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Solution of the Schrödinger Equation for Trigonometric Scarf Plus Poschl-Teller Non-Central Potential Using Supersymmetry Quantum Mechanics

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ABSTRACT

In this paper, we show that the exact energy eigenvalues and eigen functions of the Schrödinger equation for charged particles moving in certain class of noncentral potentials can be easily calculated analytically in a simple and elegant manner by using Supersymmetric method (SUSYQM). We discuss the trigonometric Scarf plus Poschl-Teller systems. Then, by operating the lowering operator we get the ground state wave function, and the excited state wave functions are obtained by operating raising operator repeatedly. The energy eigenvalue is expressed in the closed form obtained using the shape invariant properties. The results are in exact agreement with other methods.

Keyword*:* Supersymmetry, Trigonometric Scarf plus Poschl Teller, Non-central potentials

ABSTRAK

Nilai eigen dan fungsi eigen energi eksak dari persamaan Schrodinger untuk partikel bermuatan yang bergerak dalam potensial non-sentral dapat dengan mudah dihitung secara analitik menggunakan metode supersimteri (SUSYQM). Selain itu, sistem yang dibahas adalah Trigonometri *Scarf plus Poschl-Teller*. *Lowering Operator* menghasilkan fungsi gelombang keadaan dasar sedangkan *Raising Operator* menghasilkan fungsi gelombang keadaan tereksitasi. Nilai eigen energi diekspresikan dalam bentuk tertutup yang diperoleh dengan menggunakan sifat-sifat *shape invariant*. Hasil-hasil eksak yang diperoleh sesuai jika menggunakan metode-metode lainnya.

Kata kunci: *Supersymmetry, Trigonometric Scarf plus Poschl Teller, Non-central potentials*

INTRODUCTION

One of the important work in theoretical physics is to obtain exact solution of the Schrödinger equation for special potentials[1-3]. It is well known that exact solution of Schrödinger equation are only possible for certain cases. The exact solution of Schrödinger equation for a class of non-central potentials already studied in quantum chemistry. With the advent of supersymmetric quantum mechanics SUSYQM[1-3], and the idea of shape invariance [4], study of potential problems in non-relativistic quantum theory has received renewed interest. SUSYQM allows one to determine eigenvalues and eigenstates of known analytically solvable potentials using algebra operator formalism without ever having to solve the Schrödinger differential equation by standard series method. However, the operator method has so far been applied only to one dimensional and spherically symmetric theree dimensional problems. Supersymmetri is, by definition[5-8], a symmetry between fermions and boson. A supersymmetric field theoretical model consists of a set of quantum fields and of a lagrangian for them which exhibit such a symmetry. The Lagrangian determines, through the action principle, the equations of motion and hence the dynamical behaviour of the particle.

Recently, some authors have investigated the energy spectra and eigenfunction with Non-central potential[9-13], Trigonometric Poschl-Teller plus Rosen-Morse using SUSY[9], Hulthén plus Manning-Rosen potential[10], and Scarf poential plus Poschl-Teller using NU[12]. In this paper, we investigate the energy eigenvalues and eigenfunction of trigonometric Scarf plus Poschl-Teller potential non-central potentials using SUSYQM method. The trigonometric Scarf potential is also called as generalized Poschl-Teller potential[13]. The trigonometric Poschl-Teller play the essential roles in electrodynamics interatomic and intermolecular forces and can be used to describe molecular vibrations. Some of these trigonometric potential are exactly solvable or quasi – exactly solvable and their bound state solutions have been reported[14-17].

Review of Formula for Supersymmetry Quantum Mechanics

Supersymmetry Quantum Mechanics (SUSY QM)

Witten defined the algebra of a supersymmetry quantum system, there are super charge operators Q_i which commute with the Hamiltonian H_{ss} [19]

$$[Q_i, H_{ss}] = 0 \text{ with, } i = 1, 2, 3, \ldots N \tag{1}$$

and they obey to algebra $\{Q_i, Q_j\} = \delta_{ij} H_{ss}$

with H_{ss} is called Supersymmetric Hamiltonian. Witten stated that the simplest quantum mechanical system has N=2, it was later shown that the case where N = 1, if it is supersymmetric, it is equivalent to an N = 2 supersymmetric quantum system[5]. In the case where N = 2 we can define,

$$Q_1 = \frac{1}{\sqrt{2}}\left(\sigma_1 \frac{p}{\sqrt{2m}} + \sigma_2 \phi(x)\right) \text{ and } Q_2 = \frac{1}{\sqrt{2}}\left(\sigma_2 \frac{p}{\sqrt{2m}} + \sigma_1 \phi(x)\right) \tag{2}$$

Here the σ_i are the usual Pauli spin matrices, and $p = -1\hbar \frac{\partial}{\partial x}$ is the usual momentum operator. For example two component, we shall write H_{ss} as $H_{\pm}$. Using equation (1) and (2) we get,

$$H_{ss} = \begin{pmatrix} -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{\hbar}{\sqrt{2m}}\frac{d\phi(x)}{dx} + \phi^2(x) & 0 \\ 0 & -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \frac{\hbar}{\sqrt{2m}}\frac{d\phi(x)}{dx} + \phi^2(x) \end{pmatrix} = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix} \tag{3}$$

with,

$$H_- = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_-(x) \text{ with } V_-(x) = \phi^2(x) - \frac{\hbar}{\sqrt{2m}}\phi'(x) \tag{4a}$$

and,

$$H_{+}=-\frac{\hbar^{2}}{2m}\frac{d^{2}}{dx^{2}}+V_{+}(x) \text{ with } V_{+}(x)=\phi^{2}(x)+\frac{\hbar}{\sqrt{2m}}\phi'(x) \tag{4b}$$

with H_- and H_+, is defined as supersymmetry partner in the Hamiltonian. $V_-(x)$ and $V_+(x)$ are the supersymmetry partner each other.
Thus, solving equation (4a) and (4b), Hamiltonian equation can be faktorizated,

$$H_{+}(x)=A^{+}A \text{, and } H_{-}(x)=AA^{+} \tag{5}$$

were, $A^{+}=-\frac{\hbar}{\sqrt{2m}}\frac{d}{dx}+\phi(x)$ and $A=\frac{\hbar}{\sqrt{2m}}\frac{d}{dx}+\phi(x)$ (6)

with, A^{+} as raising operator, and A as lowering operator.

