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ESTIMATE MASS DENSITY VALUE AS A PRIORI INFORMATION FOR GRAVITY BY USING BAYESIAN MARKOV CHAIN MONTE CARLO (MCMC)

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ABSTRACT

In the gravity method, information about mass density value is very important because it will influence the characteristic of the 1-D gravity acceleration graph. However, it is quite difficult to guess the mass density value so that is suitable to the 1-D acceleration graph. This is called "a priori" information. Trial and error way is one way to solve this problem. It is a very random value guess also. To make sure that the initial guess of mass density is a good parameter, Bayesian Markov Chain Monte Carlo (MCMC) can be used. It generates many possibilities from the guess value and then these possibilities will be selected to the best one by likelihood way. The validation is expressed by the random graph as a consequence of the iteration number step of the possibilities. This research is started by using certain values of mass density to create a synthetic model for the field data in Banggai Sula because the area has a complex geology. The synthetic model is used because the gravity forward modelling equation has the sinusoidal form. After Bayesian MCMC is applied to the initial mass density value, it will produce a new mass density value or the estimation value with its response to the 1-D gravity acceleration synthetic graph. Finally, this information will be very useful to create the 2D or 3D inverse modelling in Gravity.

Keywords: mass density; gravity method; Bayesian Markov Chain Monte Carlo; MCMC

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INTRODUCTION

Forward and Inverse Modelling is the important thing in describing the subsurface in Geophysics based on the earth's physics parameters. Forward modelling claims these parameters in the data while inverse modelling guesses the subsurface condition based on the data. Both of them are vice versa. However, sometimes to do forward and inverse modelling, it needs some additional information depending on the method and the case. This is called "a priori" information where it is important to guess the physics information to make the modelling become easier [1].

One example method in Geophysics is gravity. By using gravity, the subsurface condition will be known based on the density parameter as the equation of Gravity:

$$g = G\frac{\rho V}{(x^2 + z^2)^{1.5}} \quad (1)$$

where:

g : gravity acceleration (m/s^2)

G : gravitational constant (N m^2/kg^2)

ρ : mass density (kg/m^3)

V : mass volume (m^3)

x : mass location (m)

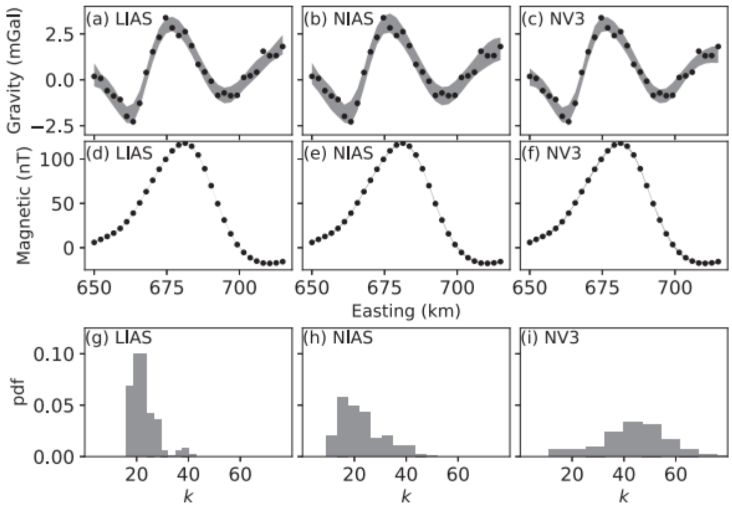
z: depth location (m)

According to equation (1), the gravity acceleration is depended on the mass density [2]. While the fact that the data can be recorded is the gravity acceleration, it means that the mass density should be predicted from the data beside of the location and the depth also.

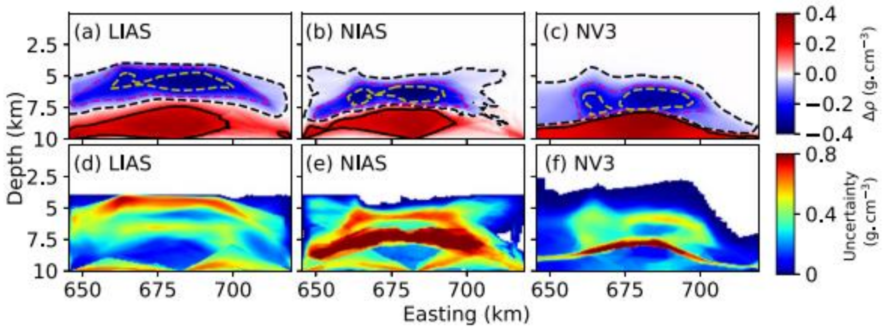
However, the location and the depth of the mass are not really influence the response of the gravity acceleration. And they are assumed to be constant, although in the other case they can be predicted by non linier inversion but in this paper we will focus on the prediction the mass density by using Bayesian Markov Chain Monte Carlo (MCMC).

