

**THE DEVELOPMENT OF LEARNING TRAJECTORY  
BASED ON SCIENTIFIC APPROACHES "PISA"  
ORIENTED PROBLEMS TO IMPROVE THE  
MATHEMATICAL REASONING ABILITIES OF EIGHT  
GRADE STUDENT OF JUNIOR HIGH SCHOOL**

**Muda Apriyanti<sup>1</sup>, Gede Suweken<sup>2</sup>, I Nengah Suparta<sup>3</sup>**

<sup>1,2,3</sup>Universitas Pendidikan Ganesha

[mudaapriyanti@gmail.com](mailto:mudaapriyanti@gmail.com), [gdsuweken5@gmail.com](mailto:gdsuweken5@gmail.com), [nengah.suparta@undiksha.ac.id](mailto:nengah.suparta@undiksha.ac.id)

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**ABSTRAK**

Penelitian ini bertujuan untuk memperoleh lintasan belajar materi sistem persamaan linear dua variabel yang dihasilkan dari lintasan belajar hipotetis yang dirancang melalui riset desain yang terdiri dari tiga tahap, yaitu persiapan eksperimen, perancangan eksperimen, dan analisis retrospektif. Lintasan belajar diperoleh dengan merevisi lintasan belajar hipotetis yang didapat dari uji coba dan diskusi dengan guru mata pelajaran. Penelitian ini difokuskan untuk meningkatkan kemampuan penalaran matematis siswa dalam menangani masalah matematika. Soal-soal setara PISA digunakan dalam penelitian ini. Kegiatan pembelajaran disusun berdasarkan pendekatan ilmiah (Mengamati, Menanya, Mengumpulkan Informasi, Mengolah Informasi, Mengomunikasikan). Pelaksanaan pembelajaran dilakukan di kelas delapan SMP PGRI 1 Denpasar tahun ajaran 2018/2019. Data terkait aktivitas dan strategi yang digunakan siswa dalam menyelesaikan masalah yang diberikan selama pembelajaran berlangsung, dikumpulkan melalui jawaban tertulis siswa di lembar kerja, hasil pasca-tes, wawancara, dan observasi. Data yang terkumpul dianalisis secara deskriptif. Lintasan belajar yang dihasilkan meliputi: (1) Membuat model matematika dari persamaan linear dua variabel dan sistem persamaan linear dua variabel dari masalah sehari-hari, (2) Mendefinisikan persamaan linear dua variabel dan sistem persamaan linear dua variabel, (3) Menyelesaikan masalah yang berkaitan dengan sistem persamaan linear dua variabel dengan berbagai metode.

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**Kata Kunci:**

Lintasan Belajar, Pendekatan Ilmiah, Sistem Persamaan Linear Dua Variabel.

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## ABSTRACT

*This study is aimed at obtaining the learning trajectory of system of two variables linear equations generated from hypothetical learning trajectory which are designed through design research consists of three steps, those are preparing for the experiment, design experimental, and retrospective analysis. Learning trajectory is obtained by revising hypothetical learning trajectory obtained from trials and discussions with teachers of the subject. The research is focused on improving students mathematical reasoning abilities for handling mathematical problems. PISA like problems were used in this research. Learning activities are arranged based on scientific approaches (Observing, Asking, Collecting Information, Processing Information, Communicating). Learning implementation was carried out in eight grade class of PGRI 1 Junior High School Denpasar 2018/2019. Data related to the activities and strategies used by students in solving problems given during learning takes place, was collected through students' written answers on the worksheets, post-test results, interviews and observations. The collected data were analyzed descriptively. The learning trajectories of: (1) Creating mathematical models of two-variable linear equations and system of two-variable linear equation from everyday problems, (2) Defining two-variable linear equations and systems of two-variable linear equations, (3) Solve problems related to the system of two-variable linear equations with various methods.*

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## Keywords:

Learning Trajectory, scientific approach, system of two-variable linear equations

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## INTRODUCTION

Many weaknesses of Indonesian students' mathematical abilities were revealed in the results of the PISA study (Wardhani, Sri dan Rumiati. 2011). Based on the PISA results Indonesia was ranked 69th out of 76 countries (OECD. 2016. PISA 2015). The weakness of students is that they couldn't able to develop reasoned abilities.

