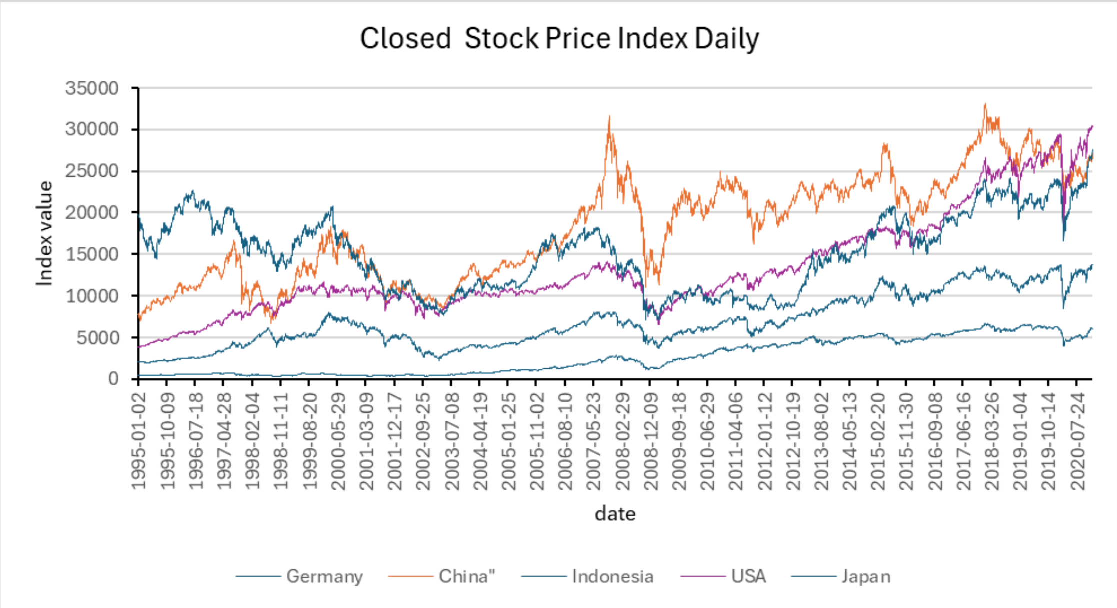

Figure 1. Stock price index daily for each index from 2 January 1995 to 31 December 2020

Table 1 The range data period

Time or data (day)	range data period	
1	3/01/1995 - 9/11/1998	Initial data
2	4/01/1995 - 10/11/1998	
.		
260	2/01/1996 - 8/11/1999	
261	3/01/1996 - 9/11/1999	
.		
550	13/02/1997 - 21/12/2000	the beginning of
551	14/02/1997 - 22/12/2000	the Asian crisis
.		
3125	2/02/2007-16/12/2010	the beginning of
3126	3/02/2007-17/12/2010	the Global crisis

The stages in this research are

1. Convert stock index time series data into return time series data

$$r_i = \ln\left(\frac{S(t_i)}{S(t_{i-1})}\right) \tag{4}$$

2. Create a probability distribution for each window.

   The entropy of the return time series is calculated for a period of every 1000 days or a window width of 1000 days (M=1000). The p_{ji}probability distribution is created for every 1000 data or 5715 windows. Example data and the range data period that uses is shown in Table 1.

3. Calculate the ntropy value for each window using equation (3).

4. Make a graph of entropy as a function of time and carry out analysis.

RESULTS AND DISCUSSION

The graph of entropy against time (day) for each stock index is shown in Figure 2.

