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PHYSICS APPLICATION IN TRADITIONAL ARCHERY: SIGHT-LINES METHOD FOR AIMING USING A TRADITIONAL BOW

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ABSTRACT

The absence of bow sight in a traditional bow can be replaced by introducing sight lines as a reference for accurate elevation angles in archery. Assuming a quadratic drag force acting on the arrow, a sight line can be calculated by using the values of target distances, initial arrow speed, arrow velocity decay, and the stance of the archer. This method can improve shooting accuracy for various target distances.

Keywords: traditional archery; arrow accuracy; distance variation archery; sight lines; physics application; drag force

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INTRODUCTION

Bow and arrows have an essential role throughout history[1-2]. In ancient times it functioned as a defense and hunting tool to sustain daily life[3]. At the other time, this tool became vital to showing superiority across society[4]. In modern times, this tool still holds a vital role as an advanced sport[5] and cultural manifestation[6]. Archery has become an essential branch of sports competitions[7]. As a traditional culture, archery is an important part that still needs conservation and development[6].

Bows can be grouped into two types: modern bows and traditional bows. One of the ways modern bows were developed to increase their accuracy and efficiency is by adding extra accessories. For example, an additional weight is added as a stabilizer, binoculars to assist the view, and a sight to determine the direction of the arrow. Traditional bows, on the other hand, only rely on the bow and arrow without any additional accessories.

From this classification, generally, traditional bows are more flexible, lightweight, and less accurate than modern bows. In the modern bow, scope or sight tools help archers aim consistently. In contrast, in the traditional bow, these accessories are unavailable and not allowed to be used in competitions. Archers are allowed to use barebow only. The absence of sighting tools makes traditional archers rely on their aim instinct. One example of this traditional bow is the Turkish bow[8] compared with recurve modern bows.

This article discusses the relationship between elevation angles and target distances. Then, by introducing some sight lines, an archer can accurately direct its arrow for specific elevation angles. This approach can improve accuracy, even for various distance targets that often occur in traditional competition.

THEORETICAL ANALYSIS

Hitting right on target is the main goal in archery. In target archery competitions, this is the key to victory. In animal hunting, this is also the key to success. There are several critical success factors in drawing, aiming, and releasing arrows to reach the target. From the kinematics of the motion, there are at least three critical factors: target distance, initial arrow speed, and elevation angle. The dynamics of the bow and the arrow motion, including air resistance, also play a significant role in the arrow motion[9–11].

Measurement of Arrow Release Speed

Arrow release speed can be measured using modern electronic equipment such as a ballistic chronograph. This tool has high accuracy and can be used repeatedly but is relatively expensive. Another alternative for measuring an arrow's release speed is a ballistic pendulum that uses the principle of conservation of momentum and mechanical energy. This method is less expensive and can be performed with simple equipment but with less accuracy[12]. In addition, the advantages of this ballistic pendulum can also be used to measure the arrow's speed when it reaches the target. At the same time, the chronograph is very difficult to use because of the narrow sensor area. Another method to measure the arrow speed is using the sound recording method proposed by Meyer[13] with simple equipment: microphones, a computer, audio recording, and analysis software with millisecond resolution.

Elevation Angle as Function of Target Distance and Arrow Release Speed

a) No Air Drag Consideration

In the case of negligible air resistance, the motion of the arrow towards the target can be explained using the motion of the projectile under the influence of gravity. Assuming the arrow has a release speed v_0 and the target distance is S, then the relationship between the arrow's elevation angle θ and the target distance:

$$S = \frac{v_0^2}{g} sin2\theta \tag{1}$$

so that

$$\theta = \frac{1}{2} arcsin\left(\frac{gS}{v_0^2}\right) \tag{2}$$

where g is the gravitational acceleration.