Therefore, in improving students' mathematical reasoning skills, a question is needed that requires a fairly high reasoning ability. One of them uses the PISA literatur. The focus on PISA is to emphasize the skills and competencies of students obtained from school and can be used in daily life and in various situations (OECD 2016). But in reality, most students were still lacking in solving problems related to ability system of two-variable linear equations. Based on the results of interviews with a mathematics teacher at SMP PGRI 1 Denpasar it is found that students were not able to change the story questions into the form of mathematical models, That's why the students couldn't able to solve problems related with two-variable linear equations. The researcher conducted the first test for analyzing the students problems in solving mathematical problems. In the first test on the material system of linear equations two variables which were given by researchers to students in SMP PGRI 1 Denpasar, it was found if there are many students were still having difficulty in solving mathematical problems.

Because, there is no appropriate learning trajectories which is ready to be used then this research is mainly intended to develop a learningtrajectories of two variable system of linear equation. The learning trajectories is developed to investigate. How do teachers start the lesson? how does the teacher know about the steps the teacher will take? How can the teacher achieve this goal? etc. How it was learning will provides guidance for the teacher to determine and formulate learning objectives to be achieved. Then the teacher can make decisions about the steps of the strategy that will be used to realize these goals. The teacher must also think about the prerequisite knowledge, the thinking strategies that will be used by the child, the level of thinking that the students show and how various activities can help students develop the thinking needed to

achieve that goal. That is the importance of the learning trajectory for the teacher. In this case, researchers consider it important to design a learning that could support students' mathematical reasoning abilities in solving the problem of two-variable linear equation systems. Because of this, researchers plan to conduct a study that aims to develop a learning trajectory which could support students' mathematical reasoning abilities in solving the problem of two-variable linear equation systems. This learning trajectory has a very large contribution related to how students learn and how students think.

We conceptualize learning trajectories as a description of thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional mental processes or hypothesized actions to move through a developmental progression of levels of thinking, created with the intent of specific supporting children's achievements in that mathematical domain (Szilágyi et al., 2013). The way of learning that will be compiled, it would be nice to pay attention to the applicable curriculum in Indonesia. The scientific framework of learning carried out by the 2013 Curriculum is a scientific approach where the learning process can be matched with a scientific process. The learning process must touch three domains, namely, attitude (attitude), skills (skills), and knowledge (knowledge). The learning process that refers to the scientific approach includes five steps, namely: observing, asking questions, gathering information / experiments, associating / processing information, and communicating.

## **RESEARCH METHODS**

The purpose of this study was to develop and obtain the learning trajectory based on scientific approaches and "PISA" oriented problems to improve the mathematical reasoning abilities of eight grade student of junior high school, and find out how the design would work in the classroom. Therefore design research was chosen as a method in this study. Three stages in design research were applied in this study, which included preparing for the experiment, implementation and retrospective analysis (Bakker, A., & Van Eerde, H.A. 2015).

At the preparing for the experiment, a Hypothetical Learning Trajectory (HLT) is designed which will become a learning reference. HLT consists of three components: learning objectives that will determine the direction of learning activities, learning activities and hypothetical learning process hypotheses that contain predictions of how students think when given the intended learning activities (Simon dan Ron Tzur. 2004). HLT then tested the validity of the content through an expert testing process. After being declared valid, HLT is then tested to the class at the stage of field implementation.

The field implementation was held in three cycles. The first cycle was conducted involving 12 students of class VIII H SMP PGRI 1 Denpasar academic year 2018/2019. The purpose of this first cycle is to find out the implementation of learning designed. For this reason, in this first cycle the researcher acts directly as a teacher. The findings in Cycle I were used to refine the design made to then be applied to Cycle II. Cycle II was implemented in VIII J SMP PGRI 1 Denpasar by involving all students and class teachers who taught in the class. The findings in the second cycle were used to refine the design made to then be applied to cycle III. The implementation of learning in the third cycle is the same as the second cycle only applied to different classes, namely class VIII I SMP PGRI 1 Denpasar.

