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Nucleon-Nucleon Potentials and Computation of Scattering Phase Shifts

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ABSTRACT

By judicious exploitation of supersymmetry formalism of quantum mechanics higher partial wave nucleon-nucleon potentials are generated from its ground state interactions. The nuclear Hulthen potential and the corresponding ground state wave function with the parameters of Arnold and MacKellar are used as the starting point of our calculation. We compute the scattering phase shifts for our constructed potentials through Phase Function Method to examine the merit of our approach to the problem.

Keywords: Supersymmetry, Nucleon-nucleon potential, Phase function method, Scattering phase shifts

INTRODUCTION

Witten[1] first developed the methodology to study the quantum mechanical system governed by an algebra identical to that of supersymmetry in field theory. Since then it becomes a popular tool to deal with hierarchy problems and calculation of related physical observables. For any Hamiltonian with one degree of freedom, a comparison Hamiltonian can be constructed such that the resulting system as a whole is supersymmetric[1,2,3,4]. The Hamiltonian hierarchy problems in supersymmetric quantum mechanics [SQM] lead to the addition of appropriate centrifugal barriers and consequently the higher partial wave potentials are generated fairly accurately in atomic physics. Naturally question may arise whether this methodology is equally applicable for nuclear cases or not as the nucleon-nucleon potentials are highly state dependent. In the recent past we have studied nucleon-nucleon scattering within the framework of SQM[5-7] for both energy-dependent and independent interactions and achieved fairly good agreement for higher partial wave phase shifts in the low and intermediate energy range. However, for high energy range ($E_{Lab} \geq 200\ MeV$) our phase shift values differ from those of standard data[8,9].

It is well known that Schrödinger equation is exactly solvable for the Coulomb interaction in all partial waves and for all energies but it is not true for the Hulthen or Hulthen-like potentials. Hulthen potential is exactly solvable for s-wave only. Thus one must take recourse to some other methods to construct higher partial wave solutions for such kind of interactions. The systems which involve two potentials like (p-p) and (p-2p), where an electromagnetic potential is added to its short-range local nuclear part, generally do not possess exact solution even for s-wave and are often treated approximately. In such a situation one adds the effect of electromagnetic interaction to its nuclear part separately. However, this is no loss of generalization. In our earlier publications [5-7] the higher partial wave strong nuclear interactions are derived by the addition of appropriate centrifugal

barrier-like terms to its s-wave counter part and the corresponding (p-p) interactions are generated by simply adding electromagnetic potential to it. Unlike our previous methods here we shall generate p-wave potentials for both the nuclear and atomic Hulthen potentials separately and add them to obtain the resultant p-wave interaction for the (p-p) system in which half of the contribution to centrifugal barrier term comes from electromagnetic part and other half from nuclear part. We also define another set of interactions by the addition of proper centrifugal term to its s-wave part and study their effectiveness in computation of nucleon-nucleon phase shifts through the phase function method (PFM)[10]. The present paper is an effort in this direction.

In Section II we develop p-wave nuclear potential for (n-p) and (p-p) systems through SQM. We shall demonstrate the usefulness of our constructed potentials in Section III by computing the values of p-p and n-p scattering phase shifts by PFM. Finally, in Section IV we put some concluding remarks.

P-WAVE NUCLEAR HULTHEN POTENTIAL

There exist experimental situations which involve scattering by additive interactions, some of which must for various physical reasons be treated exactly, whereas others may be relatively small perturbation. A typical example of this kind is the scattering of particles under the combined influence of Coulomb and nuclear forces like proton-proton (p-p) scattering. In charged particle scattering the long range nature of the electromagnetic interaction (Coulomb) is a source of special difficulties. It has been argued that pure Coulomb potential never really occurs in nature and becomes somewhat screened at a certain distance.