b) Effect of Air Drag Force on Elevation Angle

In fact, the motion of the arrow in the air experiences an air drag that depends on its speed. In the textbook, the drag force can be linearly proportional to velocity or quadratic or to any velocity function $f(v)$. In general, the force equation becomes:

$$m\frac{d\vec{v}}{dt} = -Ckf(v)\hat{v} + m\vec{g} \tag{3}$$

where m is the mass of the arrow, $\vec{v}$ is the velocity of the arrow, $\hat{v}$ is the unit vector in the direction of the arrow's velocity, C is a constant commonly referred to as the *coefficient of drag* that is affected by the details of the arrow, k is a physical constant that depends on the density of air and the size of the arrow, and $\vec{g}$ is the local acceleration due to gravity. This force equation can be projected on the x (horizontal direction) and y-axis (upward direction) to be:

$$m\ddot{x} = -Ckf(v)cos\phi, \qquad m\ddot{y} = -Ckf(v)sin\phi - mg \tag{4}$$

where ϕ is the angle of inclination of the tangent to the path at any point. To simplify writing, we set:

$$c \equiv \frac{Ck}{m}, \tag{5}$$

This constant is sometimes referred to as the *ballistic coefficient*. Equation (4) can be written as:

$$m\ddot{x} = -Ckf(v)cos\phi, \qquad \ddot{y} = -cf(v)sin\phi - g. \tag{6}$$

These two equations can be rearranged using $\dot{x} = v\ cos\ cos\ \phi$ and $\dot{y} = v\ sin\ sin\ \phi$, so:

$$\frac{d}{dt}(vcos\phi) = -cf(v)cos\phi, \qquad \frac{d}{dt}(vsin\phi) = -cf(v)sin\phi - g. \tag{7}$$

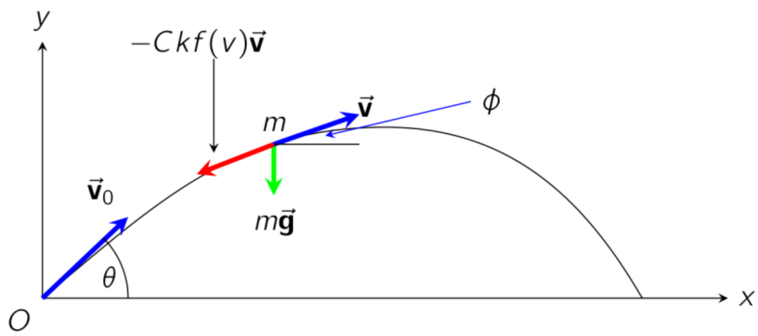

Figure 1. Force diagram on the body during the flight.

According to Figure 1, by projecting the forces acting in the direction of the tangent and the normal direction of the path at any point, we can write

$$\frac{dv}{dt} = -cf(v) - gsin\phi, \frac{v^2}{r} = gcos\phi, \tag{8}$$

where $\frac{dv}{dt}$ is the tangential acceleration and $\frac{v^2}{r}$ is the normal directional acceleration, where r is the radius of curvature of the path for any point.

The second equation in Equation (8) is free from $f(v)$ so it always applies to all types of air resistance. This equation can be written in other forms, by using the curvature equation

$$\frac{1}{r} = -\frac{d^2\frac{y}{d}x^2}{[1+(\frac{dy}{dx})^2]^{\frac{3}{2}}} \tag{9}$$

where r is the curvature radius, $\frac{dy}{dx} = tan\phi$ and $1 + tan^2\theta = \frac{1}{cos^2}\theta$, we have

$$\frac{1}{r} = -\frac{d^2y}{dx^2}cos^3\phi. \tag{10}$$

The second equation of Equation (8) becomes

$$\frac{d^2y}{dx^2} = -\frac{g}{v^2cos^2\phi}. \quad (11)$$

We can also use the $\frac{1}{r} = -\frac{d\phi}{ds}$ relation, where ds is the length element of the path so that the same second equation of Equation (8) becomes

$$v^2\frac{d\phi}{ds} = -gcos\phi \rightarrow \frac{d\phi}{dt} = -\frac{gcos\phi}{v} \quad (12)$$

by using $v = \frac{ds}{dt}$. The last three equations above can be combined with the second part of Equation (8) for later use.

Next, the first part of equation (7) can be multiplied by both sides $d\phi$ to become

$$d(vcos\phi)\frac{d\phi}{dt} = -cf(v)cos\phi d\phi \quad (13)$$

Next, by using Equation (12) the above equation becomes

$$gd(vcos\phi) = vcf(v)d\phi. \quad (14)$$

From this equation, we can relate the speed and the inclination ϕ.