Shape Invariance

Gendenshteın[4] discovered another symmetry which if the supersymmetric system satisfies it will be an exactly solvable system, this symmetry is known as shape invariance. If our potential satisfies shape invariance we can readily write down its bound state spectrum, and with the help of the charge operators we can find the bound state wave functions. It turned out that all the potentials which were known to be exactly solvable until then have the shape invariance symmetry. If the supersymmetric partner potentials have the same dependence on x but differ in a parameter, in such a way that they are related to each other by a change of of that parameter, then they are said to be shape invariant. Gendenshteın stated this condition in this way,

$$V_{+}(x;a_{j})=V_{-}(x;a_{j+1})+R(a_{j+1}) \tag{7}$$

with, $V_{+}(x;a_{j})=\phi^{2}(x;a_{j})+\frac{\hbar}{\sqrt{2m}}\phi'(x;a_{j})$ (8a)

$$V_{-}(x;a_{j})=\phi^{2}(x;a_{j})-\frac{\hbar}{\sqrt{2m}}\phi'(x;a_{j}) \tag{8b}$$

where j = 0,1,2,.., and a is a parameter in our original potential whose ground state energy is zero. $a_{j+1}=f(a_{j})$ where f is assumed to be an arbitrary function for the time being. The remainder $R(a_{j})$ can be dependent on the parametrization variable a but never on x. In this case V_- is said to be shape invariant, and we can readily find its spectrum, take a look at H,

$$H=H_{-}+E_{0}=-\frac{\hbar^{2}}{2m}\frac{d^{2}}{dx^{2}}+V_{-}(x;a_{0})+E_{0} \tag{9}$$

Acoording to equation (9) a further equation is obtained between $V_+(x)$ and $V_-(x)$ we get,

$$V(x)=V_{-}(x;a_{0})+E_{0}=\phi^{2}(x;a_{0})-\frac{\hbar}{\sqrt{2m}}\phi'(x;a_{0})+E_{0} \tag{10}$$

where $V(x)$ is often stated os effective potential V_{eff}. While $\phi(x)$ is determined hypothetically based on the shape of effective potential from the associated system.

The hamiltonian equation can be generalized,

$$H_{k}=-\frac{\hbar^{2}}{2m}\frac{d^{2}}{dx^{2}}+V_{-}(x;a_{k})+\sum_{i=1}^{k}R(a_{i}) \text{, dengan } k=0, 1, 2,\ldots \tag{11}$$

By comparing equation (8) and (9), it is found that $E_0 = \sum_{i=1}^{k} R(a_i)$. So that, in eigen energy spectra, the value of H₋, can be generalized as follows,

$$E_n^{(-)} = \sum_{k=1}^{n} R(a_k) \tag{12}$$

Furthermore, we get the total energy spectra,

$$E_n = E_n^{(-)} + E_0 \tag{13}$$

with E_0 as ground state energy in a Hamiltonian lowering partner potential.

Based on the characteristics of lowering operator, then the equation of ground state wave function can be obtained from the following equation,

$$A\psi_0^{(-)} = 0 \tag{14}$$

Meanwhile, the excited wave function, one and so forth $\psi_n^-\ (x; a_0\ )$ can be obtained by using raising operator and ground state wave function $\psi_0^-\ (x; a_0\ )$. In general, the equation of wave function can be stated as follow,

$$\psi_n^{(-)}(x;a_0) \approx A^+(x;a_0)\psi_{n-1}^{(-)}(x;a_1) \tag{15}$$

Solution of Schrödinger Equation for Trigonometric Scarf Plus Poschl-Teller Non-Central Potential Using Supersymmetry

Schrödinger equation trigonometric Scarf plus Poschl-Teller Non-central potential is the potentials present simulataneusly in the quantum system. This non-central potential is expressed as[11],

$$V(r,\theta) = \frac{\hbar^2\alpha^2}{2m}\left(\frac{b^2 + a(a-1)}{\sin^2(\alpha r)} - \frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right) + \frac{\hbar^2}{2mr^2}\left(\frac{a(a-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right) \tag{16}$$

The three dimensional Schrödinger equation for trigonometric Scarf plus Poschl-Teller non-central potential is written as,

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right]$$

$$+\frac{\hbar^2\alpha^2}{2m}\left(\frac{b^2 + a(a-1)}{\sin^2(\alpha r)} - \frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)\psi + \frac{\hbar^2}{2mr^2}\left(\frac{a(a-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right)\psi = E\psi \tag{17}$$