In SQM the supersymmetric partner H_1 with potential V_1 (p-wave potential) of the Hamiltonian H_0 is presented by Eq. 1.

$$H_0 = -\frac{\partial^2}{\partial x^2} + V_0(x) \tag{1}$$

with its ground state eigen function $\psi_0^{(0)}$ and energy eigen value $E_0^{(0)}$ is given by[1,3] Eq. 2.

$$H_1 = -\frac{\partial^2}{\partial x^2} + V_1(x) \tag{2}$$

$V_1(x)$ is given by Eq. 3.

$$V_1(x) = V_0(x) - \frac{\partial^2}{\partial x^2} \ln \psi_0^{(0)} \tag{3}$$

Since its appearance the Yamaguchi potential[11]

$$V(r,s) = \lambda e^{-\beta(r+s)} \tag{4}$$

with λ, the strength and β, inverse range parameters becomes a popular tool in dynamical calculations. The bound state wave function for the Yamaguchi potential is identical to the wave function for the first bound state of the Hulthen potential with range $(\beta-\alpha)^{-1}$ and depth $-(\beta^2-\alpha^2)$ [12]. A nuclear Hulthen potential with these parameters rewritten as Eq. 5.

$$V_{0N}(r) = -(\beta^2-\alpha^2)\frac{e^{-\beta r}}{(e^{-\alpha r}-e^{-\beta r})} \tag{5}$$

with its ground state solution is given by Eq. 6.

$$\psi_0^{(0)} \sim e^{-\alpha r} - e^{-\beta r} \tag{6}$$

Thus, for p-pscattering the s-wave effective potential $V_{0P}(r)$ is written as Eq. 7.

$$V_{0P}(r) = V_{0A}(r) + V_{0N}(r) \tag{7}$$

Where $V_{0A}(r)$ is given by Eq. 8.

$$V_{0A}(r) = V_0 \frac{e^{-r/a}}{(1-e^{-r/a})} \tag{8}$$

is the s-wave Hulthen or screened Coulomb potential with atomic parameters *V0* & *a*.The application of the above SQM relations to the Hulthen potential are now in order. In view of Eq. 3 and 5 the wave function in Eq. 6 leads to the supersymmetric partner potential.

$$V_{1N}(r) = V_{0N}(r) + \frac{(\beta-\alpha)^2 e^{-(\alpha+\beta)r}}{(e^{-\alpha r} - e^{-\beta r})^2} \tag{9}$$

In Eq. 9, the second term simulates the effect of centrifugal barrier apart from a factor of 2. Similarly, for atomic Hulthen potential one has

$$V_{1A}(r) = V_{0A}(r) + \frac{e^{-r/a}}{a^2 (1-e^{-r/a})^2} \tag{10}$$

In view of above the p-wave potential for p-p scattering is defined as

$$V_{1P}(r) = V_{1A}(r) + V_{1N}(r) \tag{11}$$

The corresponding potential for n-p scattering designated as $V_{1NP}(r)$ is obtained with $V_{0A}(r) = 0$ in Eq. 11, so that the centrifugal barrier term from atomic part of the potential comes into effect. The associated phase shifts with supersymmetry generated potentials for the (p-p) and (n-p) systems will be denoted by $\delta_{\ell P}$ and $\delta_{\ell N}$ respectively.

We also propose the following set of potentials by adding the centrifugal term to its s-wave part directly

$$V_{1NB}(r) = V_{0N}(r) + \frac{2}{r^2} \tag{12}$$

and

$$V_{1PB}(r) = V_{0A}(r) + V_{1NB}(r) \tag{13}$$

and study its impact on the scattering phase shifts. In the above the letter B in subscript denotes the potentials where the corresponding centrifugal barrier terms are added directly to the s-wave parts and the associated phase shifts will be designated as $\delta_{\ell PB}$ and $\delta_{\ell NB}$ for (p-p) and (n-p) systems respectively.

RESULTS AND DISCUSSION

The phase function method represents an efficient approach to evaluate the scattering phase shifts for quantum mechanical problems involving local[10] and non local interactions[13,14]. For a local potential the function $\delta_\ell(k,r)$, called the phase function, satisfy a first order non-linear differential equation given by Eq. 14.

$$\delta'_\ell(k,r) = -k^{-1}V(r)[\hat{j}_\ell(kr)\cos\delta_\ell(k,r) - \hat{\eta}_\ell(kr)\sin\delta_\ell(k,r)]^2 \qquad (14)$$

with $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ the Riccati Bessel functions. We shall follow the phase convention of Calogaro[10] with Hankel function of first kind written as $\hat{h}^1_\ell(x) = -\eta_\ell(x) + i\hat{j}_\ell(x)$. The scattering phase shift $\delta_\ell(k)$ is obtained by solving the equation from origin to asymptotic region with the initial condition $\delta_\ell(k,0) = 0$. During the solution of the phase equation, $\delta_\ell(k,r)$ is built up by the potential as one moves away from the origin and it reaches its asymptotic value as soon as one gets out of the range of the potential. Obviously, $\delta_\ell(k) = Lim_{r\to\infty}\delta_\ell(k,r)$.