For our case, the suitable air resistance is proportional to the square of the velocity: $cf(v) = cv^2$, so Equation (14)

$$gd(vcos\phi) = cv^3d\phi \quad (15)$$

We can solve this by using transformation $\dot{x} = vcos\phi$ and using $d\phi = -g(vcos\phi)\frac{ds}{v^3}$ from Equation (12), then Equation (15) can be changed into

$$\frac{d\dot{x}}{\dot{x}} = -cds \quad (16)$$

By setting the initial condition $(\dot{x})_{s=0} = \dot{x}_0$, we obtain

$$ln\dot{x} - ln\dot{x}_0 = -cs \qquad \dot{x} = \dot{x}_0e^{-cs}$$

Then by using Equation (11), we get

$$\frac{d^2y}{dx^2} = -\frac{g}{\dot{x}^2} = -\frac{g}{\dot{x}_0{}^2}e^{2cs}. \quad (17)$$

Let's use an approximation for arrow trajectory that $S \approx x$ so that we have

$$\frac{d^2y}{dx^2} = -\frac{g}{\dot{x}_0^2}e^{2cx} \quad (18)$$

and the integration of this is

$$\frac{dy}{dx} = -\frac{ge^{2cx}}{2c\dot{x}_0^2} + A_1 \quad (19)$$

By using $(d\frac{y}{d}x)_{x=0} = tan\theta$, we have

$$A_1 = tan\theta + \frac{g}{2c\dot{x}_0^2} \quad (20)$$

Second integration produces

$$y = -\frac{ge^{2cx}}{4c^2\dot{x}_0^2} + A_1x + A_2, \tag{21}$$

and by using the initial condition $y(x = 0) = 0$ we get

$$A_2 = \frac{g}{4c^2\dot{x}_0^2} \tag{22}$$

Hence, the path equation is:

$$y = \frac{g}{4c^2v_0^2}(1 - e^{2cx}) + \left(tan\theta + \frac{g}{2cv_0^2}\right)x \tag{23}$$

Then it can be arranged to become:

$$y = xtan\theta - \frac{gx^2}{2v_0^2cos^2\theta}\left(\frac{-1-2cx+e^{2cx}}{\frac{1}{2}(2cx)^2}\right) \tag{24}$$

This is the equation to determine the elevation angle as the function of target distance and arrow release speed. Another variable to be considered is the ballistic coefficient c that depends on the arrow.

The elevation angle for reaching the target that has same height as the initial arrow position ($y = 0$) and set $x = S$ we have:

$$\theta = \frac{1}{2}arcsin\left(\frac{gS}{v_0^2}\left(\frac{-1-2cS+e^{2cS}}{\frac{1}{2}(2cS)^2}\right)\right) \tag{25}$$

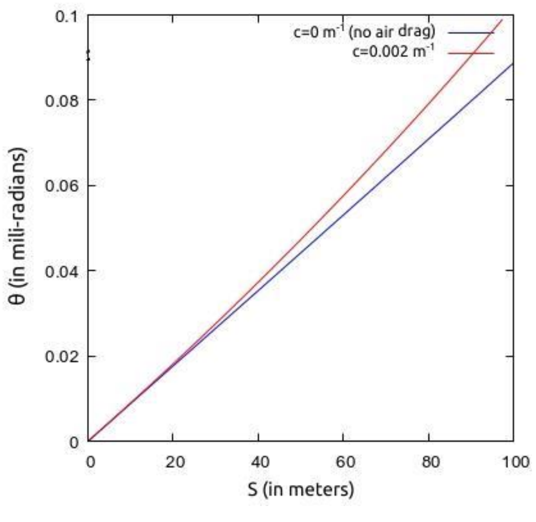

Figure 2. Plot of $\theta(S)$.

The plot (Figure 2) of this equation for $v_0 = 74.5$ m/s and a typical value $c = 0.002$/m of an arrow is in agreement with the numerical methods and experimental evidence in Meyer(2015)[13], and the formula of this constant is given by[14]

$$c = \frac{C\rho_{air}\pi R^2}{2m}. \tag{26}$$

for the case that air drag is proportional to the velocity-squared of the arrow, this constant c is also called the *velocity decay*. One can experimentally measure the value of c by utilizing the arrow sound when it's released from the bow and hits a target[13]. The coefficient of drag C for a variety of arrows commonly ranges from 1.5 to 4.0[15].

AIMING AND TARGETING

In modern competition archery, equipment called bow sight is added to help in aiming. Archers usually adjust this sight by shooting experiments (see Figure 3). If the results hit the upper (lower) part of the target, then the sight needs upward (downward) adjustment, and the reverse is also applicable for lower-hit results. The adjustments are done many times until perfect results are acquired. Here, it is assumed that the arrow's initial speed is unchanged for every shooting.