Data is collected in various ways in accordance with the principles of subject and technique triangulation. Data is collected through observation of student activities, analysis of student work, and interviews. Observations and interviews used were unstructured observations and open interviews. In the retrospective analysis phase, the data obtained from this study were analyzed qualitatively. To improve the validity of this study, several strategies were carried out, namely: (1) using triangulation data, (2) testing data compatibility between HLT and actual learning activities occurring and (3) looking for possible counterexamples during the assessment stage to test predictions made [3]. Meanwhile, to ensure reliability in data analysis the methods taken are follows: (1) documenting all learning activities, (2) explaining how learning is carried out and (3) explaining how the conclusions are carried out.

## RESULTS AND DISCUSSION

In this research, was designed for study which was consisting of 8 meetings every each cycle. The reseach was conducted as long as 3 cycles. The material focused on this research is a two-variable linear equation system. In system learning, two-variable linear equations are arranged PISA-oriented learning to support students' mathematical reasoning abilities in dealing with mathematical problems. In the following discussion we will explain about the activities carried out and some important findings obtained during learning.

### 1. Learning I

Generally, the purpose of 1st learning was to teach students about the concept of two-variable linear equations, starting from solving problems related to linear equations of two variables, modeling mathematics and defining two-variable linear equations. In general, in this 1st learning is difficulties of the students experienced when they were asked to complete tables and change problems into mathematical forms. With the provision of scaffolding carried out by the teacher in the form of questions or little hints, students are able to complete the empty tables that have been provided and change the problem into mathematical form. The following are examples of student answers

Penyelesaian:

Uang Rp 100.000	1	2	3	4	5	6	7	8	9
Uang Rp 50.000	18	16	14	12	10	8	6	4	2

**Figure 4.1** Student answers

Misalkan	$x$ : Banyak uang di ATM BNI
	$y$ : Banyak uang di ATM BCA
Model Matematika	$100.000x + 50.000y = 1.000.000 \rightarrow 2x + y = 20$

**Figure 4.2** Student answers

### 2. Learning II

The principle of the second learning was the same as learning I, in this second learning students look for mathematical models of systems of two variable linear equations, define a system of linear variables two variables, and distinguish two-variable linear equations with systems of two-variable linear equations. In this first activity, students on average have been able

to properly variable variables and change problems into mathematical forms. The following are examples of student answers.

Penyelesaian: Keterangan: Baju = x    CELANA = y				
Besaran yang Ditanya	Baju	CELANA	PESAMAANNYA	
Seorang Aia	2 2x	3 3y	$2x + 3y = 175.000$	
Seorang Ibu	3 3x	2 2y	$3x + 2y = 200.000$	

### Figure 4.3 Student answers

### 3. Learning III

This learning focused on solving the problem of the Two Variable Linear Equation System. Students founded some methods which could be used to solve the problem of Two Variable Linear Equation Systems. In general students have been able to do this activity well. In this learning students were asked to find several methods to solve the system problems of two-variable linear equations. the problems contained in the student's worksheet were made oriented "PISA" where the problems were connected to each other. For making students' understanding could be comprehensive and could develop students' mathematical reasoning abilities. On the third student's worksheet, there were several answers from the students were found by researchers as below.