Bayesian MCMC is the statistical way to estimate the probabilistic of prior parameter (then it called posterior parameter) by using certain distribution. In this paper the Gaussian distribution will be used to generate the mass density probability function. After that the good estimation will be selected by likelihood method. To validate the result, the posterior parameter will be applied to equation (1) and if the 1-D gravity graph is match with the observed data it means that the mass density information is suitable with the field condition [2].

Actually, Bayesian MCMC is a flexible method. It can apply to any method in geophysics to estimate any parameter. It is not important about the relation between one parameter to antoher like in inversion case. For example, Bayesian MCMC well in predicting the 2-D geometry anomaly of gravity and magnetic method by mixing density prediction and trans Dimensional polygon [3],[4]. In this case, Bayesian MCMC predict the density value dissemination of the polygon. The final model refers to Complete Bouger Anomaly [5]. In practice, Bayesian MCMC make the observation of non uniqueness problem in inversion case become easier [6] that is in gravity method is predicting gravity anomaly. However, Bayesian MCMC is included to the geostatistical way (probabilistic) in estimate the parameter. It is like an alternative way in inversion method especially when the inversion is quite difficult to be used in describe the subsurface [7].



(a)



(b)

Figure 1. (a) Gravity and Magnetic Anomaly with the prior model, first row graph represents Gravity anomaly at three different places (LIAS, NIAS and NV3) while the second row show the magnetic anomaly and the third row shows probability density function of both; (b) Gravity and Magnetic 2-D geometry anomaly based on Bayesian MCMC (Ghalenoei et al, 2022)

Furthermore, Bayesian MCMC can be replaced the inverse modelling because of its effectiveness. Bayesian MCMC neglecting some steps in inverse modelling and make it easier. Just by setting up the density value and looking for the graph that suitable to the original one, it can be assumed as the best model. This is the big gap between Bayesian MCMC and the inverse modelling. This is also the something new in geophysical way. Applied some statistical process not only for predicting but also can do the inversion method. Unfortunately, just a few papers use this method in geophysical method.

METHOD

This research is started by develop the synthetic model of 1-D gravity acceleration graph by assume that the mass has the circle form. The certain value (prior parameters) of density mass, location, radius, and the depth are applied to equation (1) to get the 1-D gravity acceleration graph. Then the density mass value is generated to the probability function by using Gaussian distribution. This distribution act as an input for MCMC method where the MCMC will create many possibilities distribution as the final result by equation (2)

$$Dens = \frac{counts}{\sum counts\ x\ (bin\ width)} \quad (2)$$

where counts represent the frequency of the distribution while bin width is the number of the distribution data [8].

The posterior parameter is estimated from likelihood process of Bayesian inference. The equation of Bayesian is shown in equation (3)

$$P(h|e) = \frac{P(e|h)P(h)}{P(e)} \quad (3)$$

P in equation (3) is probability while h and e are hypothesis and evidence. $P(h|e)$ states the probability of the hypothesis to evidence. Likelihood function is expressed by $P(e|h)$. The general insight of equation (3) is how far the hypothesis probability based on some evidence that given in the data [9],[10].

In other words, $P(h|e)$ in equation (3) is how the probability of specific value of mass density as a prior information can be suitable with the 1-D gravity acceleration data to the many mass density values possibilities [7]. Actually, Bayesian inference deliver the inverse problem to the statistical approach. In inverse problem, equation (1) can be rewritten as:

$$d = Km \quad (4)$$

where d is the data (g), m is the parameter model (ρ) and the other is included to K or the Kernell matrix as the forward modelling equation. To get the parameter model, equation (4) needs to be solved as:

$$m = (K^T K)^{-1} K^T d \quad (5)$$

According to equation (5), the solution of inverse problem is similar with the posterior result of Bayesian inference [11] with the likelihood represents the Root Mean Square (RMS) error [12].