In principle, the adjustment method is used to find the right elevation angle (vertical direction) and right-left direction (horizontal direction) to direct the arrow. Unfortunately, this adjustment only applies to a specific target distance. The vertical position of the sight needs to be redefined if the distance alternates, meanwhile, the horizontal direction remains intact.

In the traditional bow, no additional equipment is applied. Since adding more will reduce its flexibility for horseback archery or dynamics archery. Furthermore, most international competitions in traditional archery have similar regulations. Regarding this restriction, the same essence of bow sight can be applied by introducing a *sight line*, called h-line, in the bow.

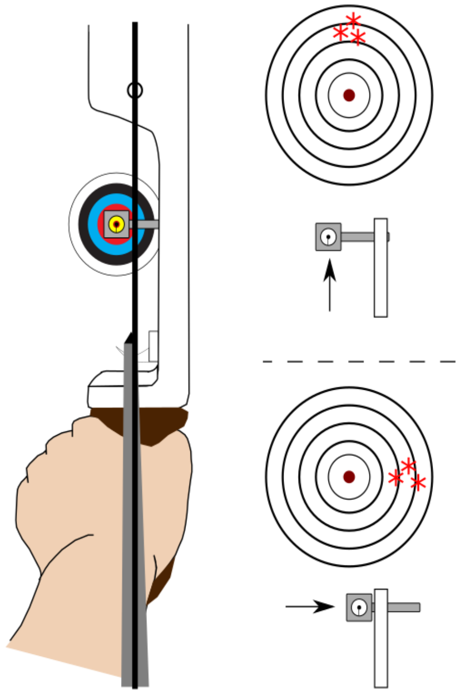

Figure 3. Aiming adjustment using a bow sight.

Figure 4 shows two conditions of aiming for a right-eye dominant and a left-eye dominant archer. To adjust the vertical elevation angle the *h-line* – a horizontal blue line on the bow-limb can be used. The h-line has the same function as the sight for vertical adjustment, as the horizontal angle adjustments are not discussed, since it can be trained by the archer.

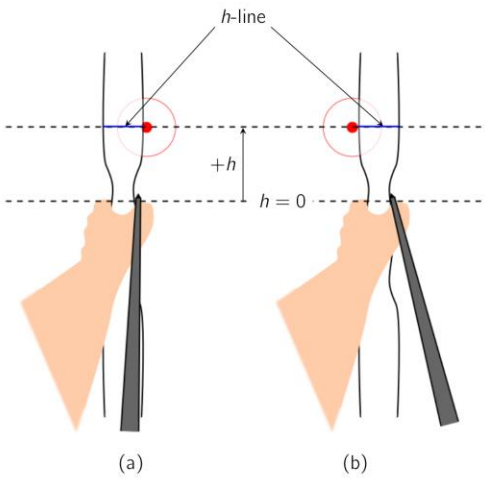

Figure 4. Distance of h-line from arrow rest line (a) right eye dominant, and (b) left eye dominant.

By the previous formula of elevation angle as a function of S in Equation (25), a value of θ as a function of S can be acquired. Then by using a scheme in Figure 5, the h-lines position as a function of S also can be calculated. The distance h is obtained by finding the distance between the point (x_{in}, y_{in}) and the point (x_r, y_r). First, set the origin of the coordinates to be at the nocking point of the arrow on the bowstring. Then the positive x-axis is on the line connecting O with the target point and the positive y-axis is perpendicular to this line upwards. The next required point is the eyepoint (x_e, y_e) and also the point of contact of the arrow on the bow grip (x_r, y_r). However, the point of contact can be obtained from the draw length L_0 and the elevation angle θ. Based on this data, the intersection point between line ET and line AB (x_{in}, y_{in}) can be calculated and also the distance h.

