Senior High School Students Problem Solving in Terms of Mathematical Abilities

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Abstract: *Practicing problem-solving can increase students' confidence outside of the mathematics field. Meanwhile, students who have different mathematical abilities have different problem-solving abilities in solving problems. Therefore, this research aims to describe students' problem-solving abilities in terms of mathematical abilities by using qualitative descriptive research methods. The subjects were three students with high, medium, and low abilities, while the main instrument was the researcher himself, supported by test and interview instruments. The results of high and moderate students exploring and understanding the problem showed the reluctance to write in detail on the answer sheet for the information known and problem asked in the questions, then delivered it in the interview session. In contrastNTS, the low-ability students wrote in great detail. At the stage of representing and formulating, high, medium, and low students used the same terms and formulas. At the stage of planning and implementing, the three students used various methods. However, the three students acquired the same answer and rechecked the results with what was known in the questions.*

Keywords: *differential, mathematical ability; problem-solving*

Abstrak: *Melatih pemecahan masalah dapat meningkatkan kepercayaan diri siswa. Sedangkan dalam memecahkan masalah, siswa yang memiliki kemampuan yang berbeda memiliki kemampuan pemecahan masalah yang berbeda pula. Sehingga tujuan dari penelitian ini adalah untuk mendeskripsikan kemampuan pemecahan masalah siswa ditinjau dari kemampuan matematika dengan menggunakan metode penelitian deskriptif kualitatif. Terdapat tiga siswa sebagai subjek pada penelitian ini, siswa berkemampuan matematika tinggi, sedang, dan rendah sedangkan instrumen utamanya adalah peneliti sendiri yang didukung dengan instrumen tes dan wawancara. Hasil penelitian siswa berkemampuan tinggi dan sedang dalam hal mengeksplorasi dan memahami masalah enggan untuk menuliskan secara rinci pada lembar jawaban atas informasi dan masalah yang terdapat pada soal dan disampaikan pada sesi wawancara, sedangkan siswa berkemampuan rendah siswa menulis secara rinci. Pada tahap merepresentasikan dan merumuskan, siswa berkemampuan tinggi, sedang, dan rendah menggunakan istilah dan rumus yang sama. Pada tahap perencanaan dan pelaksanaan, ketiga siswa tersebut menggunakan metode yang berbeda. Walaupun dengan metode yang berbeda, ketiga siswa tersebut mendapatkan jawaban yang sama dan memeriksa kembali solusi dengan apa yang diketahui dalam soal*

Kata Kunci: *Kemampuan matematika, pemecahan masalah, turunan*

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INTRODUCTION

Six functions are implemented in teaching and learning: collaborate, communicate, inform, manage, produce, and scaffold (Shamaki, 2017). Mathematics is the basic knowledge necessary to study in life. It is one of the branches of knowledge that provides the basis for technology development and has an important role in many kinds of science that can improve human mind power. In Indonesia, students from Elementary School (SD) to Senior High School (SMA). The Indonesian government has made some efforts to encourage thinking skills and creativity, productivity, critical thinking, independence, collaboration, and communication in basic and middle education (Kemendikbud, 2016a). The government expects mathematics can be more accepted and understood by changing the curriculum every five years. One of the ways is by presenting mathematical problems in a context close to everyday life to relate easily. Hopefully, the students will be more intrigued by mathematics. Since one of the purposes of mathematics is turning the learners into being responsible, responsive, and not easily giving up in solving the problems (Kemendikbud, 2016b). However, the memorization method dominates students' learning activities, so they do not hone students' problem-solving skills (Lithner, 2015). The importance of studying problem-solving in mathematics is that a student can get the way of thinking, thoroughness, curiosity, and self-confidence in solving the problem that will happen to the student outside mathematics (NCTM, 2000). Moreover, problem-solving includes other skills such as identification and the ability to search, select, evaluate, organize and consider various alternatives and interpret information about a given problem (Zubaidah, 2016)

A problem can be interpreted as a situation or statement that stimulates and challenges a student or group of students to answer. However, they do not have certain rules, logarithms/procedures or laws that can immediately be used to determine the answer (Siswono, 2018; Kurnia, 2018). The problem of mathematics involves a situation proposed in abstract or conceptual background. A person in that situation cannot immediately determine how to continue or whether the algorithm moves toward the solution (Dossey, 2017). The background of mathematical problems is notional and conceptual, non-customary, and is meant to be solved (Brookhart, 2010). Mathematical problems involve a situation, posed in an abstract or contextual setting, non-routine and aim to be found out. However, students do not have procedures available and can hardly have solutions. In other words, students must try to find solutions. (Dossey, 2017; Brookhart, 2010; Aydogdu, 2008).