If equation (17) multiplied by factor ($-\frac{2mr^2}{\hbar^2}$), and the result is solve using separation variable method since the non-central potential is separable. By setting $\psi(r,\theta,\varphi) = R(r)P(\theta)\Phi(\varphi)$,with $RP\Phi(\psi(r,\theta,\varphi) = \frac{X(r)Q(\theta)e^{im\varphi}}{r\sqrt{\sin\theta}}$ we obtain,

$$\left[\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) - r^2\alpha^2\left(\frac{b^2 + a(a-1)}{\sin^2(\alpha r)} - \frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right) + \frac{2mr^2}{\hbar^2}E\right] + \left[\frac{1}{P\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right) + \frac{1}{\Phi\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2} - \left(\frac{a(a-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right)\right] = 0 \tag{18}$$

From equation (18) we obtain radial and angular Schrödinger equation as,

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) - r^2\alpha^2\left(\frac{b^2 + a(a-1)}{\sin^2(\alpha r)} - \frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right) + \frac{2mr^2}{\hbar^2}E = \left(\frac{a(a-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right) - \frac{1}{P\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right) - \frac{1}{\Phi\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2} = \lambda = \ell(\ell+1) \tag{19}$$

with $l(l+1)$ is constanta variabel separable, where ℓ as orbital momentum number[11].

From equation (19) we get radial and angular wave function Schrödinger equation with single variable as following,

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)-r^2\alpha^2\left(\frac{b^2+a(a-1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)+\frac{2mr^2}{\hbar^2}E=\ell(\ell+1) \tag{20}$$

or equation (20) multiplied by ($\frac{R}{r^2}$), with $R(r)=\frac{\chi(r)}{r}$, so using symple algebra, we get,

$$\frac{\partial^2\chi}{\partial r^2}-\alpha^2\left(\frac{b^2+a(a-1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)\chi-\frac{\ell(\ell+1)}{r^2}X=-\frac{2m}{\hbar^2}E\chi \tag{21}$$

and than, for solve radial Schrödinger equation, we use approximation for centrifugal term[18],

$\frac{1}{r^2}\cong\alpha^2\left(d_0+\frac{1}{\sin^2(\alpha r)}\right)$ for αr <<1, with $d_0=\frac{1}{12}$, $\in^2=-\frac{2m}{\hbar^2}E$, we get,

$$\frac{\partial^2\chi}{\partial r^2}-\alpha^2\left(\frac{b^2+a(a-1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)\chi-\ell(\ell+1)\alpha^2\left(d_0+\frac{1}{\sin^2(\alpha r)}\right)\chi=\in^2\chi \tag{22}$$

From equation (22) simplied by $(-\frac{\hbar^2}{2m})$ we get radial Schrödinger equation,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\chi}{\partial r^2}+\frac{\hbar^2\alpha^2}{2m}\left(\frac{b^2+a(a-1)+\ell(\ell+1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)\chi+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0\chi=-\frac{\hbar^2}{2m}\in^2\chi \tag{23}$$

and we have the angular and Schrödinger equation as,

$$\left(\frac{a(a-1)}{\sin^2\theta}+\frac{b(b-1)}{\cos^2\theta}\right)-\frac{1}{P\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right)-\frac{1}{\Phi\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2}=\ell(\ell+1) \tag{24}$$

and we have set $\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}=-m^2$ that give azimuthalwave function as

$$\Phi=\sqrt{\frac{1}{2\pi}}e^{im\varphi},\ m=0,\pm1,\pm2,... \tag{25}$$

Equation (24) will be,

$$\left(\frac{a(a-1)}{\sin^2\theta}+\frac{b(b-1)}{\cos^2\theta}\right)-\frac{1}{P\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right)+\frac{m^2}{\sin^2\theta}=\ell(\ell+1) \tag{26}$$

With m^2 as variable separation and we get angular Schrödinger equation one dimentional,

$$-\frac{\hbar^2}{2m}\frac{d^2}{d\theta^2}+\frac{\hbar^2}{2m}\left(\frac{a(a-1)+m^2-\frac{1}{4}}{\sin^2\theta}+\frac{b(b-1)}{\cos^2\theta}\right)H=\frac{\hbar^2}{2m}(\ell(\ell+1)+\tfrac{1}{4})H \tag{27}$$

The Solution of Radial Scrodinger Equation Trigonometric Scarf plus Poschl Teller Potential

Factor *R* in equation (20) is defined as wave function ψ , then the Schrödinger equation for trigonometric Scarf plus Poschl Teller non-central potential in radial with the assumption of $\in' = -\frac{\hbar^2}{2m}\in^2$ can be rewritten as follow,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\chi}{\partial r^2}+\frac{\hbar^2\alpha^2}{2m}\left(\frac{b^2+a(a-1)+\ell(\ell+1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)\chi+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0\chi=\in'\chi \tag{28}$$

Based on equation (28), the effective potential of radial SE trigonometric Scarf potential plus Poschl Teller can be rewritten as follow,

$$V_{eff}=\frac{\hbar^2\alpha^2}{2m}\left(\frac{b^2+a(a-1)+\ell(\ell+1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0 \tag{29}$$

or,

$$V_{eff}=\frac{\hbar^2\alpha^2}{2m}\left(\frac{a'(a'-1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0 \tag{30}$$

with $a'=\sqrt{b^2+a(a-1)+\ell(\ell+1)+\frac{1}{4}}+\frac{1}{2}$

By inserting effective potential in equation (30) into equation (10), its obtained

$$\phi^2(x)-\frac{\hbar}{\sqrt{2m}}\phi'(x)=\frac{\hbar^2\alpha^2}{2m}\left(\frac{a'(a'-1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0-\in \tag{31}$$

