COSMOLOGICAL CONSEQUENCE OF VARYING SPEED OF LIGHT AND GRAVITATIONAL CONSTANT

Gabriel Wirdzelii Joseph*[1], Lucky Peter Kenda[1], Maxwell Obia Kanu[1], Muslim Usman Abdullahi[2]

[1]Department of Physics, Taraba State University, P. M. B. 1167, Jalingo, Taraba State, Nigeria
[2] Department of Chemistry, College of Education Zinc, Taraba State, Nigeria
* gabrielwjoseph@gmail.com

ABSTRACT

The speed of light is taken to be a constant in a vacuum. This forms the essential tool for the principle of General Covariance, which asserts that all laws of Physics should take the same form in all frames of reference. Without considering inflation, the theory of the varying speed of light (VSL) would solve fundamental problems of cosmology in the early universe. Furthermore, the gravitational constant G that occurred in the Friedmann Equations may not have been a fundamental constant in the early universe but may have some variation with the universe scale factor. Cosmological models with varying physical constants have been of interest in recent years, with few works in the literature. A cosmological solution obtained by incorporating variable speed of light and gravitational constant gives a cosmic model free from the initial Big Bang singularity and horizon problem. It is also observed here that the early universe was dominated by radiation; however, as the scale factor increases, the dark energy becomes dominant.

Keywords: VSL; energy density; dark energy; dark matter; Hubble parameter

Cite this as: Joseph, G. W., Kenda, L. P., Kanu, M.O., & Abdullahi, M. U. 2024. Cosmological Consequence of Varying Speed of Light and Gravitational Constant. *IJAP: Indonesian Journal of Applied Physics, 14*(1), 120-126. doi: https://doi.org/10.13057/ijap.v14i1.74838

INTRODUCTION

The principle of general covariance, which asserts that the laws of Physics take the same form in all reference frames, derives its base from the constancy of the speed of light in a vacuum [1]. However, astronomical studies in recent times from quasars linked the fine structure constant to depend on redshift. This suggests a variation of the redshift cosmic time [2]. Observation from cosmic microwave background (CMB) and supernova type-Ia suggests that the universe's expansion is accelerating [3].

An outstanding challenge confronting the history of cosmology is the implications of the supernovae data, which revealed that cosmic expansion is now accelerating [4–6]. Afterward, to deepen the understanding of the origin of the comic acceleration, simpler models of cosmological expansion became insufficient, and efforts were being made to connect theory with observation to give us what transpired before, during, and after the Big Bang. Observational data from the measurement of CMB radiation revealed that 4.9% of the universe's constituents come from ordinary matter, while dark energy and dark matter constitute 68.3% and 26.8%, respectively [7]. The simplest and most well-known explanation is the cosmological constant (a minimal and constant amount of energy with enough negative pressure which acts as a repulsive

force and causes accelerated expansion). Although this appears to explain the cosmic acceleration, it suffers from the problem of reconciliation of the observed and theoretical energy density; this is called the cosmological constant problem [8]. This has created room for dynamic dark energy models, possible variations of the cosmological constant, and other physical constants of nature [9-10]. Ref [11] derived the generalized Einstein-Schrodinger theory, gravitational field equations incorporated into a general perfect fluid. The spacetime was considered immersed in a larger eight-dimensional space, and this construction leads to a geometrical origin of the velocity vectors due to the immersion. This theory aimed to give a geometric origin of matter in the universe through the obtained perfect fluid. Applying the above theory to cosmology has been made as an attempt to give a geometrical origin to the cosmological constant. A time-dependent cosmological constant is derived in that model where its energy density decays via the scale factor.

The so-called standard model of cosmology (SCM) [12] suffers from many problems, such as the singularity problem, the inflation problem, the horizon problems, and the flatness problems [13]. Further, the standard model asserts that the universe is decelerating. This is contrary to the recent astronomical data from modern telescopes showing that the universe is in a state of accelerating expansion. Recent studies revealed that dark energy is responsible for this accelerating expansion of our universe. One such study is the varying speed of light theory. To fully understand the nature and forms of dark energy, there is a need to develop other models of dark energy to add to the existing literature.