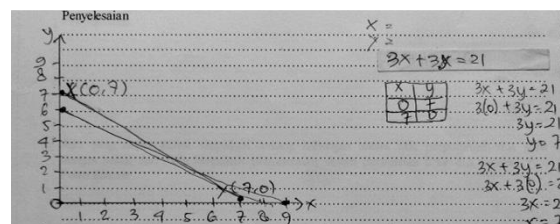
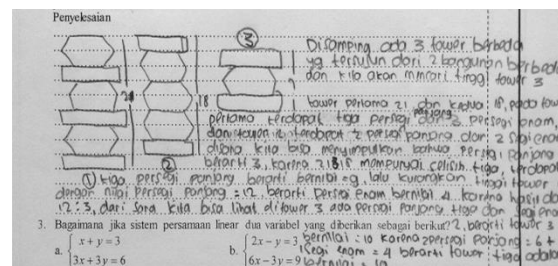
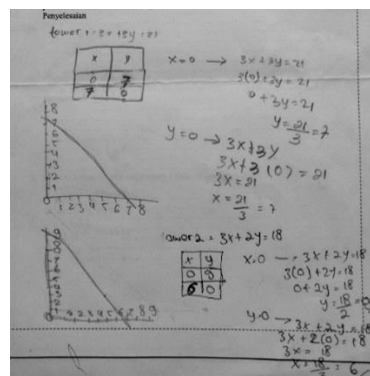


Figure 4.4 Example of student answers



**Figure 4.5** Example of student answers



**Figure 4.6** Example of student answers



Seen from the students' answers, there are several obstacles found (1) students do not use a ruler or relevant measuring instruments in drawing graphics so students do not find the cutoff point, (2) students still use the tries system in solving problems given, (3 ) students do not draw graphics in a coordinate system so that this will result in students never finding the intersection of the two lines. Until finally students can find the right answer in finding a solution to the problem given. The following are examples of student answers.

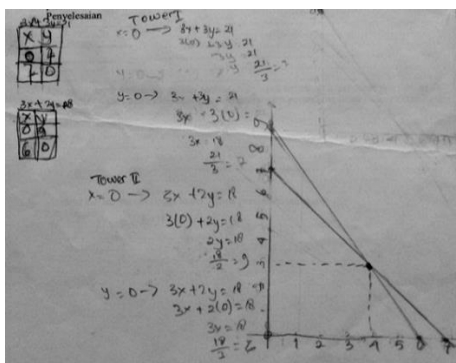


Figure 4.7 Example of student answers

#### 4. Learning IV

Learning IV focused on the system of two variable linear equations with various methods. In this fourth lesson, the problems given were connected with previous learning. Where the problem was nearly as same as the previous problem, only large numbers were used. With prior knowledge, students used graphical methods to solve this problem. This showed that students submit allegations in solving problems given. However, obstacles arise when students will determine the second intersection of the line. Because the numbers were used quite large, so the students have difficulty determining the intersection. The following are examples of answers from students.

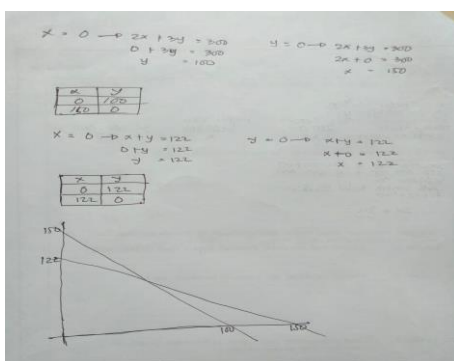


Figure 4.8 Example of student answers

And then the students were asked for looking other methods in order to find the right solution. With the help of literature and the provision of scaffolding from

researchers, students were able to find substitution methods even though it takes a long time. The following are examples of student answers.

Penyelesaian:  
 $2x + 3y = 300$   $x + y = 122$   
 $x = 122 - y$   
Substitusikan  $x = 122 - y$  ke metode 2  
 $2x + 3y = 300$   
 $2(122 - y) + 3y = 300$   
 $= 244 - 2y + 3y = 300$   
 $= 244 + y = 300$   
 $+y = 300 - 244$   
 $+y = 56$   $y = 56$

**Figure 4.9** Example of student answers

## 5. Learning V

Learning V focused on solving systems of two variable linear equations with various methods. Just like previous learning, the problems given were connected with previous learning. In finding solutions to the problems given, students took advantage of methods or methods which were taught previously. But students were not able to find solutions to these problems by means of graphics or substitutions. So it's need a method or other method to find a solution with the problem given. The following are examples of student answers.