By using incisive hypothesis, it is assumed that superpotential in equation (30) is,

$$\phi(x)=A\cot(\alpha r)-B\csc(\alpha r) \tag{32}$$

Where *A* and *B* are indefinite constantans that will be calculated. From equation (32), we can determine the value of $\phi'(x)$ and $\phi^2(x)$, then the result is distributed into equation (31), then the following is obtained,

$$\frac{A^2}{\sin^2(\alpha r)}-A^2+\frac{B^2}{\sin^2(\alpha r)}+2AB\frac{\cos(\alpha r)}{\sin^2(\alpha r)}+\frac{\hbar}{\sqrt{2m}}\frac{\alpha A}{\sin^2(\alpha r)}=\frac{\hbar^2\alpha^2}{2m}\left(\frac{a'(a'-1)}{\sin^2(\alpha r)}-\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right)+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0-\in \tag{33}$$

By analysing the similar concept between left flank and right flank, from equation (33), it is obtained,

$$\left(A^2+B^2+\frac{\alpha\hbar}{\sqrt{2m}}A\right)\frac{1}{\sin^2(\alpha r)}=\frac{\hbar^2\alpha^2}{2m}(a'(a'-1))\frac{1}{\sin^2(\alpha r)};\left(2AB+\frac{\alpha\hbar}{\sqrt{2m}}B\right)\frac{\cos(\alpha r)}{\sin^2(\alpha r)}=-\frac{\hbar^2\alpha^2}{2m}\left(\frac{2b(a-\frac{1}{2})\cos(\alpha r)}{\sin^2(\alpha r)}\right);$$

and $$A^2+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0=\in=E_0 \tag{34}$$

From the three equation in equation (34), it is obtained,

$$A_+=\frac{\alpha\hbar}{\sqrt{2m}}\left(\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{((a'-\frac{1}{2})^2)^2-4(b(a-\frac{1}{2}))^2}}{2}}-\frac{1}{2}\right) \text{ and } A_-=-\frac{\alpha\hbar}{\sqrt{2m}}\left(\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{((a'-\frac{1}{2})^2)^2-4(b(a-\frac{1}{2}))^2}}{2}}+\frac{1}{2}\right) \tag{35a}$$

$$B = \frac{\alpha\hbar}{\sqrt{2m}}\left(\frac{b(a-\frac{1}{2})}{\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}}\right) \quad (35b)$$

$$E_0 = \frac{\hbar^2\alpha^2}{2m}\left(\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}+\frac{1}{2}\right)+\frac{\hbar^2\alpha^2}{2m}\ell(\ell+1)d_0 \quad (35c)$$

The value of A and B are determined in certain way so the value of E_0 is equal to zero, so,

$$\phi(r) = -\frac{\alpha\hbar}{\sqrt{2m}}\left(\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}+\frac{1}{2}\right)\cot\alpha r+\frac{\alpha\hbar}{\sqrt{2m}}\left(\frac{b(a-\frac{1}{2})}{\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}}\csc\alpha r\right) \quad (36)$$

By using equation (6) and (36), we get

$$A^+ = -\frac{\hbar}{\sqrt{2m}}\frac{d}{dr}+\phi(r) = -\frac{\hbar}{\sqrt{2m}}\frac{d}{dr}-\frac{\alpha\hbar}{\sqrt{2m}}\left(\left(\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}+\frac{1}{2}\right)\cot\alpha r-\left(\frac{b(a-\frac{1}{2})}{\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}}\csc\alpha r\right)\right) \quad (37)$$

And,

$$A = \frac{\hbar}{\sqrt{2m}}\frac{d}{dr}+\phi(r) = \frac{\hbar}{\sqrt{2m}}\frac{d}{dr}-\frac{\alpha\hbar}{\sqrt{2m}}\left(\left(\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}+\frac{1}{2}\right)\cot\alpha-\left(\frac{b(a-\frac{1}{2})}{\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}}\cos\alpha r\right)\right)$$

$$A = \frac{\hbar}{\sqrt{2m}}\frac{d}{dr}-\frac{\alpha\hbar}{\sqrt{2m}}\left(\left(M+\frac{1}{2}\right)\cot\alpha r-\left(K\csc\alpha r\right)\right) \quad (38)$$

with, $M = \sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}$; and, $K = \frac{b(a-\frac{1}{2})}{\sqrt{\frac{(a'-\frac{1}{2})^2-\sqrt{\left((a'-\frac{1}{2})^2\right)^2-4\left(b\left(a-\frac{1}{2}\right)\right)^2}}{2}}}$

The ground state wave function can be obtained from equation (14) and (38), which are,

$$\left\{\frac{\hbar}{\sqrt{2m}}\frac{d}{dr}+\phi(r)\right\}\psi_0^- = 0$$

Then the ground state wave function of Scarf potential is as follow,

$$\int\frac{d\psi_0^-}{\psi_0^-}(r,a_0) = \left(M+\frac{1}{2}\right)\int\cot(\alpha r)d(\alpha r) - K\Big)\int\csc\alpha r d(\alpha r)$$

$$\ln\psi_0^-(r,a_0) = \left\{\left(M+\frac{1}{2}\right)\ln\sin(\alpha r)-K\ln\left(\csc\alpha r-\cot\alpha r\right)\right\}+C$$

$$\psi_0^-(r,a_0) = C\left(\sin(\alpha r)\right)^{\left(M+\frac{1}{2}\right)}\left(\frac{1}{\sin(\alpha r)}-\frac{\cos\alpha r}{\sin(\alpha r)}\right)^{(-K)}$$

$$= C\left(\sin(\alpha r)\right)^{\left(M+K+\frac{1}{2}\right)}\left(1-\cos\alpha r\right)^{(-K)} \quad (39)$$