METHODS

Varying speed of light (VSL) cosmology [14] has recently received considerable attention as an alternative to cosmological inflation to provide a different basis for resolving the problems of the standard model. Barrow [15] shows that the conception of VSL can lead to the resolution of horizon, flatness, and monopole problems if the speed of light falls at an appropriate rate. Instead of adopting the inflationary idea that the very early universe experienced a period of superluminal expansion, in these universes, one assumes that light traveled faster in the early universe. In such cosmological models, the known puzzles of the standard early universe cosmology are absent at the cost of breaking the general covariance of the underlined gravity theory. We do not enter into a discussion here on the foundations of VSL theories and the conceptual problems arising from the very meaning of varying the speed of light.

The Newtonian gravitational constant G occurs in the source term of Einstein's field equation of the general theory of relativity, which is a fundamental equation for developing every model of cosmology. In Einstein's field equation, G acts as a coupling constant between the geometry of spacetime and matter. In quantum mechanics, G is essential in defining the Planck constant [16]. Meanwhile, in SCM, G is an invariant quantity. It has been noted that there is significant evidence that the gravitational constant G can vary in time [17]. The Einstein field equation is still preserved when either G or the cosmological constant Λ is varied with time. It has been shown that the variable Newtonian gravitation constant can account for dark energy and most of its effects, and current dynamical dark energy models using time-dependent cosmological constant terms are being considered [18].

In this research, we apply the ansantz of varying speed of light and gravitational constant to the Friedmann equations obtained in [19] to study the cosmic evolution of energy density in the early universe and other cosmic parameters. The study shows that the universe's energy density is the combination of matter-energy density and dark energy density. The study also revealed that in the very early universe.

RESULT AND DISCUSSION

Cosmology with VSL and Variable G

In varying speed of light (VSL) theories, the action is [15]:

$$s = \int dx^4 \sqrt{-g} \left(\frac{R+\lambda}{16\pi G} + L_m \right) \tag{1}$$

Varying the action concerning the metric and ignoring surface terms leads to [15]:

$$R_{\mu v} - g_{\mu v} \Lambda = \left(\frac{8\pi G}{c^4} \right) T_{\mu v} \tag{2}$$

In a cosmological context, let dark energy scenario allow for a varying Newton's constant G and speed of light c, the Friedmann metric can be written as [15-20]:

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right] \tag{3}$$

Where $a(t)$ is the scale factor, t is the comoving time, and K=0,1,-1 represents a flat, closed, and open FRW universe, respectively. The Friedmann equations are:

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G(t)}{3}(\rho_D + \rho_m) - \frac{Kc(t)^2}{a^2} \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3} \left\{ (\rho_D + \rho_m) + 3\frac{P}{c(t)^2} \right\} \tag{5}$$

Where $\rho_{eff} = \rho_D + \rho_m$ is the effective energy density. Note that H is the Hubble parameter, and the dot is the differentiation concerning time. Moreover, $\rho_m = \frac{m_0}{a^3}$ (index 0 marks the present value of a quantity) and ρ_D denote the energy densities of matter and dark energy (DE), respectively. The equation of state parameter is given by denoted by $\omega = \frac{P_D}{\rho_D}$. Now, assuming the speed of light c and G are in the power-law form of scale factor, ref [15] ansatz is given by:

$$dG = G_0 a(t)^m \tag{6}$$
$$c = c_0 a(t)^n \tag{7}$$

Where c_0 and G_0 are both positive constants. Since we know that the speed of light decreases with time and G increases with time, n must be negative, and m must be positive. The conservation equation is given by [21]:

$$\dot{\rho}_{eff} + 3\frac{\dot{a}}{a} \left(\rho_{eff} + \frac{P}{c^2} \right) = -\rho_{eff} \frac{\dot{G}}{G} + 3\frac{Kc\dot{c}}{4\pi G a^2} \tag{8}$$

For the case of flat FRW (K=0), the above conservation equation can be written as forms:

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m + \rho_m \frac{\dot{G}}{G} = 0 \tag{9}$$

$$\dot{\rho}_D + 3\frac{\dot{a}}{a} \left(\rho_D + \frac{P_D}{c^2} \right) + \rho_D \frac{\dot{G}}{G} = 0 \tag{10}$$