In Figures 1-4 we portray the (p-p) and (n-p) potentials as a function of distance for $\ell = 0\&1$ partial waves with $\lambda = -5.237\ fm^{-3}$ and $\beta = 1.4054\ fm^{-1}$ for $^1s_0$ scattering and, $\lambda = -7.533\ fm^{-3}$ and $\beta = 1.4054\ fm^{-1}$ for $^3s_1$ scattering[12]. It is observed that in figures 2 and 4 repulsive cores develop in the generated potentials. These potentials, generated from their $^1s_0$ and $^3s_1$parts, correspond to $^1p_1$ and $^3p_1$ states respectively.

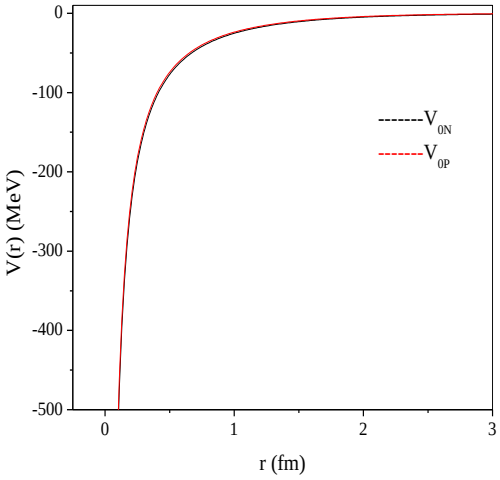

Figure 1. *(Color Online)* $^1s_0$ potential as a function of r

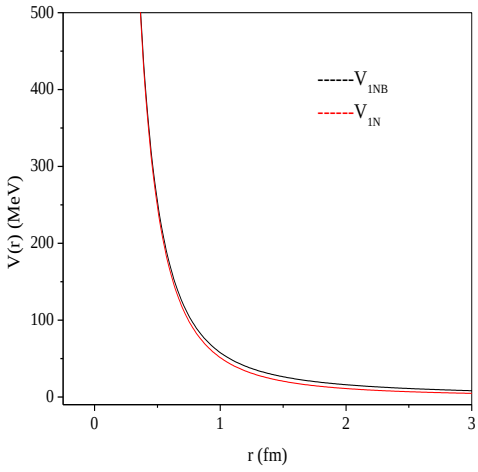

Figure 2. *(Color Online)* Constructed $^1p_1$ potential as a function of r

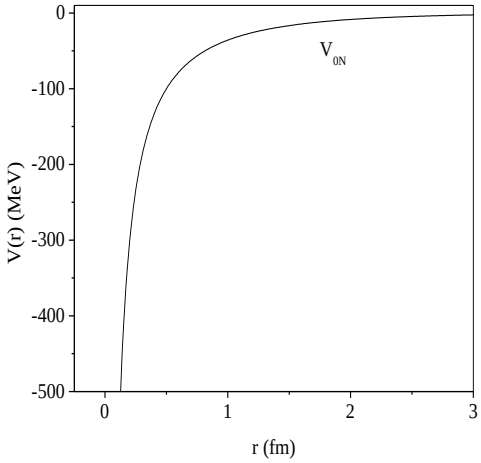

Figure 3. $^3s_1$ potential as a function of r

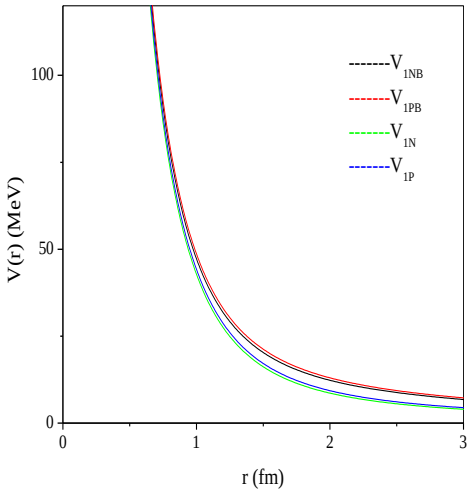

Figure 4. *(Color Online)* Constructed $^3p_1$ potential as a function of r

The corresponding singlet and triplet state phase shifts have been computed using the PFM and plotted in Figures 5-8 as a function of laboratory energy up to 300 MeV along with the values of Arndt et. al.[8] and Gross-Stadler[9] for comparison. Note that the results for the pure nuclear phase shifts (n-p) have been obtained by turning off the atomic Hulthen interaction $V_{0A}(r)$ in the associated numerical routine for generating (p-p) phase shifts.