By using equation (15) we can obtain excited wave function on the first level as follow,

$$\psi_1^{(-)}(r;a_0) \approx A^+(x;a_0)\psi_0^{(-)}(r;a_1) \quad (40)$$

where $a_0 = M = a_1 = M+1$, and $a_0 = M = a_1 = M+1 \quad \cdots, a_n = M+n$, is the independent parametre to variable "r". By inserting the value of the parametre to equation (39) and (37) and by using equation (40), the following we get,

$$\psi_1^{(-)}(r;a_0) \approx \left(-\frac{\hbar}{\sqrt{2m}}\frac{d}{dr} - \frac{\alpha\hbar}{\sqrt{2m}}\left(\left(M+\frac{1}{2}\right)\cot\alpha r - K\csc\alpha r\right)\right)\left\{C(\sin(\alpha r))^{\left(M+K+\frac{3}{2}\right)}(1-\cos\alpha r)^{(-k+1)}\right\}$$

$$\approx \left(-\frac{\hbar}{\sqrt{2m}}\left((2M+K+2)(\cos(\alpha r)) - K\frac{(\sin^2(\alpha r))}{(1-\cos(\alpha r))} - K\right)\right)\left\{C(\sin(\alpha r))^{\left(M+K+\frac{1}{2}\right)}(1-\cos(\alpha r))^{(-K)}\right\} \quad (41)$$

The breakdown in equation (41) can be continued to find wave function $\psi_2^{(-)}(r;a_0), \psi_3^{(-)}(x;a_0)$, ... and so on.

The determination of the potential partner which have shape invariant, by using equation (8a) and (8b) results,

$$V_-(r,a_0) = \frac{\hbar^2\alpha^2}{2m}\frac{\left(\left(M+\frac{1}{2}\right)^2 + K^2 - \left(M+\frac{1}{2}\right)\right)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}K\left(2\left(M+\tfrac{1}{2}\right)-1\right)\frac{\cos(\alpha r)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}\left(M+\tfrac{1}{2}\right)^2 \quad (42a)$$

and,

$$V_+(r,a_0) = \frac{\hbar^2\alpha^2}{2m}\frac{\left(\left(M+\frac{1}{2}\right)^2 + K^2 + \left(M+\frac{1}{2}\right)\right)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}K\left(2\left(M+\tfrac{1}{2}\right)+1\right)\frac{\cos(\alpha r)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}\left(M+\tfrac{1}{2}\right)^2 \quad (42b)$$

If we have choose parameters $a_0 = M\,;\, a_1 = M+1,\ \ldots$
then $V_-(r, a_1)$ Obtained if on equation (42) the value of v', changed into $v'+1$, i.e

$$V_-(r,a_1) = \frac{\hbar^2\alpha^2}{2m}\frac{\left((M+1)^2 + K^2 - \frac{1}{4}\right)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}K\left(2(M+1)\right)\frac{\cos(\alpha r)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}\left(M+\tfrac{3}{2}\right)^2 \quad (43)$$

From those equation (42b) and (43) can be seen that V_+ (r,a_0) have similar shape with $V_-(r, a_1)$, and with using shape invariance relation on equation (8) obtained $R(a_1)$ i.e,,

$$R(a_1) = V_+(r,a_0) - V_-(r,a_1) = \frac{\hbar^2\alpha^2}{2m}\left(\left(M+\tfrac{3}{2}\right)^2 - \left(M+\tfrac{1}{2}\right)^2\right) \quad (44)$$

We repeat the step as on the determination of equation (48) by using the steps equation (42a), (42b), and (43), to obtain equation $V_+(r, a_1)$ and V_- (r, a_2), so obtained,

$$V_+(r,a_1) = \frac{\hbar^2\alpha^2}{2m}\frac{\left(M^2 + 4M + \frac{15}{4} + K^2\right)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}K\left(2(M+2)\right)\frac{\cos(\alpha r)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}\left(M+\tfrac{3}{2}\right)^2 \quad (4.5)$$

$$V_+(r,a_2) = \frac{\hbar^2\alpha^2}{2m}\frac{\left(M^2 + 4M + \frac{15}{4} + K^2\right)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}K\left(2(M+2)\right)\frac{\cos(\alpha r)}{\sin^2(\alpha r)} - \frac{\hbar^2\alpha^2}{2m}\left(M+\tfrac{5}{2}\right)^2 \quad (45b)$$

From equation (45a) and (45b) so obtained,

$$R(a_2) = V_+(r,a_1) - V_-(r,a_2) = \frac{\hbar^2\alpha^2}{2m}\left(\left(M+\tfrac{5}{2}\right)^2 - \left(M+\tfrac{3}{2}\right)^2\right) \quad (46)$$

Then, the determination steps on equation (44) or equation (46) above are repeated until parameters heading to n, a_n to determinate $R(a_n)$ and finally obtained,

$$E_n^{(-)} = \sum\nolimits_{k=1}^{n} R(a_k) = \frac{\hbar^2\alpha^2}{2m}\left(\left(M+\tfrac{1}{2}+n\right)^2 - \left(M+\tfrac{1}{2}\right)^2\right) \quad (47)$$