Using the equation of state parameter $P_D = \omega\rho_D$, the conservation equation (10) becomes:

$$\sigma(x) = \frac{1}{1+e^{-x}} \tag{11}$$

Evolution of Energy Density of The Universe

With varying gravitational constant, the matter-energy density of the universe obtained from equation (9) is:

$$\rho_m = \frac{\rho_{m0}}{a^{3+m}} \tag{12}$$

Where m_0 is a constant and denotes the present matter content of the universe. For $m = 0$, described radiation-dominated universe and $m = 1$ marks matter-dominated universe respectively with energy densities given by:

$$\rho_r = \frac{\rho_{m0}}{a^3} \tag{13}$$

$$\rho_m = \frac{\rho_{m0}}{a^4} \tag{14}$$

Putting equations (6) and (7) into (9), the dark energy density is gotten to be:

$$\rho_D = e^{\frac{3\omega a^{-2n}}{2nc_0^2}} a^{-3-m} \rho_{m0} \tag{15}$$

Or very large scale factor $a(t)$ $\rho_D = e^{\frac{3\omega a^{-2n}}{2nc_0^2}} \to 1$ and $\rho_D \sim \rho_m$. Therefore, effective energy density $\rho_{eff} = \rho_D + \rho_m$ becomes:

$$\rho_{eff} = \left(1 + e^{\frac{3\omega a^{-2n}}{2nc_0^2}}\right) \frac{\rho_{m0}}{a^{3+m}} \tag{16}$$

The exponential term denotes the dark energy density responsible for the present accelerating universe expansion.
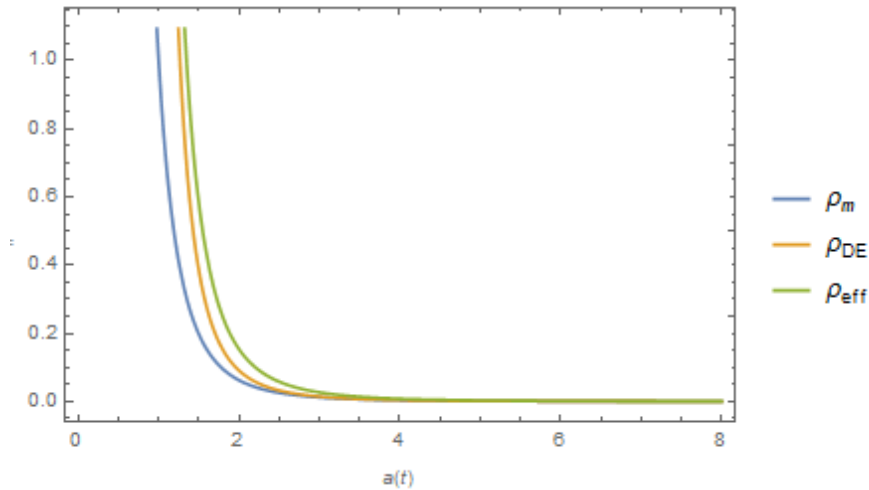

Figure 1. Evolution of Energy Density of the Universe

The variation of the energy density concerning scale factor *a* is shown in Figure 1. From the plot, it is clear that the universe's density decreases with the expansion of the universe. With dark energy, the expansion rate is slightly high and higher with effective energy density, and at large times, the expansion rate starts increasing again. Substituting equation (12) into the first Friedmann's equation (4), we obtained the scale factor of the universe given by:

$$a[t] = \frac{(3t_0 + 2\sqrt{6\pi G_0 m_0} t)^{2/3}}{2^{2/3}} \tag{17}$$

Where t_0 is the time of the Big Bang. The evolution of the scale factor for radiation and the matter-dominated universe is shown in Figure 2 below, which gives the de Sitter universe.
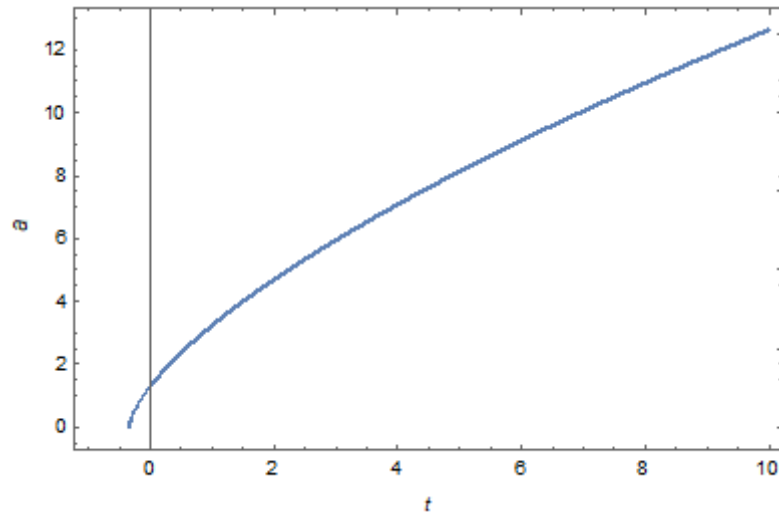

Figure 2. Variation of Scale Factor

The Hubble Parameter

The Hubble parameter from equation (17) is obtained as:

$$H = \frac{4\sqrt{\frac{2}{3}}\sqrt{G_0 \pi m_0}}{3t_0 + 2\sqrt{6}t\sqrt{G_0 \pi m_0}} \tag{18}$$

Figure 3 shows that the Hubble parameter diverges at the beginning and end of the universe. Such a universe starts with a positive deceleration parameter indicating deceleration expansion and transits to a negative deceleration parameter indicating accelerating expansion phase.
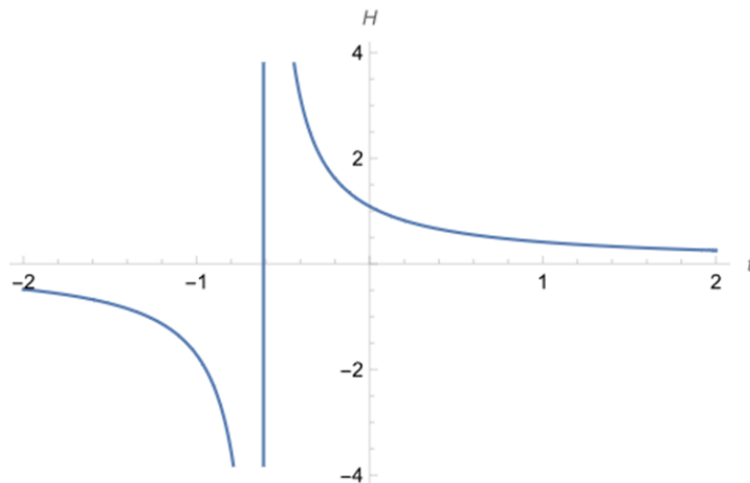

Figure 3. The evolution of the Hubble Parameter

The Deceleration Parameter

The deceleration parameter is a measure of the rate at which the expansion of the universe is changing. It provides information about the acceleration or deceleration of the universe's expansion over time. The deceleration parameter in terms of the Hubble parameter is given as:

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{1}{2} \qquad (19)$$

Since the deceleration parameter is negative (q < 0), it indicates that the universe's expansion is accelerating. This situation has been observed in recent cosmological observations and is attributed to the influence of dark energy.

CONCLUSION

One can approach it numerically when it is difficult to obtain the analytical solution to the Friedmann equation. How each matter component affects the universe's evolution is described on its dependencies on the value of t for each density parameter. The $\frac{1}{a^4}$ term describes that the radiation density affects the evolution at small values of t ($m = 1$), or the early universe. At this point, the radiation causes rapid expansion. The influence of matter with $\frac{1}{a^3}$ ($m = 0$) dependency increase starts surfacing with the decreasing effect of radiation on the universe. The dark energy term, $e^{\frac{3\omega a^{-2n}}{2nc_0^2}} a^{-3-m}$, became dominant at a larger value of t, causing the accelerating expansion of the universe. At the current time, both the matter density and dark energy densities $[(1 + e^{\frac{3\omega a^{-2n}}{2nc_0^2}}) \frac{\rho_{m0}}{a^{3+m}}]$ are playing the largest role in the evolution and acceleration of the universe, and they also have much more of a role in the age of the universe. The process of how the universe will quickly contract back into a singularity results from the large expansion caused by these two components. It is shown that in the early universe, radiation dominated. However, as the scale factor increases, dark energy becomes dominant. This is in line with the work of ref [11].