Mathematical problem-solving attempts to identify the gap between the problem and its solution using knowledge and reasoning (Hesse, 2015; Siswono, 2018). Thus, the problem solving of mathematics is an effort to figure out something unknown and the finishing method. One famous step of mathematical problem-solving is PISA 2012 (OECD, 2013), which includes exploring and understanding, representing and formulating, planning and executing, monitoring and evaluating. Students explore, investigate, and determine what is known and asked in the exploring and understanding step. After recognizing information contained in the problem, students are expected to understand concepts about the problem. In the representing and formulating step, students represent problems by making tables, graphs, symbols or reiterating verbally. Afterward, the students manage the information obtained, think and integrate it with previous knowledge to put together the best formula. In planning and executing steps, students plan the strategies used in solving the problem and then include the steps in executing the plan. Furthermore, in monitoring and evaluating, students re-examine the solutions implemented and evaluate the solutions from a different perspective.

Mathematical abilities are also vital for the students as the abilities are potential to use. Students' mathematical abilities are categorized into three: students with high mathematical ability, students with average mathematical ability, and students with low mathematical ability. Students also need mathematical abilities to obtain, process information and store information. The ability is also beneficial

in learning and mastering ideas and potentials related to mathematical concepts. (Liu, 2016; Karsentyon sing, 2020). Mathematical abilities are also considered as multidimensional constructs, including quantitative abilities (number understanding and pre-algebra reasoning), causal abilities (examination of cause-and-effect relationships), spatial abilities (spatial perspective and rotational abilities), and inductive/deductive abilities (Kattau, 2012). In addition, students' mathematical abilities can be grouped into three categories: students with high mathematical abilities, students with moderate mathematical abilities, and students with low mathematical abilities. By the various degrees of ability, students have different abilities to solve the problem (Rahima, 2019). Mathematical ability is not inherent. It can be obtained in life by continuous learning, including(1) The ability to obtain the mathematical information; (2) The ability to process the mathematical information; and (3) The ability to save the mathematical information (Szabo, 2018). Based on the background, this study aims to identify the problem-solving capability of Senior High School students in terms of mathematical ability in the material of algebra derivative formula.

RESEARCH METHODS

This research is a qualitative study with the descriptive research method to describe how senior high school students solved mathematical problems based on the steps of PISA in terms of mathematical abilities in derivative material. The primary data of the mathematical ability test aimed to categorize the level of the student's mathematical ability. The second instrument was a written test consisting of two problems in the application of algebra derivative formula aiming at identifying the students' problem-solving capability, while the third instrument was an interview aiming to scrutiny the thinking process of the students.

Furthermore, the data collection procedure in this research included (a) selecting the research subject in terms of mathematical ability by using a test. In the test, there were 20 problems in an essay adapted from the national examination to categorize the level of mathematical ability of the students. The grouping of the student's ability depended on the student's score. It was designed by the researcher based on Permendikbud number 23, 2016, as follow:

Tabel 1. Category of Mathematical Ability

Score	Group
$80 \leq score \leq 100$	High
$60 \leq score \leq 79$	Average
$0 < score \leq 59$	Low

The next procedure included (b) selecting the research subject from each mathematical ability category and giving the written test about applying algebra derivative formula; and (c) conducting an interview session with each research subject to understand their thinking process. The next step was data analysis.

Some processes were carried out in the data analysis stage to draw credible conclusions, including (a) data reduction. In this stage, the data obtained were summarized and selected to obtain the main and important aspects supporting the research and eliminating unnecessary data. The next step was (b) the stage of data serving. In this stage, data reduced were categorized into each level of mathematical ability to identify the difference of each level of the ability in solving the problem based on the steps of PISA, such as exploring and understanding, representing and formulating, planning and

implementing, also monitoring and evaluating (OECD, 2013). The final step was (c) the stage of conclusion and verification. The initial conclusion obtained from the data reduction and data serving stage was strengthened with strong evidence. Furthermore, valid and credible data were used to identify how the students solved the problem by using the steps of PISA in terms of each mathematical ability in the material of the algebra derivative formula.

RESULTS AND DISCUSSION

Based on the score of the mathematical ability test, the students with high mathematical ability were 10 students (27.8%), the students with average mathematical ability were 18 students (50%), and those with low mathematical ability were 8 students (22.2%), as shown in the following table:

Tabel 2. Group of Mathematical Ability

Group	The Number Of Student	Percentage
High	10	27.8%
Average	18	50%
Low	8	22.2%

This research described students' problem-solving skills based on three levels of their mathematical abilities. Next, the selected students were given the initial name, namely, SH (representative of students with high mathematical ability), RY (representative of students with average mathematical ability), FK (representative of students with low mathematical ability).