As mention before that this paper develop synthetic data first to get the good description of Bayesian MCMC of equation (1). After that, the method will be applied to the Gravity Data from Topex XYZ [13] around Banggai Sula area in Indonesia. This area is chosen because it has complex geological condition with high and low 1-D gravity acceleration data as refer to Usman and Panuju (2013) research [14].

Based on previous explanation, probability is an important thing in Bayesian MCMC. In other words, Bayesian MCMC plays in non stationary case, when the model can change anytime. Because it is very flexible, it can be so wild in producing the posterior model/data. To cover this problem, it is required to have a good knowledge of the estimation parameter itself based on the prior data [15].

RESULTS AND DISCUSSION

The simple synthetic 1-D gravity acceleration is developed with the prior density mass value about 750 kg/m^3. By using equation (1) the 1-D gravity acceleration can be seen in Figure 2.

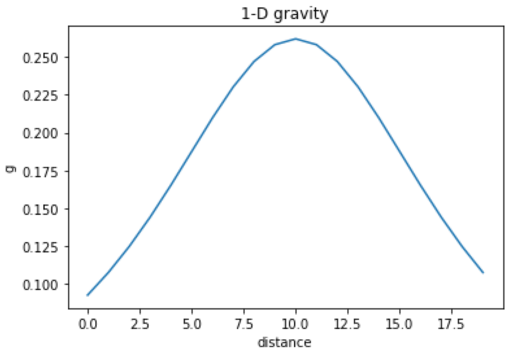


Figure 2. Simple 1-D gravity acceleration

Then The Bayesian MCMC is applied to the synthetic model and the new mass density value is observed to be 747.5 kg/m^3 like in Figure 3.

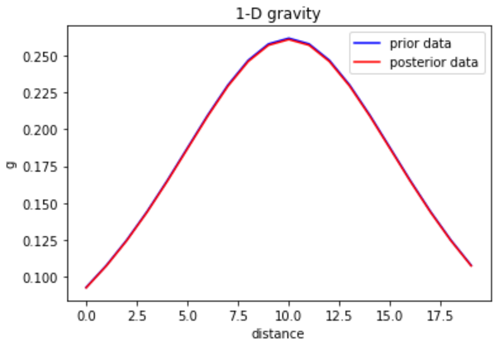
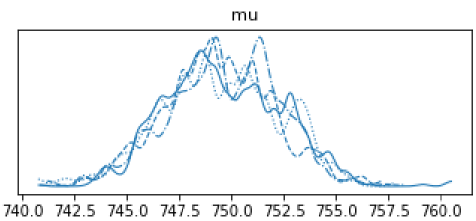
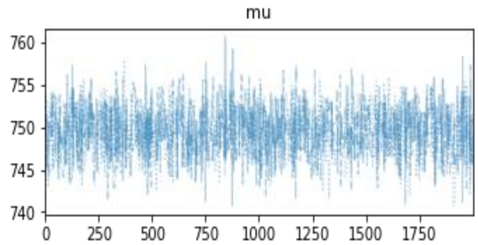


Figure 3. Simple 1-D gravity acceleration with prior mass density (750 kg/m^3, blue) and posterior mass density (747.5 kg/m^3, red)

Based on Figure 3, posterior mass density as a result of Bayesian MCMC produce 1-D gravity acceleration coincide with the prior data. The posterior data is observed with 2000 iteration to result best model (high possibility).



(a)



(b)

Figure 4. (a) Posterior mass density (b) Validation graph

Bayesian MCMC validation is shown in Figure 4 (b). In Bayesian concept, the best model is observed from the random graph.

The same way is applied to the “real” gravity data refer to Usman and Panuju (2013) research with the area of Banggai Sula. The gravity map is shown in Figure 5.

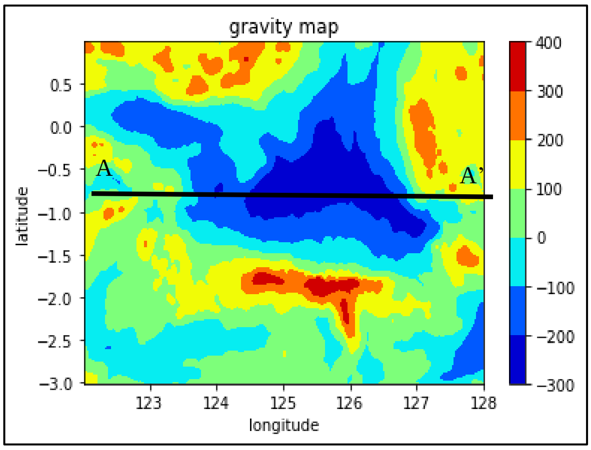


Figure 5. Banggai Sula gravity map from Topex

A slice that is named A-A’ is digitized from west to east on the map to get the 1-D gravity model (Figure 6) and the g value is converted to single number to make the Bayesian calculation simpler.