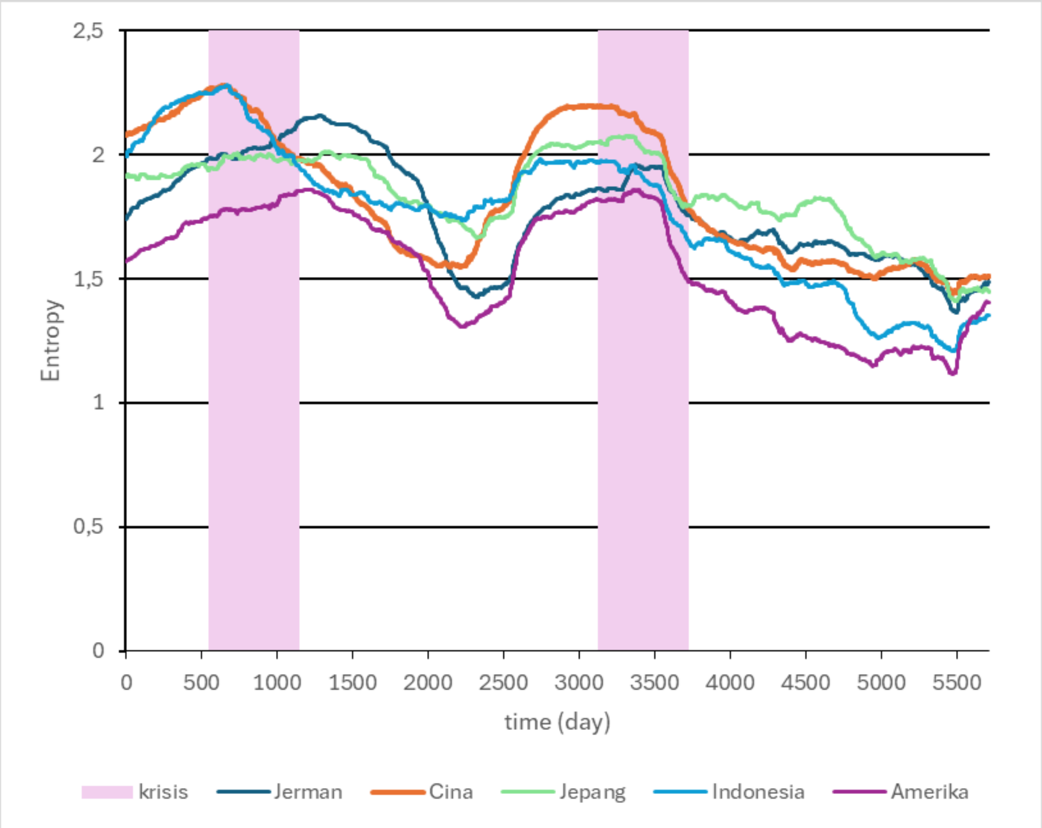

Figure 2. Entropy as a function of time.

The graph in Figure 1 shows that during the Asian crisis, BG entropy increased. Data that uses data during the crisis is shown in areas with pink blocks. The Asian crisis began in July 1997 and ended at the end of 1998. The data period is shown in Table 1. It was seen that an increase in entropy began to occur when we started using data in 1997. The increase in entropy occurred on the 550th day using data from February 13, 1997, to December 21, 2000. In other words, entropy on day 550 uses index data at the time of the crisis. Countries affected by the Asian crisis, namely China and Indonesia, appeared to experience an increase in entropy during the Asian crisis, while Japan's entropy did not change much during the Asian crisis.

The increase in entropy during the Asian crisis shows that entropy increased when there was an increase in stock price uncertainty during the crisis. When the crisis begins to end and the index begins to stabilize or uncertainty decreases in stock prices on the index, it appears that entropy also decreases. Thus, it can be stated that an increase in entropy indicates an increase in disorder, in this case, there is an increase in fluctuations in the index value or an increase in uncertainty in the index value. Meanwhile, the index value describes the average share price of companies listed in the index.

A similar relationship also occurred during the global crisis that occurred in 2007-2008. During the global crisis, almost all countries experienced a financial crisis. There appears to be an increase in index entropy in Germany, China, Japan and also Indonesia. Entropy using data during a global crisis appears to have increased. It can be seen that starting from the 3125th data (using the index value on February 2, 2007 to December 16, 2010) which uses data when the crisis occurred, the entropy increases, then starts to decrease when the crisis ends or uses data after the crisis ends. It can be said that an increase in index entropy indicates an increase in index or stock price irregularities or an increase in fluctuations which indicates uncertainty in the stock market during the global crisis, such as an increase in entropy during the Asian crisis.

Both in the Asian crisis and in the global crisis, namely, when there was increased fluctuation and uncertainty, Boltzmann-Gibbs entropy increased. On the other hand, when the crisis begins to subside, the Boltzmann-Gibbs entropy value decreases. Thus the Boltzmann-Gibbs Entropy can be used to measure or characterize the increase or decrease in irregularities in the stock market. When the Boltzmann-Gibbs entropy increases, irregularity in the stock market increases.

CONCLUSION

Based on the graphic pattern in Figure 1, it appears that entropy increases when a crisis occurs and decreases when the crisis begins to end, both during the Aisa crisis and during the Global crisis. On the other hand, when a crisis occurs, there is an increase in stock price fluctuations or stock price irregularities occur. It appears that entropy increases when there is an increase in disorder, thus the Boltzman Gibbs-Shanon entropy can be used as a measure of stock price disorder.

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