REFERENCE

1   G. W. Joseph, T. V. Targema, and M. O. Kanu, "Analytical Solution to FRW Metric with Variable Speed of Light," *INDONESIAN JOURNAL OF APPLIED PHYSICS*, vol. 11, no. 1, pp. 105–112, 2021.
2   H. E. S. Velten, R. F. Vom Marttens, and W. Zimdahl, "Aspects of the cosmological 'coincidence problem,'" *The European Physical Journal C*, vol. 74, pp. 1–8, 2014.
3   S. Capozziello and V. Faraoni, *Beyond Einstein gravity: A Survey of gravitational theories for cosmology and astrophysics*, vol. 170. Springer Science & Business Media, 2010.
4   S. Perlmutter *et al.*, "Measurements of Ω and Λ from 42 high-redshift supernovae," *Astrophys J*, vol. 517, no. 2, p. 565, 1999.
5   Thabsheera, A. A., Thasleema, T. M., & Rajesh, R. (2019). Lung cancer detection using CT scan images: A review on various image processing techniques. In *Data Analytics and Learning*, 413-419. Springer, Singapore.
6   R. A. Knop *et al.*, "New constraints on ΩM, $\Omega \Lambda$, and w from an independent set of 11 high-redshift supernovae observed with the Hubble Space Telescope," *Astrophys J*, vol. 598, no. 1, p. 102, 2003.
7   P. A. R. Ade *et al.*, "Planck 2013 results. XVI. Cosmological parameters," *Astron Astrophys*, vol. 571, p. A16, 2014.
8   S. M. Carroll, "The cosmological constant," *Living Rev Relativ*, vol. 4, no. 1, pp. 1–56, 2001.
9   Setiawan, W., Utoyo, M. I., & Rulaningtyas, R. 2019. Classification of neovascularization using convolutional neural network model. *TELKOMNIKA*, *17*(1), 463-472.
10 H. Terazawa, "Environment-Dependent Fundamental Physical Constants in the Theory of General Inconstancy," *arXiv preprint arXiv:1202.1859*, 2012.
11 H. Azri and A. Bounames, "Geometrical origin of the cosmological constant," *Gen Relativ Gravit*, vol. 44, pp. 2547–2561, 2012.
12 B. A. Robson, "Introductory Chapter: Standard Model of Cosmology," *Redefining Standard Model Cosmology*, pp. 1–4, 2019.

13 C. Bambi and A. D. Dolgov, *Introduction to particle cosmology: The standard model of cosmology and its open problems*. Springer, 2016.

14 J. W. Moffat, "Quantum gravity, the origin of time and time's arrow," *Found Phys*, vol. 23, no. 3, pp. 411–437, 1993.

15 J. D. Barrow, "Cosmologies with varying light speed," *Physical Review D*, vol. 59, no. 4, p. 043515, 1999.

16 L. Kantha, "A time-dependent and cosmological model consistent with cosmological constraints," *Advances in Astronomy*, vol. 2016, 2016.

17 J. D. Barrow, "Varying G and other constants," *Current Topics in Astrofundamental Physics: Primordial Cosmology*, pp. 269–305, 1998.

18 R. Dungan and H. B. Prosper, "Varying-G cosmology with type Ia supernovae," *Am J Phys*, vol. 79, no. 1, pp. 57–62, 2011.

19 H. Azri and A. Bounames, "Geometrical origin of the cosmological constant," *Gen Relativ Gravit*, vol. 44, pp. 2547–2561, 2012.

20 M. Ibison, "On the conformal forms of the Robertson-Walker metric," *J Math Phys*, vol. 48, no. 12, p. 122501, 2007.

21 A. Albrecht and J. Magueijo, "Time varying speed of light as a solution to cosmological puzzles," *Physical Review D*, vol. 59, no. 4, p. 043516, 1999.

22 M. V. Sazhin and O. S. Sazhina, "The scale factor in a Universe with dark energy," *Astron. Rep.*, vol. 60, no. 4, pp. 425–437, Apr. 2016, doi: 10.1134/S1063772916030136.