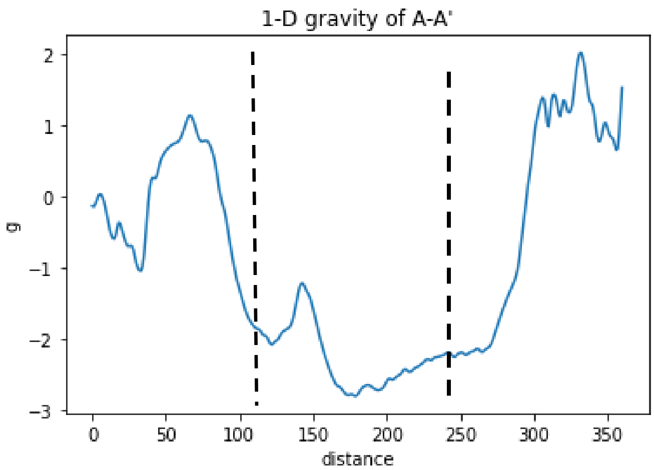


Figure 6. 1-D gravity acceleration graph of A-A’. There are two dashed line dividing the graph into three parts to be more analyzed (Figure 7, 8 and 9)

Because the graph of A-A’ is not smooth, while the graph from equation (1) is usually in sinusoidal form and, the synthetic model of the graph then made. To make it easier, the graph is divided into 3 parts.

The first part is included 100 first data. The synthetic model with mass density is about 800 kg/m^3 has the closest 1-D gravity acceleration graph to the original data.

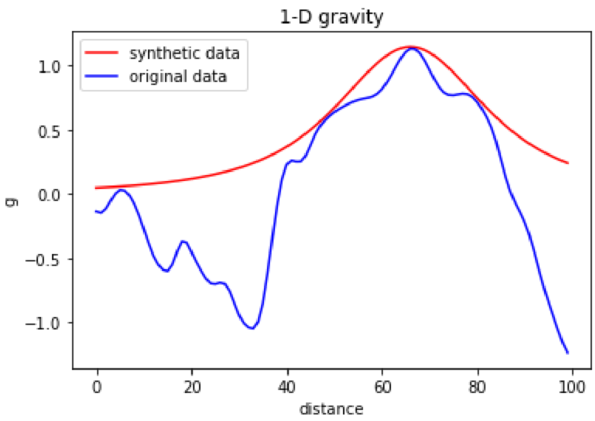


Figure 7. The synthetic data of first part

The second part continue the first part with 150 data. For the prior mass density information is -1000 kg/m^3. The negative sign means the low mass density.

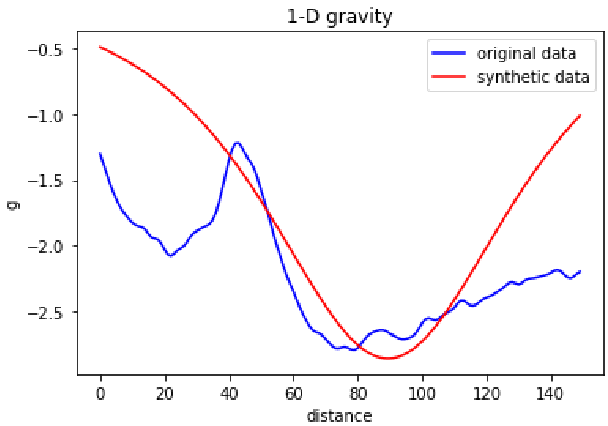


Figure 8. Second part synthetic data

The last part consist of 111 data which is not included to the first and second part. The prior mass density value of this part is about 1000 kg/m^3.