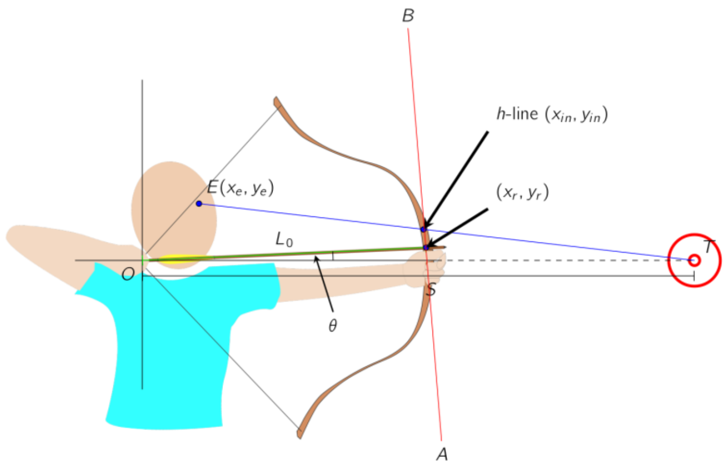

Figure 5. The schematic figure for calculating h as a function of θ and S.

In detail, the equation of the arrow line y_{arr} from the two points (0,0) and (x_r, y_r) is:

$$y_{arr}(x) = tan\theta \cdot x \tag{27}$$

then also the equation of the line of sight y_{ET} obtained from the two points $E(x_e, y_e)$ and $T(S, 0)$:

$$y_{ET}(x) = -\frac{y_e}{S}x + y_e\left(1 + \frac{x_e}{S}\right) \approx -\frac{y_e}{S}x + y_e \tag{28}$$

noting that x_e is negligible compared to S. Then the equation of the line y_{AB} which is a perpendicular to the line y_{arr} and both have a point of intersection at (x_r, y_r) so we get

$$y_{AB} = -\frac{1}{tan\theta}x + \frac{L_0}{sin\theta} \tag{29}$$

From the intersection of the two line equations y_{AB} and y_{ET}, it is then obtained the location of the point (x_{in}, y_{in}).

To accurately calculate h-line, these parameters need to be measured precisely: target distances S, initial arrow speed v_0, vertical distance of the archer's dominant eye y_e, draw length L_0, gravitational acceleration g, and the velocity decay of the arrow c. Assume that y_e=10 cm, $L_0 = 75$ cm, and $g = 9.8$ m/s^2. By using computer code executed in Maxima[16], a graph h as a function of distance and velocity of the arrow for the case of no air friction and for velocity decay $c = 0.002$ m^{-1} is shown in Figure 6. For comparison, a higher air drag $c = 0.005$ m^{-1} is also included in the graph.

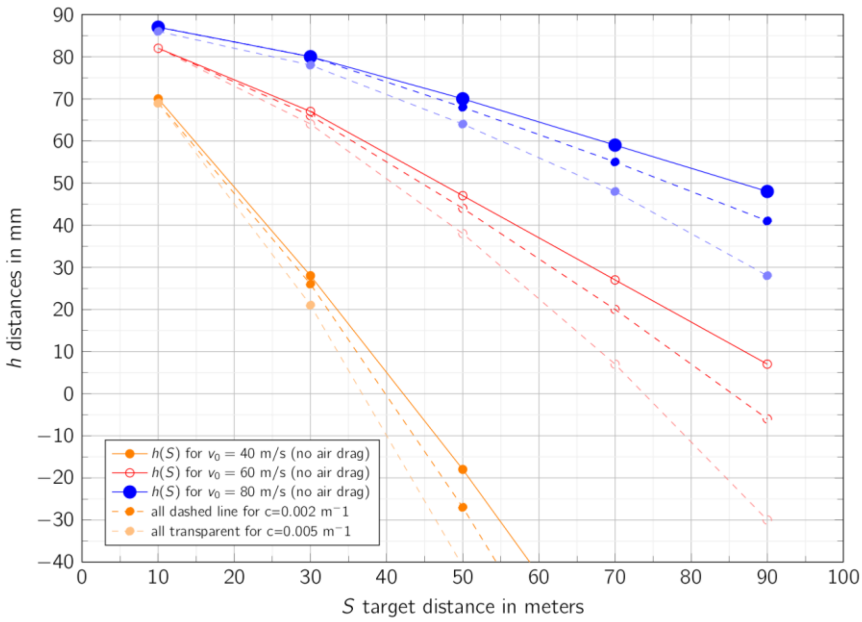

Figure 6. The plot of h as a function of distance S for some value v_0, with various air drag.

After completing the calculation of some h-lines, the archer can put a mark on the bow as guidance in aiming. With the varying h-lines as a function of target distances, the accuracy of shooting can be improved, even for varying target distances.