If equation (47) and equation (36c) incorporated to equation (13) obtained energy spectrum for Scarf system i.e.,

$$E_n = E_n^{(-)} + E_0 = \frac{\hbar^2\alpha^2}{2m}\left(\left(M+n+\tfrac{1}{2}\right)^2 + \ell(\ell+1)d_0\right) \quad (48)$$

with, $E_n = \in'$, and $\upsilon' = \sqrt{\upsilon'(\upsilon'+1)+\ell(\ell+1)+\frac{1}{4}} - \frac{1}{2}$, so equation (48) can be rewritten energy solution of Schrödinger equation for trigonometric Scarf plus Poschl-Teller non central potential as,

$$-\frac{\hbar^2}{2m}\in^2 = E_n$$

$$-\frac{\hbar^2}{2m}\left(-\frac{2m}{\hbar^2}E\right) = \frac{\hbar^2\alpha^2}{2m}\left(\left(M+n+\tfrac{1}{2}\right)^2 + \ell(\ell+1)d_0\right)$$

$$E_{nr} = \frac{\hbar^2\alpha^2}{2m}\left(\left(M+n+\tfrac{1}{2}\right)^2 + \ell(\ell+1)d_0\right) \tag{49}$$

Equation (49) showed the energy spectra of trigonometric Scarf plus Poschl-Teller non central potential. The results are in exact agreement with derived using NU method [11] with,

$\hbar$: planck constants,

m : mass of particle

a and b : constants potential depth,

n : principe quatum numbers, n=1,2,3…

n_r : radial quantum numbers, n_r=0,1,2…

l : orbital quantum numbers (the value same with polar wave function solving) l=0,1,2…n−1.

The Solution of Angular Schrödinger Equation Trigonometric Scarf Plus Poschl-Teller Non-Central Potential.

To ease the solution of angular Schrödinger Equation, i.e.,

$$\frac{\hbar^2}{2m}\left(\ell(\ell+1)+\tfrac{1}{4}\right)H = EH \tag{50}$$

If equation (50) incorporated to equation (28) so angular Schrödinger equation of Trigonometric Scarf plus Poschl-Teller non central potential chanced into,

$$-\frac{\hbar^2}{2m}\frac{d^2H}{d\theta^2} + \frac{\hbar^2}{2m}\left(\frac{a(a-1)+m^2-\frac{1}{4}}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right)H = EH \tag{51}$$

Based on equation (51), effective potential of angular Poschl-Teller plus Scarf non central potential describe as,

$$V_{eff} = \frac{\hbar^2}{2m}\left(\frac{a(a-1)+m^2-\frac{1}{4}}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right) \tag{52}$$

if, $a(a-1)+m^2-\frac{1}{4} = a'(a'-1)$, we get

$$V_{eff} = \frac{\hbar^2}{2m}\left(\frac{a'(a'-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right) \tag{53}$$

with $a' = \sqrt{a(a-1)+m^2} + \frac{1}{2}$

According to the form of those effective potential equations, then superpotential equation of angular Scarf plus Poschl-Teller non central potential can be describe as,

$$\phi(\theta) = A\tan\theta + B\cot\theta \tag{54}$$

where A and B are unstable constant that will be counted. From equation (54), determinated value of $\phi_0'(x)$ and $\phi_0^2(x)$, thus the results are subtituted into equation (6), obtained relation,

$$A^2\tan^2\theta + B^2\cot^2\theta + 2AB - \frac{\hbar}{\sqrt{2m}}\left(A\sec^2\theta - B\csc^2\theta\right) = \frac{\hbar^2}{2m}\left(\frac{a'(a'-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right) - \in \tag{55}$$

By using in common concept of coefficient between left and right internode, so that from equation (55), value is obtained,

$$\left(A^2 + \frac{\hbar}{\sqrt{2m}}A\right)\frac{1}{\cos^2\theta} = \frac{\hbar^2}{2m}(b(b-1))\frac{1}{\cos^2\theta};\left(B^2 + \frac{\hbar}{\sqrt{2m}}B\right)\frac{1}{\sin^2\theta} = \frac{\hbar^2}{2m\alpha^2}a'(a'-1)\frac{1}{\sin^2\theta};$$

and $\in = E_0 = A^2 + B^2 - 2AB$ (56)

from those third equation on equation (56) is obtained,

$$A_+ = \frac{\hbar}{\sqrt{2m}}b \quad \text{atau} \quad A_- = -\frac{\hbar}{\sqrt{2m}}(b-1) \tag{57a}$$

$$B_+ = \frac{\hbar}{\sqrt{2m}}(a'-1) = \frac{\hbar}{\sqrt{2m}}\sqrt{a(a-1)+m^2} - \tfrac{1}{2}$$

$$\text{or } B_- = -\frac{\hbar}{\sqrt{2m}}a' = -\frac{\hbar}{\sqrt{2m}}\sqrt{a(a-1)+m^2} + \tfrac{1}{2} \tag{57b}$$

$$E_0 = \in = \frac{\hbar^2}{2m}(b+a')^2 \tag{57c}$$

A and B value are chosen so that $E_0^{(-)}$ value is zero. By using equation (8a) and (8b) are obtained,

$$\phi(\theta) = \frac{\hbar}{\sqrt{2m}}b\tan\theta - \frac{\hbar}{\sqrt{2m}}a'\cot\theta \tag{58}$$

$$V_-(\theta; a'_0\, b_0) = \phi^2(\theta; a'_0\, b_0) - \frac{\hbar}{\sqrt{2m}}\phi'(\theta; a'_0\, b_0)$$

$$= \frac{\hbar^2}{2m}\left(\frac{a'(a'-1)}{\sin^2\theta} + \frac{b(b-1)}{\cos^2\theta}\right) - \frac{\hbar^2}{2m}(b+a')^2 \tag{59a}$$

$$V_+(\theta; a'_0\, b_0) = \phi^2(\theta; a'_0\, b_0) + \frac{\hbar}{\sqrt{2m}}\phi'(\theta; a'_0\, b_0)$$

$$= \frac{\hbar^2}{2m}\left(\frac{a'(a'+1)}{\sin^2\theta} + \frac{b(b+1)}{\cos^2\theta}\right) - \frac{\hbar^2}{2m}(b+a')^2 \tag{59b}$$