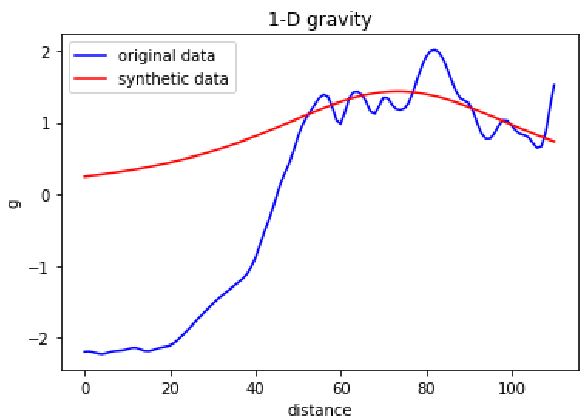


Figure 9. The last part of synthetic data

Each of prior mass density above then processed with 2000 iteration with Bayesian MCMC method and the result are 797.5 kg/m^3, -899 kg/m^3 and 1047 kg/m^3. Comparing to the prior data, so the error calculation for three densities are 0.3 %, 10.1 % and 4.7 %. The final 1-D gravity graph can be seen in Figure 10

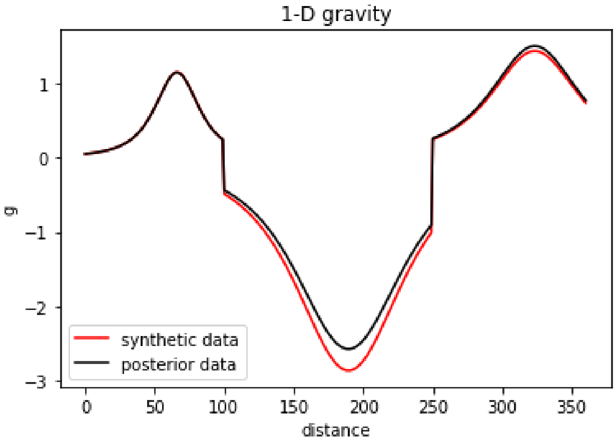
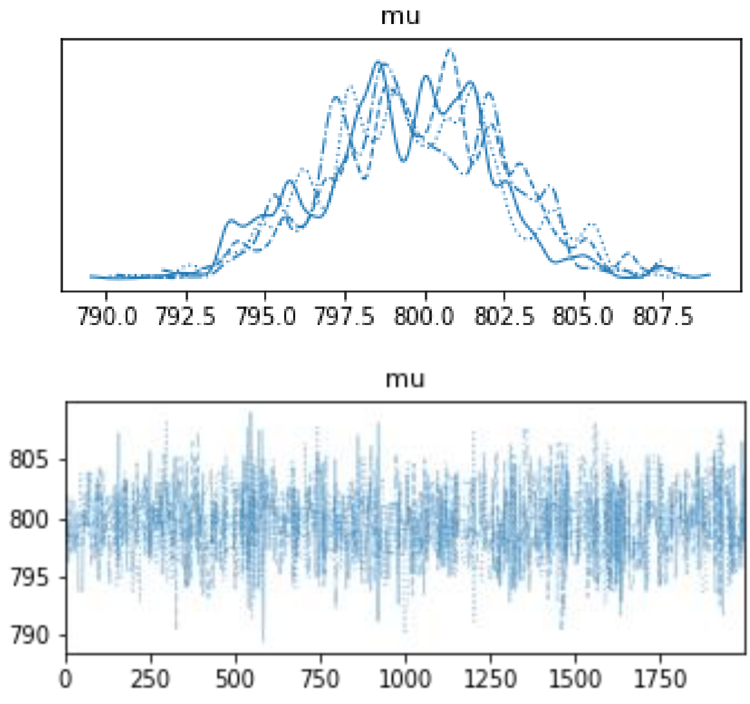


Figure 10. Posterior result of the synthetic data

While the posterior and validation graph (traces) can be seen in Figure 11 (each of Figure 11. (a), (b) and (c) consist of two graphs, the top graph is the posterior data while the bottom is the traces). In Bayesian MCMC, more bayes the traces means that the posterior data reaches to the best result.