SENSITIVITY STUDY

The shooting accuracy using h-lines sensitively depends on the measurement of the parameters, without counting human errors from the archer. Assume the correct values of S=70 m, v_0=60 m/s, and $c = 0.002$ m^{-1} which gives h-line h_0=20 mm, then if (a)the value of c is incorrectly measured to $c = 0.001$ m^{-1}, then the incorrect h-lines $h_i = 24$ mm will result in a missed target of 35 cm below the center or it will land on the ground at a distance 77.1 m. If (b)$c = 0.003$ m^{-1} is used, then the h-line becomes $h_i = 16$ mm and will make the arrow hit 38 cm above the center of the target or it will land on the ground at 82.5 m.

The initial speed of the arrow also affects the accuracy. If (c)the measurement incorrectly gives 3 m/s lower than 60 m/s, the arrow will miss 80 cm above the target, or it will land at 85.7 m. For (d) the case where the measurement speed is 3 m/s higher, the arrow will be missed 69 cm below the target, or it will land at 74.5 m. See Figure 7 for the illustration using a 1 m target radius.

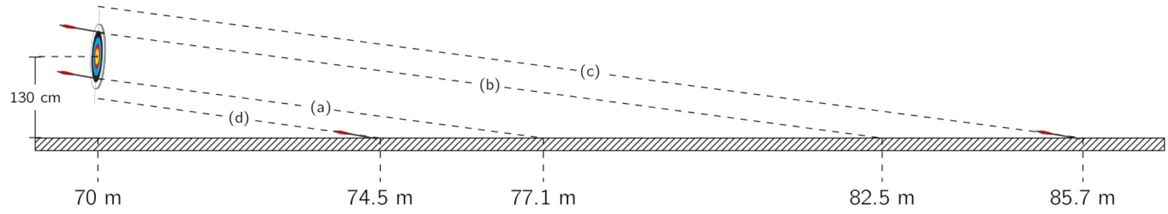

Figure 7. Effect of incorrect measurement of the correct velocity decay c=0.002 m^{-1} and initial speed v_0=60 m/s at a distance of 70 m. (a) for c=0.001 m^{-1}, (b) for c=0.003 m^{-1}, (c) for v_0=57 m/s, and (d) for v_0=63 m/s.

The above calculation assumes that the nock point of the arrow has the same height as the center of the target usually 130 cm above the ground. An archer with 150 cm tall and a nock point of 140 cm will hit the target 10 cm above the target center points. The h-line needs a 1 mm increase to correct this. If a teen has a 100 cm height of nock point, then a 3 mm decrease of h-line is appropriate. The computer program for the calculations is shown in Figure 8.

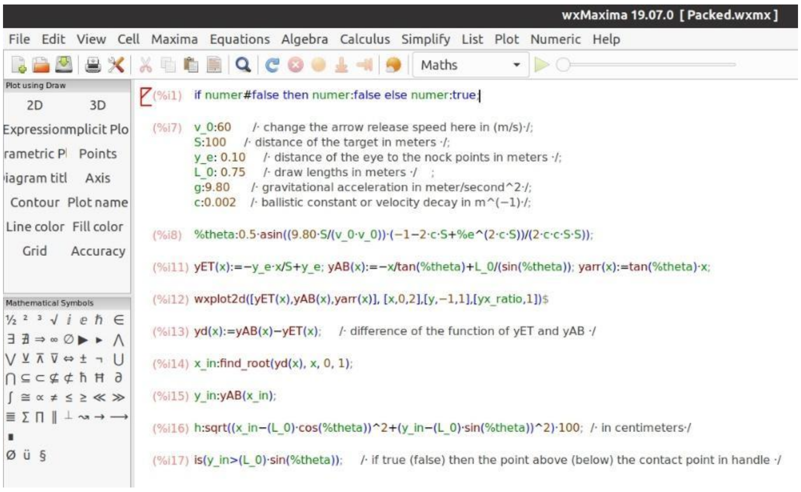

Figure 8. wxMaxima code that is used for the calculation (without output).

CONCLUSION

In the absence of air drag, the h-lines as a function of distances target can be calculated directly provided the initial speed of the arrow. For the more common situation where the air drag of the arrow cannot be neglected then the additional parameter of the ballistic coefficient is needed. For a more specific case where the arrow speed is still high, the ballistic parameter is reduced to velocity decay. For more accurate h-lines as a function of distance targets, archers need to examine their arrows to get the value of velocity decay. Then, by assigning some h-lines as a function of target distances in the bow, the accuracy of an arrow can be improved.

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