Student SH did problem-solving toward the two problems that had been prepared. In exploring and understanding the problems, SH figured out what was known and questioned, although he did not write in great detail on the answer sheet.
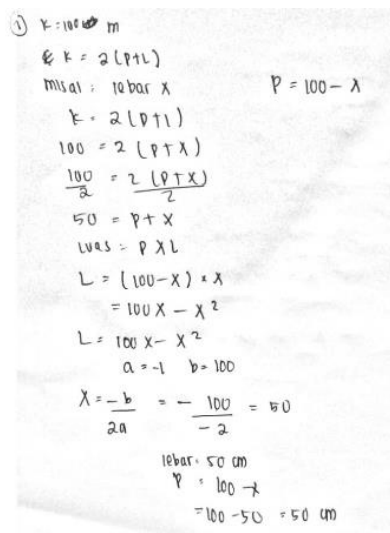

Figure 1. Answer Sheet of High Mathematical Ability Student

Next, SH represented and formulated what was known in the problems by writing $k = 100$ and the formula of the rectangle perimeter $k = 2(p + l)$. Based on the result of the interview with SH, k was the perimeter of a rectangle, p was the length, and l was the width of the cardboard. Next, in the stage of planning and solving, SH used a quadratic equation by representing the width of the

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cardboard. After that, SH substituted it for the cardboard area, as seen in picture 1. Based on the interview result, SH used a quadratic equation as it was the fastest way to solve the problems. In the monitoring and evaluation stage, SH rechecked the result of the perimeter known in the problem towards the result obtained.

Student RY did problem-solving toward the two problems that had been prepared. In the stage of exploring and understanding the problems, RY was able to figure out, investigate, and decide what was known and questioned in the problem, although RY did not write in great detail on the answer sheet. Based on the interview result, RY understood the problems based on the information that the perimeter of the rectangle was 100, and the aspect questioned was the maximum area of the cardboard. However, he did not write the formation on the answer sheet. Afterward, RY represented and formulated what was known in the problems by writing $k = 100$ and the formula of the rectangle perimeter $k = 2\,(p + l)$.
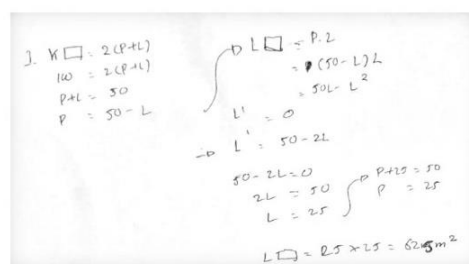

Figure 2. Answer Sheet of Average Mathematical Ability Student

Based on the interview with RY, k was the perimeter of a rectangle, p was the length, and l was the width of the cardboard. Next, in the planning and the solving stage, student RY used the previous derivative concept. RY substituted one of the variables in the perimeter equation into the equation area. Based on the interview result, student RY used the derivative concept as it was the fastest way to solve the problems, namely, getting the maximum value with $f'(x)$ 0. Afterward, in the stage of monitoring and evaluating, RY rechecked the questions in the problem after finding out the maximum area of the cardboard.

Student FK did problem-solving toward the two problems that had been prepared. In the exploration and understanding stage, FK figured out, investigated, and decided what was known and questioned. After that, FK wrote it on the answer sheet. Based on the interview result, FK understood the problems and the information that the perimeter of the rectangle was 100, and the aspect questioned was the maximum area of the cardboard. However, he did not write it in great detail on the answer sheet.
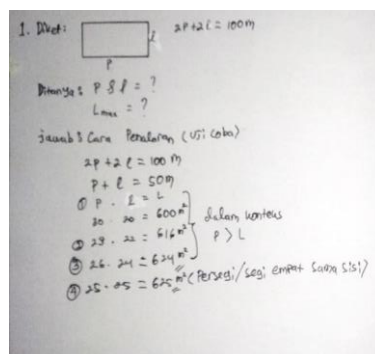

Figure 3. Answer Sheet of Low Mathematical Ability Student

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Next, FK represented and formulated what was known in the problems by drawing a rectangle and putting a label of p of cardboard's length. Next, he wrote $2p + 2l = 100$. Based on the result of the interview with FK, p was the length, and l was the width of the cardboard. Afterward, in the planning and implementing stage, FK used guessing and testing to identify the most suitable length and width number to fulfill the rectangle perimeter equation. Based on the interview result, FK used such a method as, in his opinion, it was the fastest way. He did not use the concept of mathematics since he did not know the proper concept of mathematics related to the questions. Next, in the monitoring and evaluation stage, FK rechecked the question in the problems based on what was known after finding out the maximum area of the cardboard.