From those two equations (59a) and (59b) is obtained $a_0 = a'$; $b_0 = b$;

$a_1 = a' + 1;\ b_1 = b + 1; \ldots$

$$V_-(\theta; a'_1\, b_1) = \frac{\hbar^2}{2m}\left(\frac{a'(a'+1)}{\sin^2\theta} + \frac{b(b+1)}{\cos^2\theta}\right) - \frac{\hbar^2}{2m}(b+a'+2)^2 \tag{60}$$

From those two equation (59b) and (60) can be seen that $V_+ (\theta, a_0 b_0)$ have the same form with $V_-(\theta, a_1 b_1)$, and by using shape invariance relation on equation (8), is obtained $R(a_1 b_1)$ i.e.,

$$R(a'_1\, b_1) = V_+(\theta; a'_0\, b_0) - V_-(\theta; a'_1\, b_1) = \frac{\hbar^2}{2m}(b+a'+2)^2 - \frac{\hbar^2}{2m}(b+a')^2 \tag{61}$$

We repeated the step as on determination of equation (61) with using steps equation (59), and equation (60) to obtain $V_+(\theta, a_1 b_1)$ and $V_-(\theta, a_2 b_2)$ equations, so obtained,

$$V_{+}(\theta; a'_1 b_1) = \frac{\hbar^2}{2m}\left(\frac{(a'+1)(a'+2)}{\sin^2\theta} + \frac{(b+1)(b+2)}{\cos^2\theta}\right) - \frac{\hbar^2}{2m}(b+a'+2)^2 \quad (62a)$$

$$V_{-}(\theta; a'_2 b_2) = \frac{\hbar^2}{2m}\left(\frac{(a'+1)(a'+2)}{\sin^2\theta} + \frac{(b+1)(b+2)}{\cos^2\theta}\right) - \frac{\hbar^2}{2m}(b+a'+4)^2 \quad (62b)$$

By repeated the step from equation (62a) to (62b) we often,

$$R(a'_2 b_2) = V_{+}(\theta; a'_1 b_1) - V_{-}(\theta; a'_2 b_2) = \frac{\hbar^2}{2m}(b+a'+4)^2 - \frac{\hbar^2}{2m}\left(b+a'+2\right)^2 \quad (63)$$

$$R(a'_n b_n) = V_{+}(\theta; a'_{n-1} b_{n-1}) - V_{-}(\theta; a'_n b_n) = \frac{\hbar^2}{2m}(b+a'+2n)^2 - \frac{\hbar^2}{2m}\left(b+a'+2n-2\right)^2 \quad (64)$$

Then determination steps on equation (61) or equation (63) on above are repeated until parameters heading to n, $a_n b_n$ to deteminate $R(a_n b_n)$ as on equation (64) and finally obtained the order of energy parameters that described,

$$E_n^{(-)} = \sum_{k=1}^{n} R(a_k) = \frac{\hbar^2}{2m}(b+a'+2n)^2 - \frac{\hbar^2}{2m}\left(b+a'\right)^2 \quad (65)$$

If equation (65) and equation (57c) are inserted into equation (13) we obtain,

$$E_n = E_n^{(-)} + E_0 = \frac{\hbar^2}{2m}(b+a'+2n)^2 - \frac{\hbar^2}{2m}\left(b+a'\right)^2 + \frac{\hbar^2}{2m}\left(b+a'\right)^2$$

so $E_n = \frac{\hbar^2}{2m}(b+a'+2n)^2$ (66)

with $a' = \sqrt{a(a-1)+m^2} + \frac{1}{2}$

By using the same order of energy parameters with eigen value of angular square momentum as mentioned on equation (66) so obtained angular quantum numbers that described as,

$$\left(\ell(\ell+1) + \tfrac{1}{4}\right) = \left(\sqrt{a(a-1)+m^2} + \tfrac{1}{2} + b + 2n\right)^2$$

then $\ell = \sqrt{a(a-1)+m^2} + b + 2n$ (67)

angular quantum numbers on equation (67) is used to calculate energy spectrum equation (49) with potential non central system.

By using equation (6) and (58) are obtained

$$A^{+} = -\frac{\hbar}{\sqrt{2m}}\frac{d}{d\theta} + \phi(\theta) = -\frac{\hbar}{\sqrt{2m}}\frac{d}{d\theta} + \frac{\hbar}{\sqrt{2m}} b\tan\theta - \frac{\hbar}{\sqrt{2m}} a'\cot\theta \quad (68a)$$

and $A = \frac{\hbar}{\sqrt{2m}}\frac{d}{d\theta} + \phi(\theta) = \frac{\hbar}{\sqrt{2m}}\frac{d}{d\theta} + \frac{\hbar}{\sqrt{2m}} b\tan\theta - \frac{\hbar}{\sqrt{2m}} a'\cot\theta$ (68b)

By using decreasing operator on equation (68b), determinated basic wave function for angular trigonometric Poschl-Teller plus Scarf non-central potential as follows,

$$\left\{\frac{\hbar}{\sqrt{2m}}\frac{d}{d\theta} + \frac{\hbar}{\sqrt{2m}} b\tan\theta - \frac{\hbar}{\sqrt{2m}} a'\cot\theta\right\}\psi_0^{-} = 0$$

$$\int \frac{d\psi_0^{-}}{\psi_0^{-}}(\theta, a_0) = a'\int \cot\theta d\theta - b\int \tan\theta d\theta$$

$$\psi_0^{-}(r, a_0) = C\left((\cos\theta)^b\right)(\sin\theta)^{a'} = C\left((\cos\theta)^b\right)(\sin\theta)^{\sqrt{a(a-1)+m^2}+\frac{1}{2}} \quad (69)$$