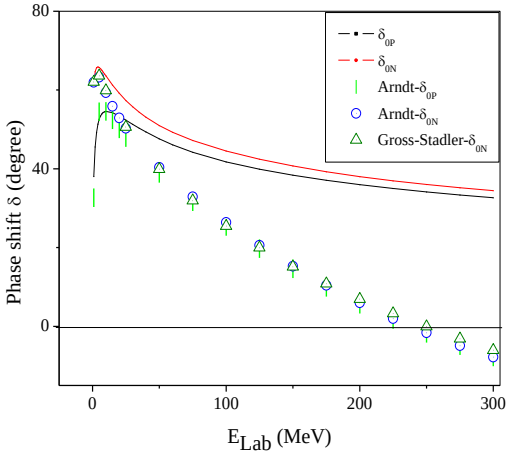

Figure 5. (*Color Online*) $^1s_0$ phase shifts as a function of E_{Lab}

Figure 5: The $^1s_0$ phase shifts δ_{0P} and δ_{0N} for (p-p) and (n-p) systems respectively agree well with that of refs. 8 & 9 for $E_{Lab} \leq 30\,MeV$. Beyond 30 MeV the phase shifts differ significantly with energy. This is quite expected because $^1s_0$ phase shifts change sign beyond 225 MeV and a one term potential is not capable of producing such effect. Therefore, it is expected that our potential for $^1p_1$ state developed from $^1s_0$ part will be able to generate reasonable fit to phase shifts at least up to 30 MeV.

Figure 6: The two sets of phase shift values δ_{1NB} and δ_{1N} along with the values of refs. 8 & 9 are displayed. All our phase shifts show correct trend for $^1p_1$ pure nuclear phase shift but differ in their numerical values. Among the two sets of phase shift values namely δ_{1NB} and δ_{1N}, δ_{1N} is more consistent than δ_{1NB} upto 250 MeV. However, beyond 250 MeV δ_{1NB} compares well with the standard data[8,9]. This is due to the fact that the r^{-2} term plays a crucial role in the high energy range. Therefore, our supersymmetry generated $^1p_1$ potential is more realistic in the low and intermediate energy scattering compared to the proposed one with direct addition of centrifugal barrier.