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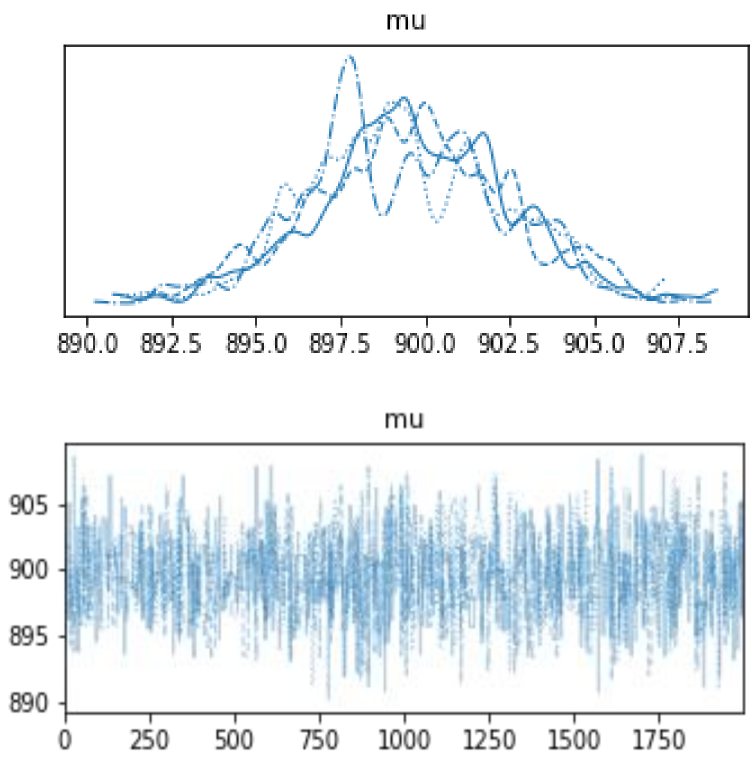

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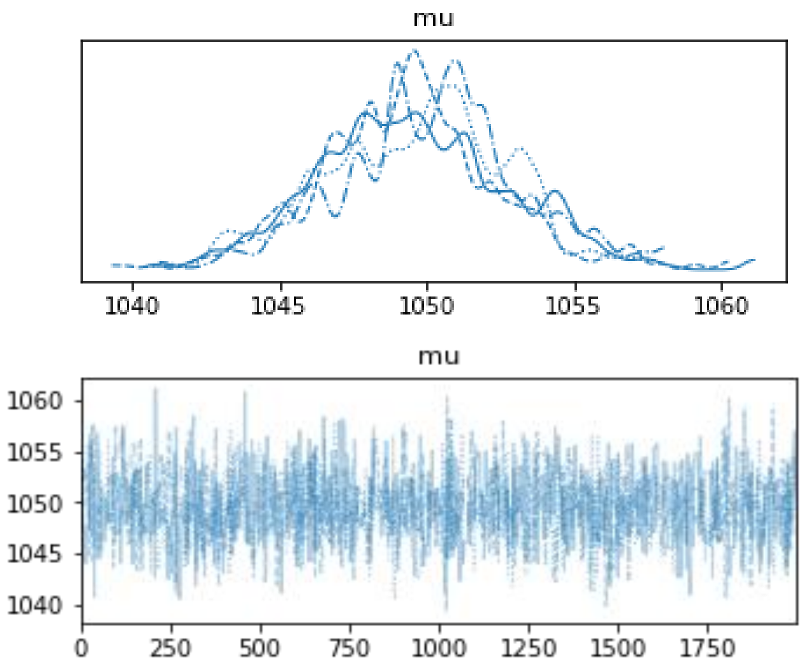

(c)

Figure 11. Posterior data and the validation in traces for (a) first part (b) second part and (c) the last part

Based on Figure 10, although the synthetic and posterior gravity graph are not suitable precisely to the original 1-D gravity graph, they still have similar characterization. It can be seen that the low gravity graph is dominated with the low mass density also and vice versa. The posterior mass density result is just the prediction of the mass density value around research area and it will be useful to create the advance modelling like 2D or 3D model. To make sure that the posterior model is the best result, it is important to look at the traces in Figure 11. In accordance with the name, that is Bayesian, this method is very statistical and has a tend to result the bayes / chaos result as the best result.

CONCLUSION

Mass density prediction as "a priori" information is an important thing in gravity. It can give the general description about the lithology around research area. To make sure that the mass density information is good enough, Bayesian MCMC can be used to guess the mass density exactly. Bayesian MCMC generates many possibilities of initial guess parameter and by using likelihood method, the best one will be selected from some iteration step. Nevertheless, this method also has weakness. Bayesian MCMC is very bayes and can resulting many posterior data as the result. So it is important to the user to do some trial and errors test until the best result is covered (like suitable to the geological condition in some cases).

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