② Jadi harga satu snack dan susu di indomaret adalah  
1. Susu = 4.650  
2. Snack = 1.200  
Toko Alfamart  $4x + 3y = 19000$   $1.3 = 12x + 9y = 57.000$   
 $3x + 4y = 20650$   $1.4 = 12x + 16y = 90.600$   
 $-7y = -33.600$   
 $y = 4800$   
Jadi harga satu snack dan susu di alfamart adalah  
1. Susu = 9800  
2. Snack = 1150

**Figure 4.10** Example of student answers

## 6. Learning VI

Learning VI focused on solving systems of two-variable linear equations with various methods. In this case, students were directed to combine 2 methods there were substitution methods and elimination methods. In this case mostly the students would ask which method should be done first. Because of that the researcher would ask the students for trying both of them firstly. The following are examples of student answers.

Penyelesaian:

Eliminasi:

$$\begin{array}{r} V_0 + 5a = 25 \quad 1 \\ V_0 + 12a = 46 \quad 2 \\ \hline -7a = 21 \quad 1 \\ a = 21 \div -7 \\ a = -3 \end{array}$$

Substitusi:

$$\begin{array}{r} V_0 + 5a = 25 \\ V_0 + 5(-3) = 25 \\ V_0 + 15 = 25 \\ V_0 = 25 - 15 \\ V_0 = 10 \end{array}$$

Figure 4.11 Example of student answers

## 7. Learning VII

Learning VII focused on solving fractional two-variable linear equation systems using various methods. In general students have been able to do this activity well. Students were able to model the problems given. students find obstacles when the equation obtained was a fraction. However, researchers and teachers provide guidance to students so that students were able to solve problems well. Although previously students were still hesitant to use which method was used in solving the problem. The following are examples of student answers.

Penyelesaian:

Eliminasi:

$$\begin{array}{r} 2x + 3y = 52.000 \quad 1 \\ 4x + 5y = 60.000 \quad 2 \\ \hline -2x + 2y = 118.000 \quad -1 \\ 2x + 3y = 52.000 \quad 1 \\ \hline -16y = 118.000 \\ y = 118.000 \div -16 \\ y = -7.375 \end{array}$$

Substitusi:

$$\begin{array}{r} 2x + 3y = 52.000 \\ 2x + 3(-7.375) = 52.000 \\ 2x - 22.125 = 52.000 \\ 2x = 52.000 + 22.125 \\ 2x = 74.125 \\ x = 74.125 \div 2 \\ x = 37.0625 \end{array}$$

Figure 4.12 Example of student answers

## 8. Learning VIII

Learning VIII focused on completing a system of two variable nonlinear equations with various methods. With prior knowledge, various methods could be used by students to solve these problems such as graph methods, substitution methods, elimination methods, and joint methods in solving these problems. But students found an obstacle when the equation obtained was a form of quadratic equation. Students feel confused whether they could immediately search for the x and variable variables y. However, researchers and teachers provide guidance to students so that students were able to solve problems well.

## CONCLUSIONS AND SUGGESTIONS

Based on the results of the research and discussion described earlier, it was obtained that from the three cycles carried out, it was founded, generally the concluded that HLT realized through applied learning could develop students' mathematical reasoning abilities in dealing with the problems of two-variable linear equation systems. Students who was firstly difficult to understand mathematical problems related to the system of two-variable linear equations become more able to understand because the learning which was oriented from improvement various possibilities from concept or problem. In the meantime, there were many improvement from students' mathematical reasoning.



Researcher founded if there were some students have problems in dealing with mathematical problems in two-variable linear equation systems were the lack of students' understanding of the system problems of two variable linear equations in the PISA context and the lack of mathematical reasoning abilities students have. There are some problems which could be resolved due to there are still some students who provide mathematical reasoning skills that are unefficient with tries system. Besides that, the very important thing need to be noticed is the aspect of connected of learning. It's meant, in compiling HLT to develop students' mathematical reasoning abilities in dealing with the problems of two-variable linear equation systems, it should be connected one to each other.

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