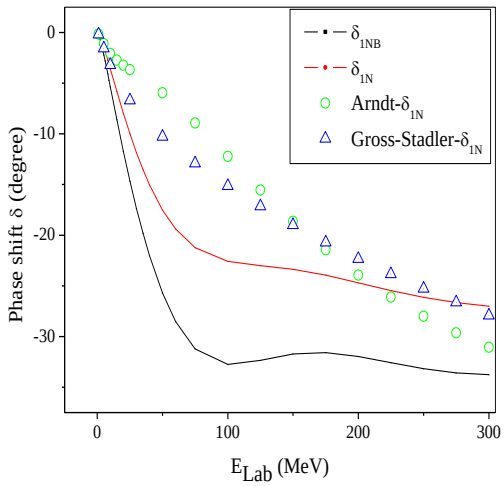

Figure 6. (*Color Online*) $^1p_1$ phase shifts as a function of E_{Lab}

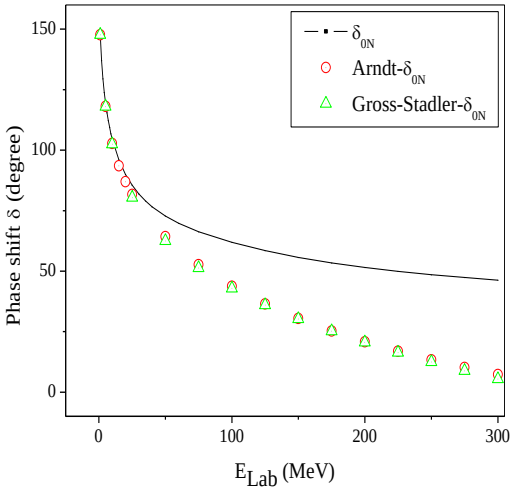

Figure 7. (*Color Online*) $^3s_1$ phase shifts as a function of E_{Lab}

Figure 7: Our phase shift values for $^3s_1$ state with the parameters of Arnold and MacKellar[12] agree well with that of refs. 8 & 9 up to $E_{Lab} = 25\,MeV$.

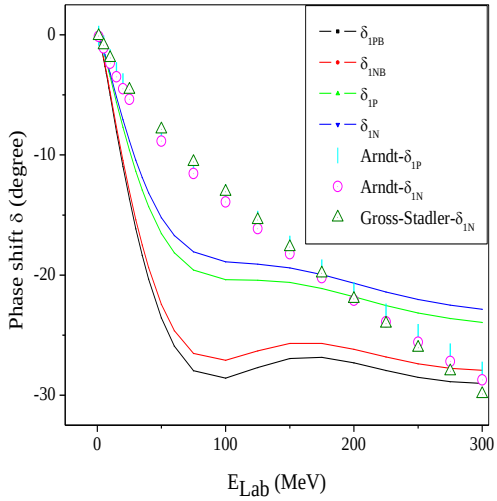

Figure 8. (*Color Online*) $^3p_1$ phase shifts as a function of E_{Lab}

Figure 8: Looking closely into this figure we notice that the phase shifts $\delta_{1P}, \delta_{1N}, \delta_{1PB}$ *and* δ_{1NB} produce correct nature of (p-p) and (n-p) scattering phase shifts for $^3p_1$ state. Among these δ_{1P} *and* δ_{1N} are superior to their counter parts δ_{1PB} and δ_{1NB} and are quite capable for comparison with Arndt et. al.[8] and Gross-Stadler[9] up to $E_{Lab} = 250\,MeV$. Beyond 250 MeV, however, δ_{1PB} and δ_{1NB} are more consistent than their counter parts. It is worthwhile to mention that our higher partial wave potentials are generated from their ground states with the addition of repulsive centrifugal potentials which make the p-p interaction more repulsive than n-p one.

CONCLUSION

From our observation it is reflected that our constructed potentials are quite capable of producing the nature of phase shifts of respective states but differ in their numerical values. This is due to the fact that unlike atomic cases the nuclear potentials are highly state dependent and cannot be generated in a proper way from any known interaction. The higher partial wave potentials that are generated here belong to Eckart class of potentials; the second terms in them behave as centrifugal barrier. Our supersymmetry operation for developing p-wave interactions from its s-wave part corresponds to the removal of one bound state and thereby produces a shallow potential with repulsive core from a deep s-wave interaction. Both deep and shallow potentials have been using in nuclear physics calculations, particularly, in nucleus-nucleus interactions. The bound states of the shallow potentials are related to the actual physical states of the fused nucleus. Michel and Reidemeister[15] nicely explained that it is possible to construct phase equivalent shallow potentials for $\alpha+^{16}O$ deep potential through SQM. Baye[16] has also found a good agreement between α-α shallow potential of Ali and Bodmer[17] and the deep potential of Buck et al.[18]. Our phase shift calculation through SQM and PFM involves only two parameters while those of Arndt et al.[8] and Gross-Stadler[9] are associated with 52 parameters and 27 parameters interactions respectively. Therefore, by comparing our phase shifts with those of Arndt et al.[8] and Gross-Stadler[9] it can be concluded that this simpleminded combined approach of SQM and PFM to compute nucleon-nucleon scattering phase shifts will be of quite interesting to a wide variety of physicists and graduate students. Also our method of computing the scattering phase shifts by the use of the variable phase method deserves serious attention.

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