CONCLUSIONS AND RECOMMENDATIONS

Based on the result of the research, it can be concluded that in solving the problems of algebra derivative formula, the students with high, average, and low mathematical abilities had their own way of solving the problems. In the stage of exploring and understanding the problems, the students with high and average mathematical abilities did not write the information known and questioned in the answer sheet in great detail. However, during the interview session, they were able to reiterate the mathematical problems and their way of thinking in detail. Whereas the students with low mathematical ability wrote everything in detail on the answer sheet. During the interview session, it was discovered that the students with low ability could not find or recollect the suitable formula to solve the problem other than the guessing method he used. Furthermore, in the stage of representing and formulating, the students with high, average, and low mathematical abilities used the same terms and formulas. In the planning and implementing stage, the three students used different methods. Despite utilizing various ways, the three students acquired the same answer and re-confirmed the result with what was known in the problem. All in all, the researcher suggested further research identify the strategies and processes of students' problem-solving in terms of their mathematical ability.

REFERENCES
Aydogdu, M., & Ayaz, M. F. (2008). The importance of problem solving in mathematics curriculum. *Physical Sciences*. 3(4), 538-545. Online Publication. Di Unduh dari https://dergipark.org.tr/en/pub/nwsaphysic/issue/20040/213558

Brookhart, S. M. (2010). *How to assess higher-order thinking skills in your classroom*. Alexandria: ASCD.

Dossey, J. (2017). Problem solving from a mathematical standpoint. In B. Csapó & J. Funke (Eds.), The nature of problem solving: Using research to inspire 21st century learning (pp. 59–72). Paris: OECD Publishing.

Dossey, J. (2017). Problem solving from a mathematical standpoint. In B. Csapó & J. Funke (Eds.), The nature of problem solving: Using research to inspire 21st century learning (pp. 59–72). Paris: OECD Publishing.

Hesse, F., Care, E., Buder, J., Sassenberg, K., & Griffin, P. (2015). A framework for teachable collaborative problem solving skills. *In Assessment and teaching of 21st century skills* (pp. 37-56). Dordrecht: Springer. DOI: 10.1007/978-94-017-9395-7

Kania, N. (2018). *Proses Pemecahan Masalah Matematis Calon Guru Sekolah Dasar*. Prosiding Seminar Nasional Pendidikan Matematika: Pendidikan Matematika Di Era Digital, Bandung. 337-344.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *Zdm Mathematics Education*, 45(2), 167-181. DOI: 10.1007/s11858-012-0467-1

Kementerian Pendidikan dan Kebudayaan Republik Indonesia. (2016a). *Salinan Lampiran Peraturan Menteri Pendidikan dan Kebudayaan Nomor 20, Tahun 2016, tentang Standar Kompetensi Lulusan Pendidikan Dasar dan Menengah*. Jakarta: Kemendikbud.

Kementerian Pendidikan dan Kebudayaan Republik Indonesia. (2016b). *Salinan Lampiran Peraturan Menteri Pendidikan dan Kebudayaan Nomor 21, Tahun 2016, tentang Standar Kompetensi Lulusan Pendidikan Dasar dan Menengah*. Jakarta: Kemendikbud

Lithner, J. (2015). *Learning mathematics by creative or imitative reasoning*. In 12th International Congress on Mathematical Education, Seoul. 487-506.

Liu, H. K. J. (2016). Correlation research on the application of e-learning to students' self-regulated learning ability, motivational beliefs, and academic performance. *EURASIA Journal of Mathematics, Science and Technology Education*, 12(4), 1091-1100.

NCTM (2000). *Standards for school mathematics*. Reston: National Council of Teachers of Mathematics.

Organisation for Economic Co-operation and Development. (2013). *PISA 2012 assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy*. Paris: OECD Publishing.

Shamaki, Timothy Ado (2015). Influence of Learning Environment on Students' Academic Achievement in Mathematics: A Case Study of Some Selected Secondary Schools in Yobe State – Nigeria. *Journal of Education and Practice*, 6 (34), 40-44.

Siswono, T. Y. E. (2018). *Pembelajaran Matematika Berbasis Pengajuan dan Pemecahan Masalah*. Bandung: PT Remaja Rosdakarya.

Szabo, A., & Andrews, P. (2018). Uncovering the relationship between mathematical ability and problem solving performance of swedish upper secondary school students. *Scandinavian Journal of Educational Research*, 62(4), 555-569.

Zubaidah, S. (2016). Keterampilan abad ke-21: Keterampilan yang diajarkan melalui pembelajaran. Makalah disajikan dalam Seminar Nasional Pendidikan dengan Tema "isu-isu strategis pembelajaran MIPA Abad 21, Sintang.