Then, by using increasing operator on equation (68a) and basic wave function determinated first level excited wave function,

$$\psi_1^{(-)}(\theta;a_0) \approx A^+(\theta;a_0)\psi_0^{(-)}(\theta,a_1)$$

$$\psi_1^{(-)}(\theta;a_0) = \left(-\frac{\hbar}{\sqrt{2m}}\frac{d}{d\theta} + \frac{\hbar}{\sqrt{2m}} b\tan\theta - \frac{\hbar}{\sqrt{2m}} a'\cot\theta\right)\left((\cos\theta)^{b+1}(\sin\theta)^{a'+1}\right)$$

$$= \frac{\hbar}{\sqrt{2m}}\left\{\left((2b+1)(\sin\theta)^2 - (2a'+1)(\cos\theta)^2\right)\right\}(\cos\theta)^b(\sin\theta)^{a'} \quad (70)$$

$$\psi_2^{(-)}(\theta;a_0) = \frac{\hbar^2}{2m}\left(-\frac{d}{d\theta} + b\tan\theta - a'\cot\theta\right)\left\{(2b+3)(\sin\theta)^2 - (2a'+3)(\cos\theta)^2\right\}(\cos\theta)^{b+1}(\sin\theta)^{a'+1}$$

$$= \frac{\hbar^2}{2m}\left\{\left((4a'^2+8a'+3)(\cos\theta)^4 + (4b^2+8b+3)(\sin\theta)^4\right) - (8ba'+12a'+12b)(\sin\theta)^2(\cos\theta)^2\right\}\left((\cos\theta)^b(\sin\theta)^{a'}\right) \quad (71)$$

To determinate excited eigenfunction above can be done as on determination of first level excited wave function as follows,

$\psi_2^{(-)}(\theta;a_0), \psi_3^{(-)}(\theta;a_0)$, and so on.

Therefore obtained wave function level that is wanted, with $a' = \sqrt{a(a-1)+m^2} + \frac{1}{2}$

RESULT AND DISCUSSION

It has been shown that the eigen spectra and eigenfunction Schrodinger equation of Scarf potential plus Poschl-Teller non-central potential is solved exactly using Supersymmetric method. The energy spectrum of the system is obtained in the closed form, showed by equation (49) and the radial ground state wave function by equation (29), and angular wave function by equation (69).The presence of Poschl-Teller non-central potential causes the decrease in energy spectrum of Scarf potential and increases the orbital quantum number. Where the complete eigenfunction in form $\psi_{tot}(n_r, n_l, m)$, with m positif, we obtain,

$$\psi_{tot}(000) = C\left(\sin(\alpha r)\right)^{\left(M+K+\frac{1}{2}\right)}\left(1-\cos\alpha r\right)^{(-K)}\left((\cos\theta)^b(\sin\theta)^{\sqrt{a(a-1)+m^2}+\frac{1}{2}}\right)\left(\sqrt{\frac{1}{2\pi}}\right)$$

$$\psi_{tot}(011) = \frac{\hbar}{\sqrt{2m}} C\left(\sin(\alpha r)\right)^{\left(M+K+\frac{1}{2}\right)}\left(1-\cos\alpha r\right)^{(-K)}$$
$$\left\{\left((2b+1)(\sin\theta)^2 - (2a'+1)(\cos\theta)^2\right)\right\}(\cos\theta)^b(\sin\theta)^{a'}\left(\sqrt{\frac{1}{2\pi}}e^{i\varphi}\right)$$

$$\psi_{tot}(021) = \frac{\hbar}{\sqrt{2m}} C\left(\sin(\alpha r)\right)^{\left(M+K+\frac{1}{2}\right)}\left(1-\cos\alpha r\right)^{(-K)}$$
$$\frac{\hbar^2}{2m}\left\{(4a'^2+8a'+3)(\cos\theta)^4 + (4b^2+8b+3)(\sin\theta)^4 - (8ba'+12a'+12b)(\sin\theta)^2(\cos\theta)^2\right\}$$
$$\left((\cos\theta)^b(\sin\theta)^{a'}\right)\left(\sqrt{\frac{1}{2\pi}}e^{i\varphi}\right)$$

$$\psi_{tot}(111) = \left(-\frac{\hbar}{\sqrt{2m}}\left((2M+K+2)(\cos(\alpha r)) - K\frac{(\sin^2(\alpha r))}{(1-\cos(\alpha r))} - K\right)\right)\left\{C(\sin(\alpha r))^{\left(M+K+\frac{1}{2}\right)}(1-\cos(\alpha r))^{(-K)}\right\}$$
$$\left\{\left((2b+1)(\sin\theta)^2 - (2a'+1)(\cos\theta)^2\right)\right\}(\cos\theta)^b(\sin\theta)^{a'}\left(\sqrt{\frac{1}{2\pi}}e^{i\varphi}\right)$$

CONCLUSION

Based on the describtion, on III and IV point, proved that the energy spectra and eigenfunction for trigonometric Scarf plus Poschl Teller non central potential with group of shape invariance potential can be solved using Supersimmetric method (SUSYQM). By operating the lowering operator we get the ground state wave function, and the excited state wave functions are obtained by operating raising operator repeatedly. The energy eigenvalue is expressed in the closed form obtained using the shape invariant properties. The results are in exact agreement with NU methods.

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